Electric Circuits for Physicists

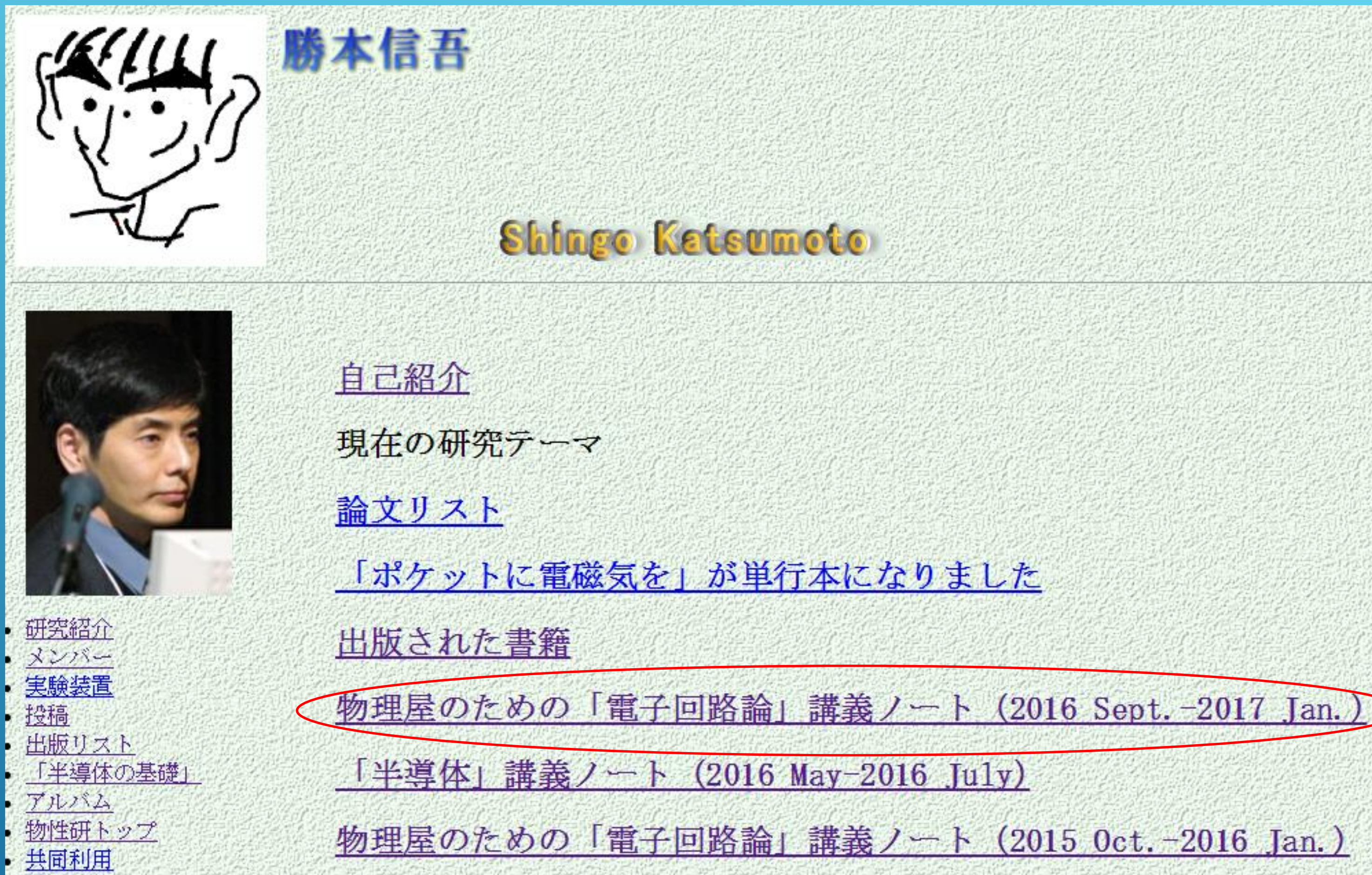
2016

Shingo Katsumoto

2016年度
電子回路論　第１回

東京大学
理学部・理学系研究科
物理学専攻
物性研究所
勝本信吾

ノート・資料等の置き場



2週に１回簡単な練習問題を出題　→　2週間以内に解答を提出

TAが採点してコメントをメール送付します

試験は期末レポート．練習問題と合わせて採点します

シラバス

1. 電磁場と電子回路
   1.1 この講義について
   1.2 電子回路とは
   1.3 ２端子素子
   1.4 回路図
   1.5 抵抗器
   1.6 キャパシタ
   1.7 インダクタ

2. 線形回路序論
   2.1 線形システムと電子回路
   2.2 電源
   2.3 回路網
   2.4 4端子(2端子対)回路
   2.5 端子対回路の諸定理
   2.6 双対性
   2.7 受動素子と能動素子

3. 伝達関数と周波数応答・過渡応答
   3.1 受動素子2端子回路の伝達関数
   3.2 ２端子受動素子回路
   3.3 受動素子回路の過渡応答

4. 増幅回路
   4.1 増幅回路と系の制御
   4.2 OPアンプ
   4.3 トランジスタ
   4.4 電場効果トランジスタ

5. 分布定数回路
   5.1 伝送路
   5.2 伝送路の伝播現象
   5.3 S行列(Sパラメタ)
   5.4 シュレディンガー方程式とLC伝送路

シラバス２

6. 信号，雑音，波形解析
- 6.1 ゆらぎ
- 6.2 増幅器の雑音
- 6.3 変調とアナログ信号伝送
- 6.4 離散化信号

7. ディジタル信号とディジタル回路
- 7.1 ディジタル信号序論
- 7.2 論理ゲート
- 7.3 論理ゲートの実装
- 7.4 論理演算の回路化と簡単化
- 7.5 A-D/D-A コンバータ
- 7.6 ディジタルフィルター
- 7.7 ハードウェア記述言語：HDL

Syllabus

1. Electromagnetic field and electric circuits
   1.1 About this lecture
   1.2 What is electric circuit?
   1.3 Two-terminal devices
   1.4 Circuit diagrams
   1.5 Resistors
   1.6 Capacitors
   1.7 Inductors
2. Introduction to linear circuits
   2.1 Linear systems and electric circuits
   2.2 Power sources
   2.3 Networks
   2.4 4-terminal (2-terminal pair) circuits
   2.5 Theorems in terminal pair circuits
   2.6 Duality
   2.7 Passive devices, active devices

Syllabus

3. Transfer function and transient response
   3.1 Transfer function of passive two-terminal pair circuits
   3.2 Two-terminal passive circuits
   3.3 Transient response of passive circuits
4. Amplifiers
   4.1 System control and amplifiers
   4.2 Operational amplifiers
   4.3 Transistors
   4.4 Field effect transistors
5. Distributed constant circuits
   5.1 Transmission lines
   5.2 Propagation through transmission lines
   5.3 S matrix (S parameters)
   5.4 Schrodinger equation and LC transmission circuit

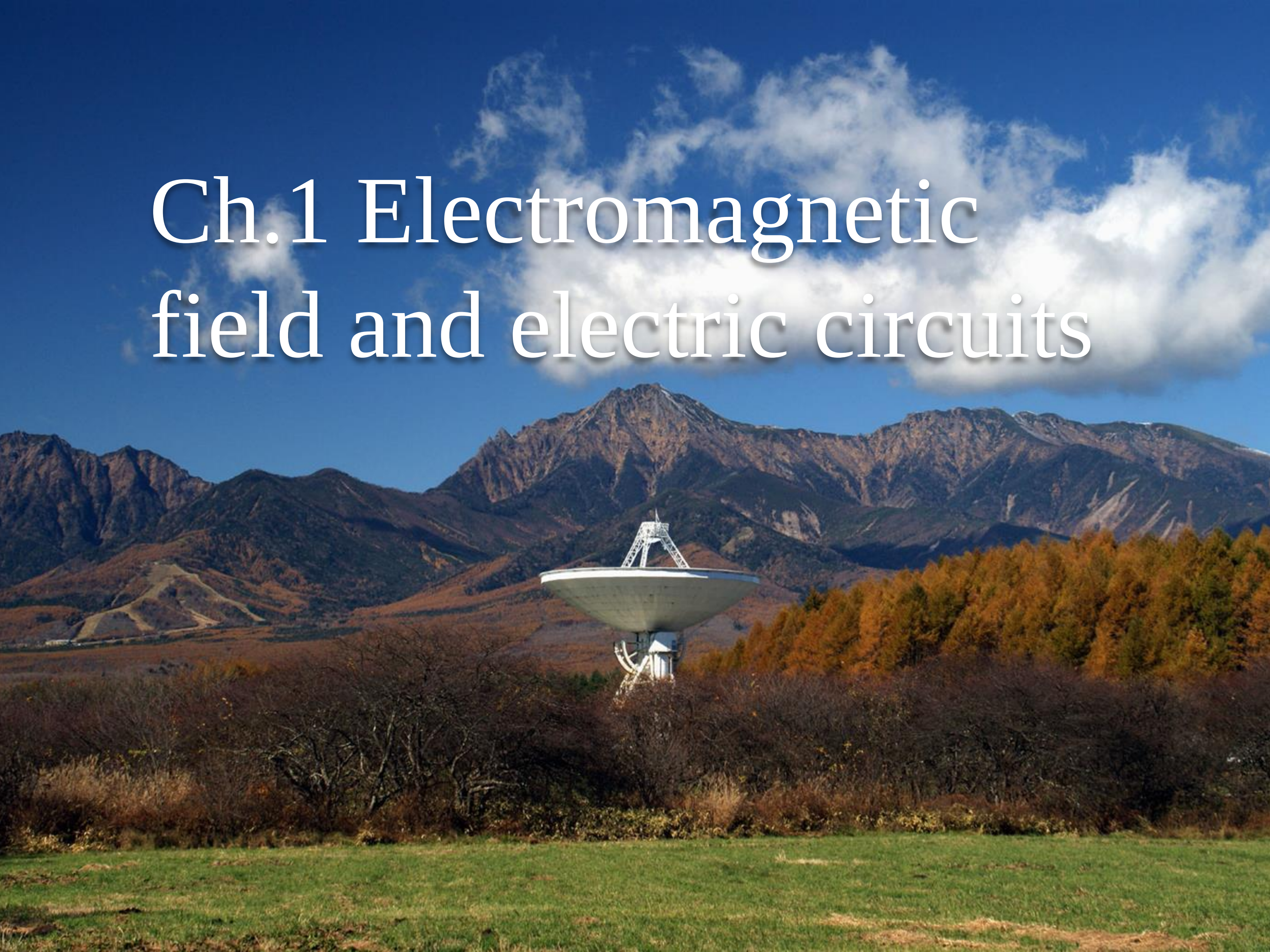
Syllabus

6. Signal, noise, waveform analysis
   6.1 Fluctuation
   6.2 Noise from amplifiers
   6.3 Modulation and analog signal transfer
   6.4 Discrete signal
7. Digital signal and digital circuits
   7.1 Introduction to digital signal
   7.2 Logic gates
   7.3 Logic circuits implementation
   7.4 Circuit implementation and simplification of logic operation
   7.5 AD/DA converters
   7.6 Digital filters
   7.7 Language to describe hardware: HDL

Outline Today

1. Electromagnetic field and electric circuits
   1.1 About this lecture
   1.2 What is electric circuit?
   1.3 Circuit diagrams
   1.4Two-terminal devices
   1.5 Resistors
   1.6 Capacitors
   1.7 Inductors
2. Introduction to linear circuits
   2.1 Linear systems and electric circuits

Ch.1 Electromagnetic field and electric circuits



For what this lecture is?

Experimentalists:

Knowledges on electric circuit are indispensable.

Purposes:

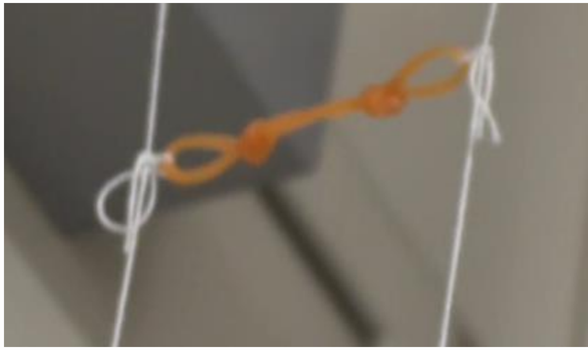
Understand how circuits work.

Design circuits along research plans

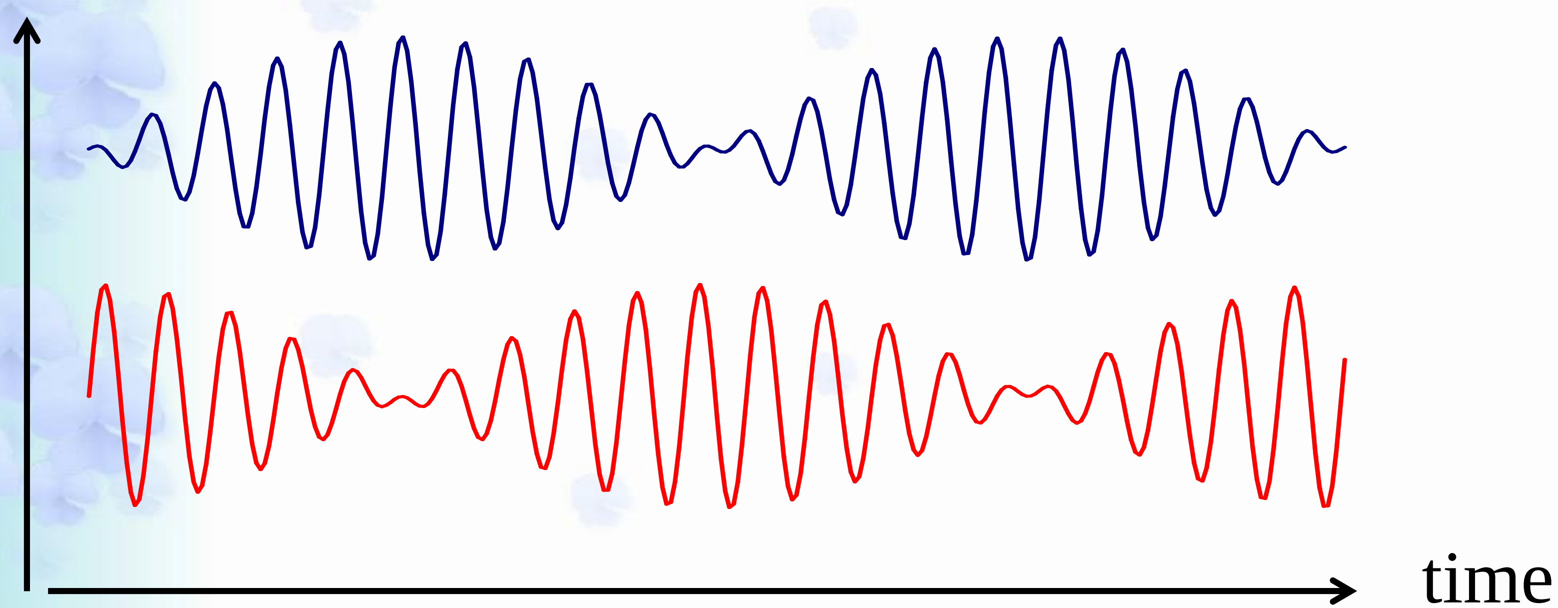
General physicists:

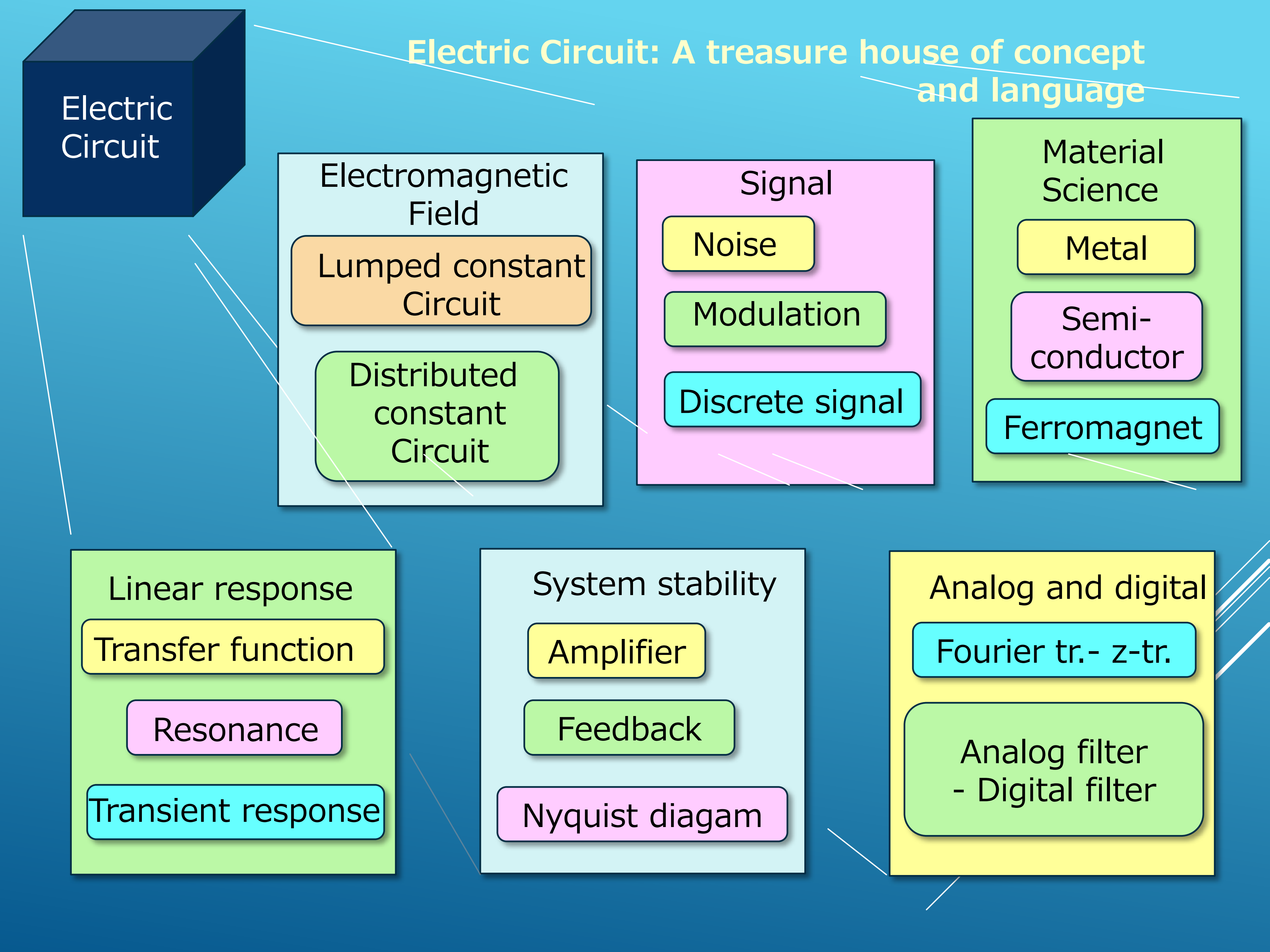
Meta physics

Coupled pendulum and neutrino oscillation



Pendulum oscillation





Electric Circuit: A treasure house of concept and language
Electric Circuit
Electromagnetic Field
Lumped constant Circuit
Distributed constant Circuit
Signal
Noise
Modulation
Discrete signal
Material Science
Metal
Semi-conductor
Ferromagnet
Linear response
Transfer function
Resonance
Transient response
System stability
Amplifier
Feedback
Nyquist diagam
Analog and digital
Fourier tr.- z-tr.
Analog filter - Digital filter

1.2 What is electric circuits?: Thunderbolt struck a plane!

Your answer?

1.

Fell down and crashed

2.

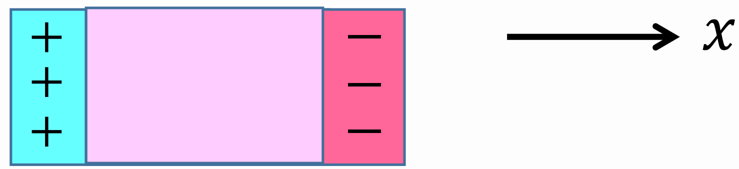
Damaged

3.

Nothing happened

Plasma frequency

$$m\frac{d^2x}{dt^2} = -eE$$



$$E = E_0 e^{-i\omega t}, \quad x = x_0 e^{-i\omega t} \rightarrow m\omega^2 x_0 = eE_0$$

Electric polarization: $P = -nex_0 = -\dfrac{ne^2 E_0}{m\omega^2}$

$$\epsilon(\omega) = \frac{D(\epsilon)}{\epsilon_0 E(\omega)} = 1 + \frac{P(\omega)}{\epsilon_0 E(\omega)} = 1 - \frac{ne^2}{\epsilon_0 m\omega^2} = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}$$

$\omega_{\mathrm{p}}^2 \equiv \dfrac{ne^2}{\epsilon_0 m}$: Plasma frequency

Cu: $n = 8.5 \times 10^{22}$ /cc $\quad m^* = 1.3m_0$

$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = 2.3 \times 10^{15}$ Hz $\quad$ λ = 130nm Near ultra-violet

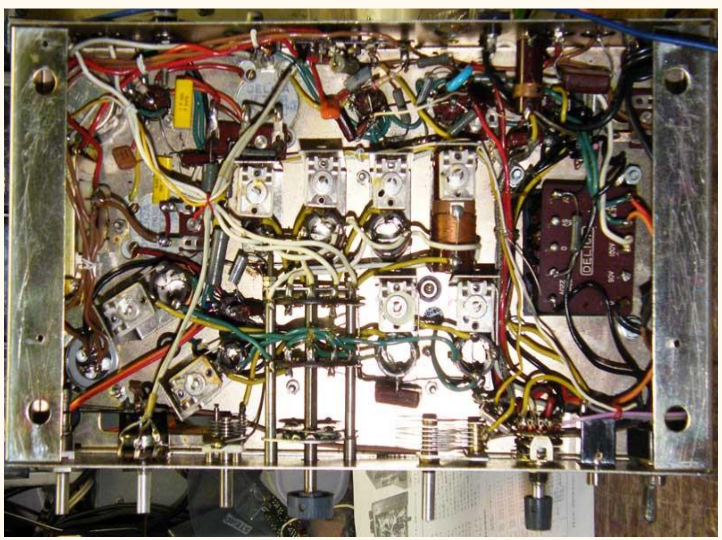
Metals are super-screening materials!

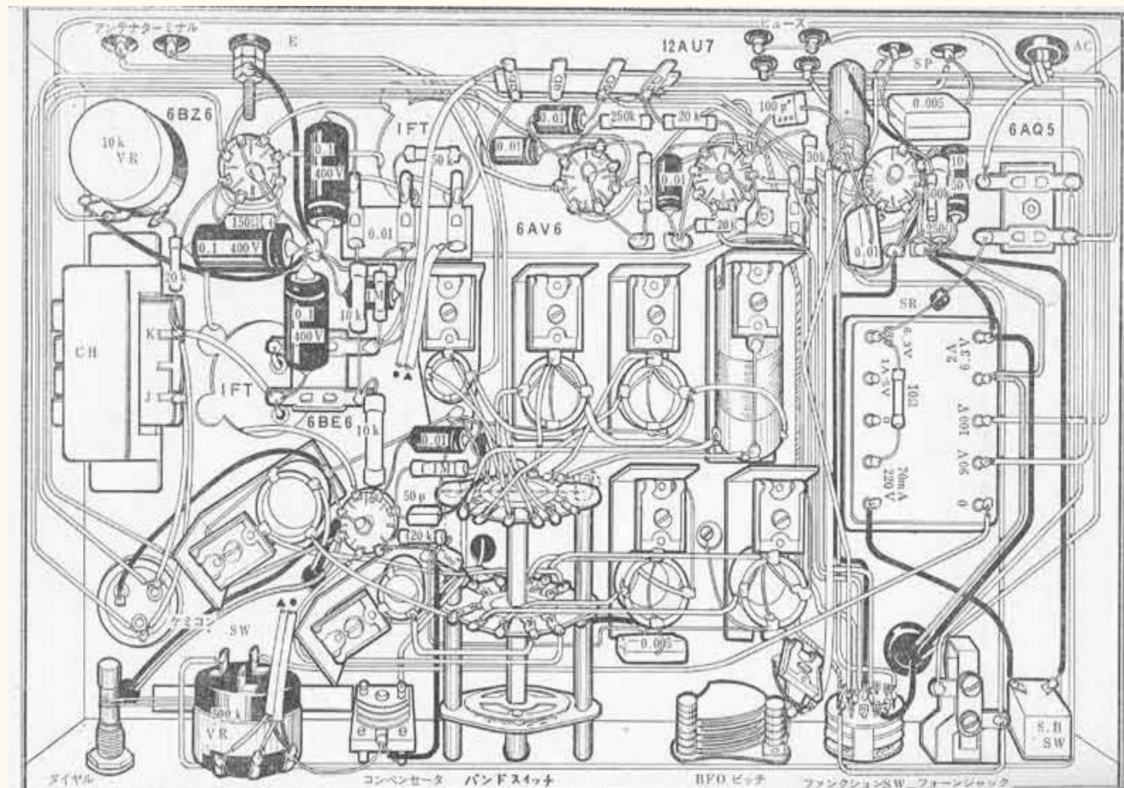
Equipotential lines

1.3 Various circuit diagrams

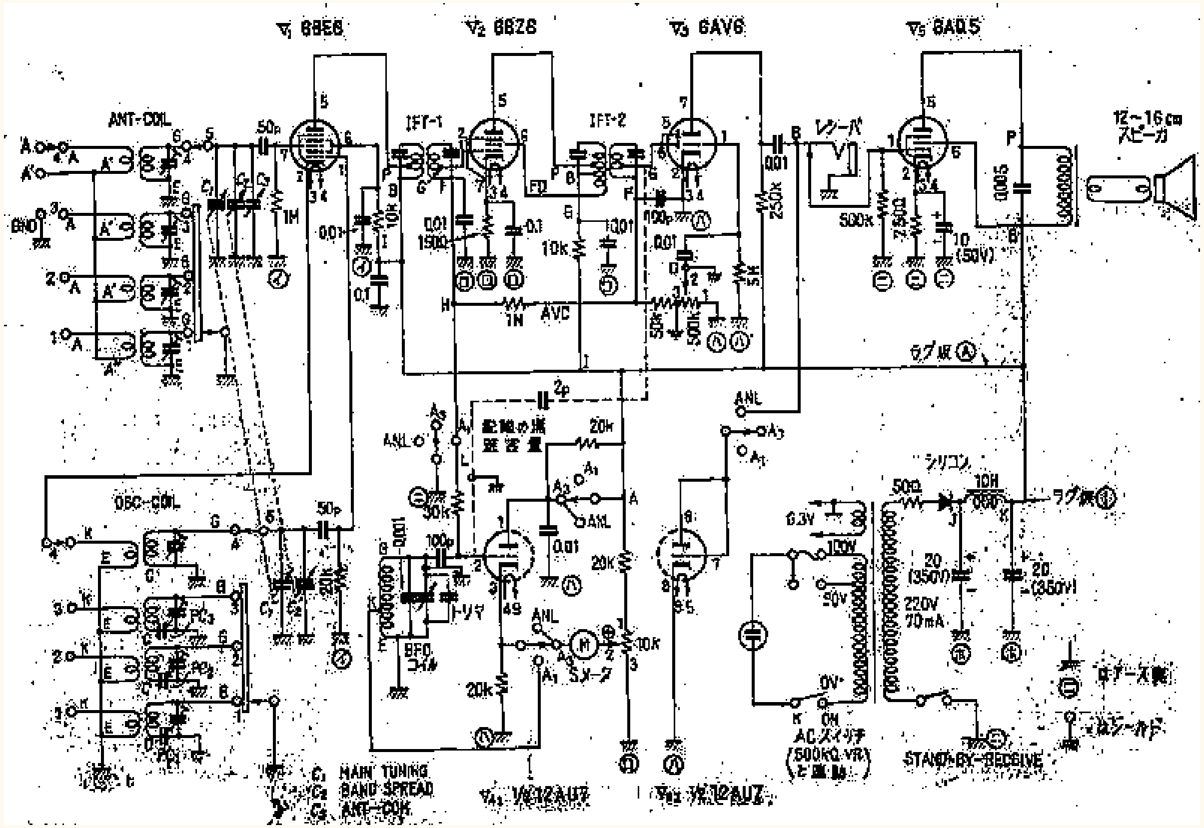
Picture diagram

三田無線研究所
DELICA DX-CS-7



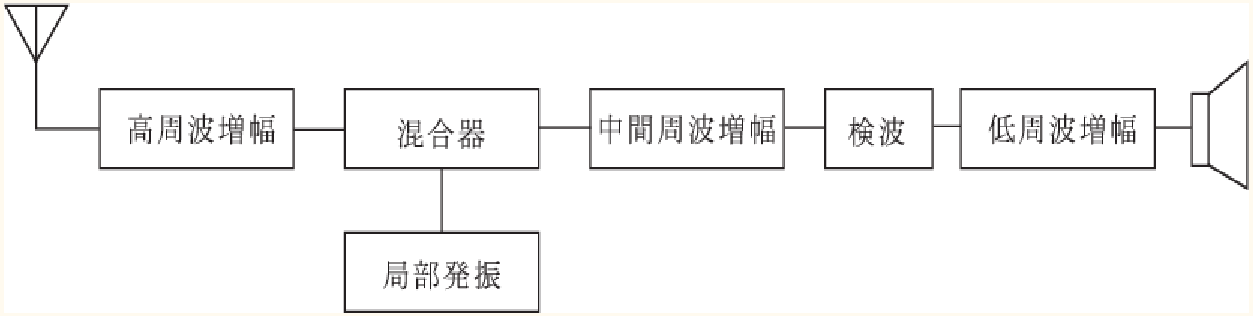


Various circuit diagrams

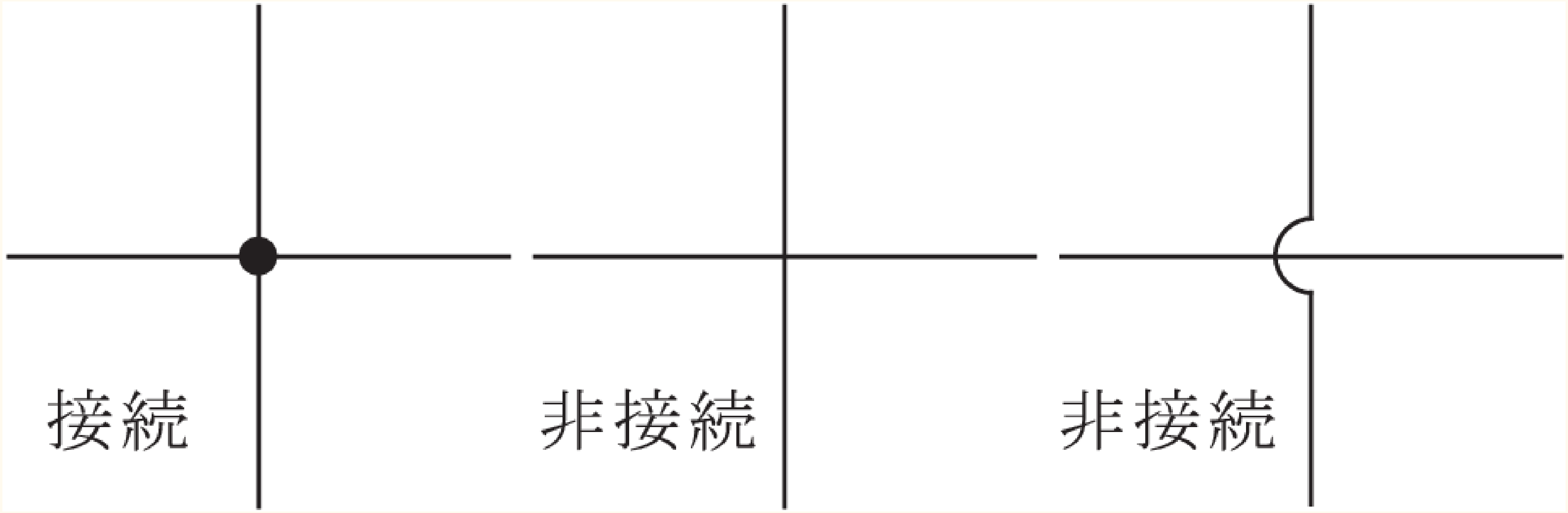


Parts + Wiring
= Wiring (Circuit) diagram

Block diagram



Wirings in electric circuits

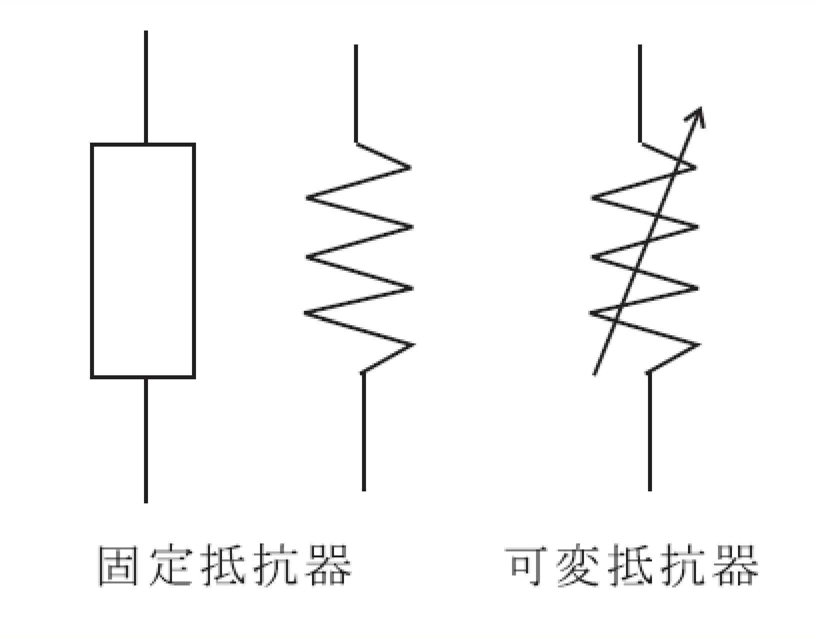


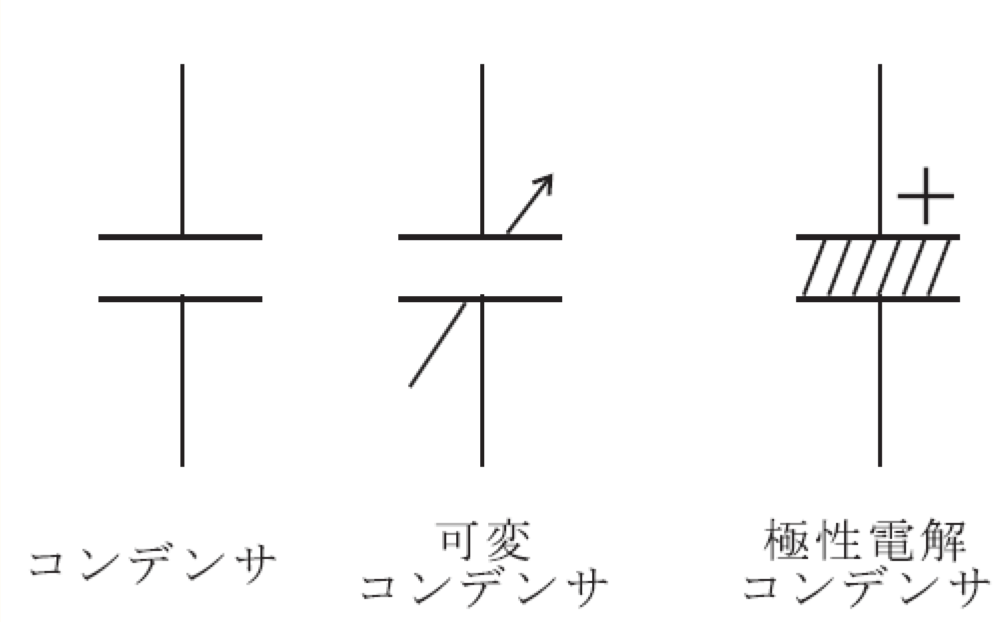
Connected Not connected Not connected

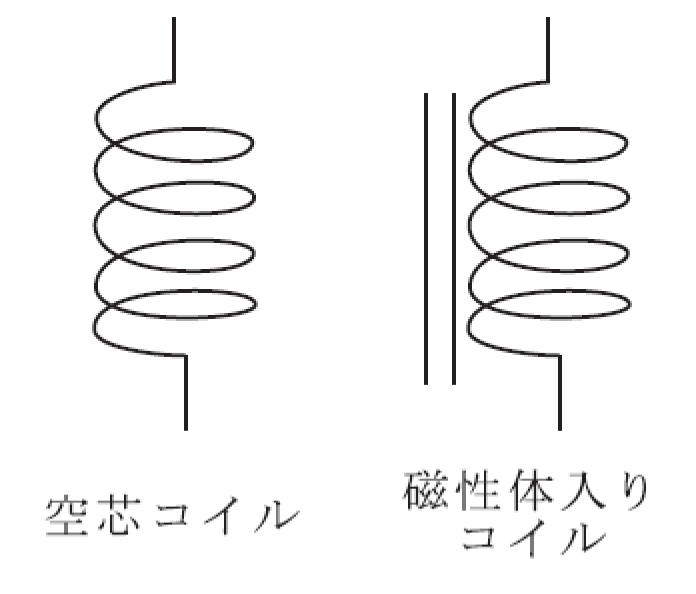
Violate electromagnetism theory

Concept of local electromagnetic field
= Lumped constant circuits (集中定数回路)

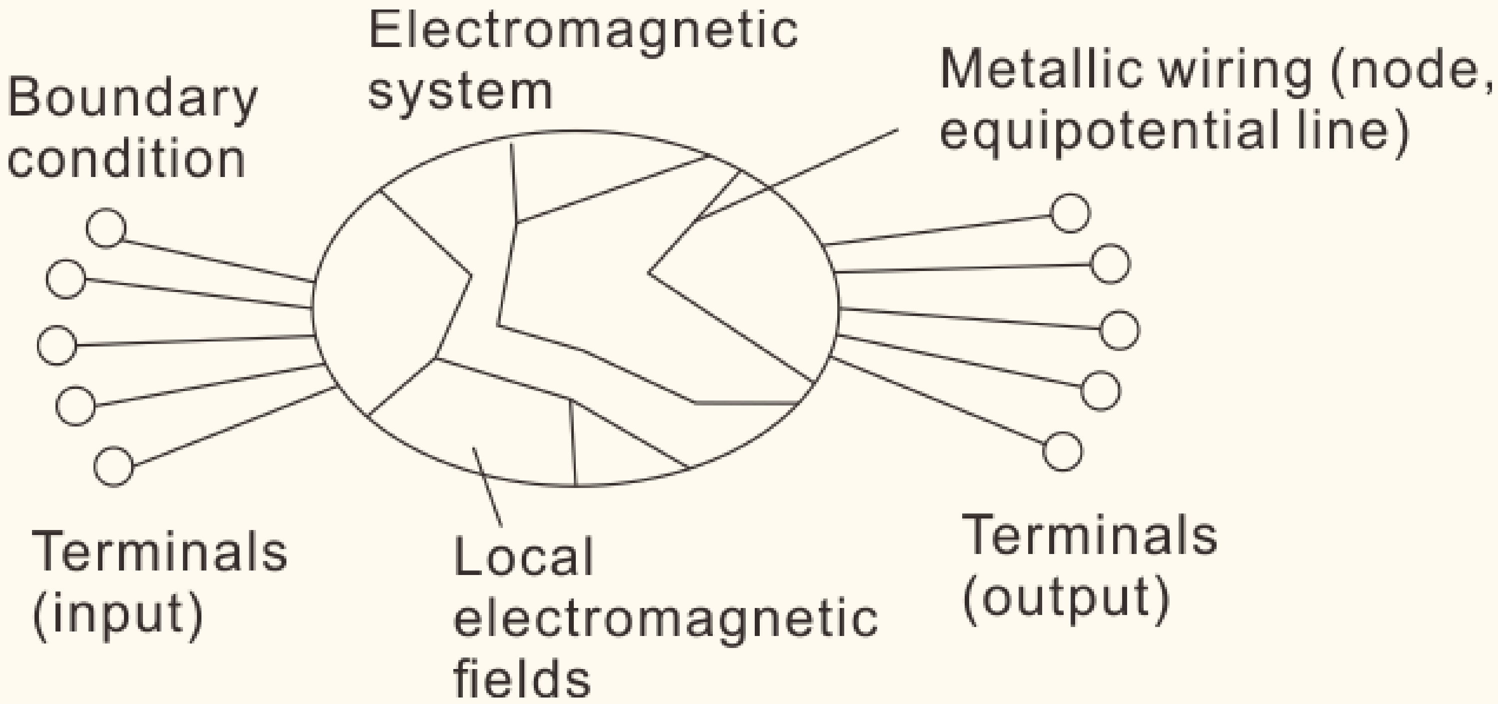
Circuit symbols for two-terminal devices



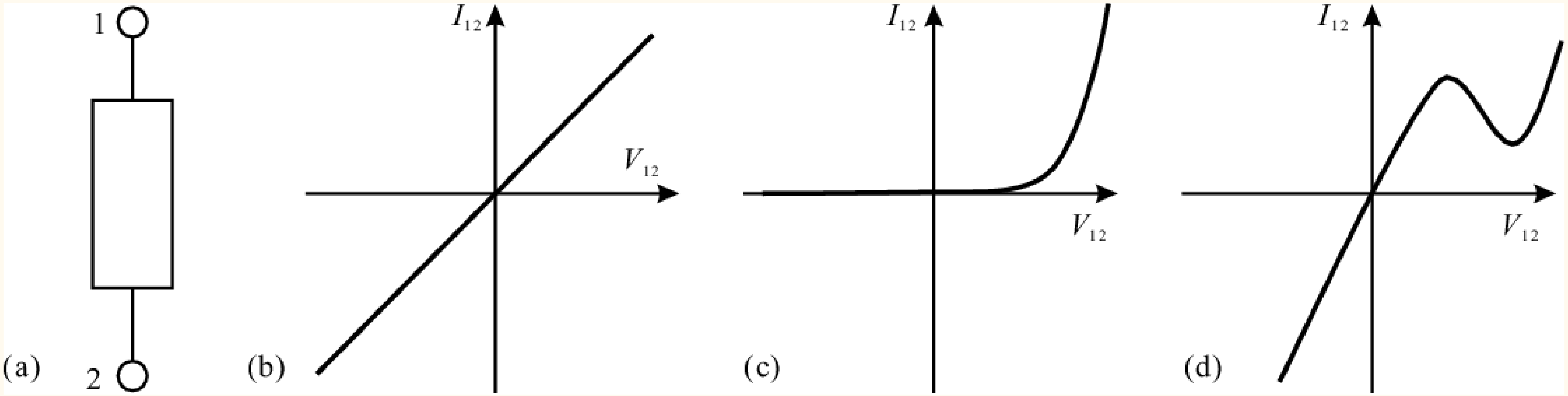




Basic concepts in electric circuits



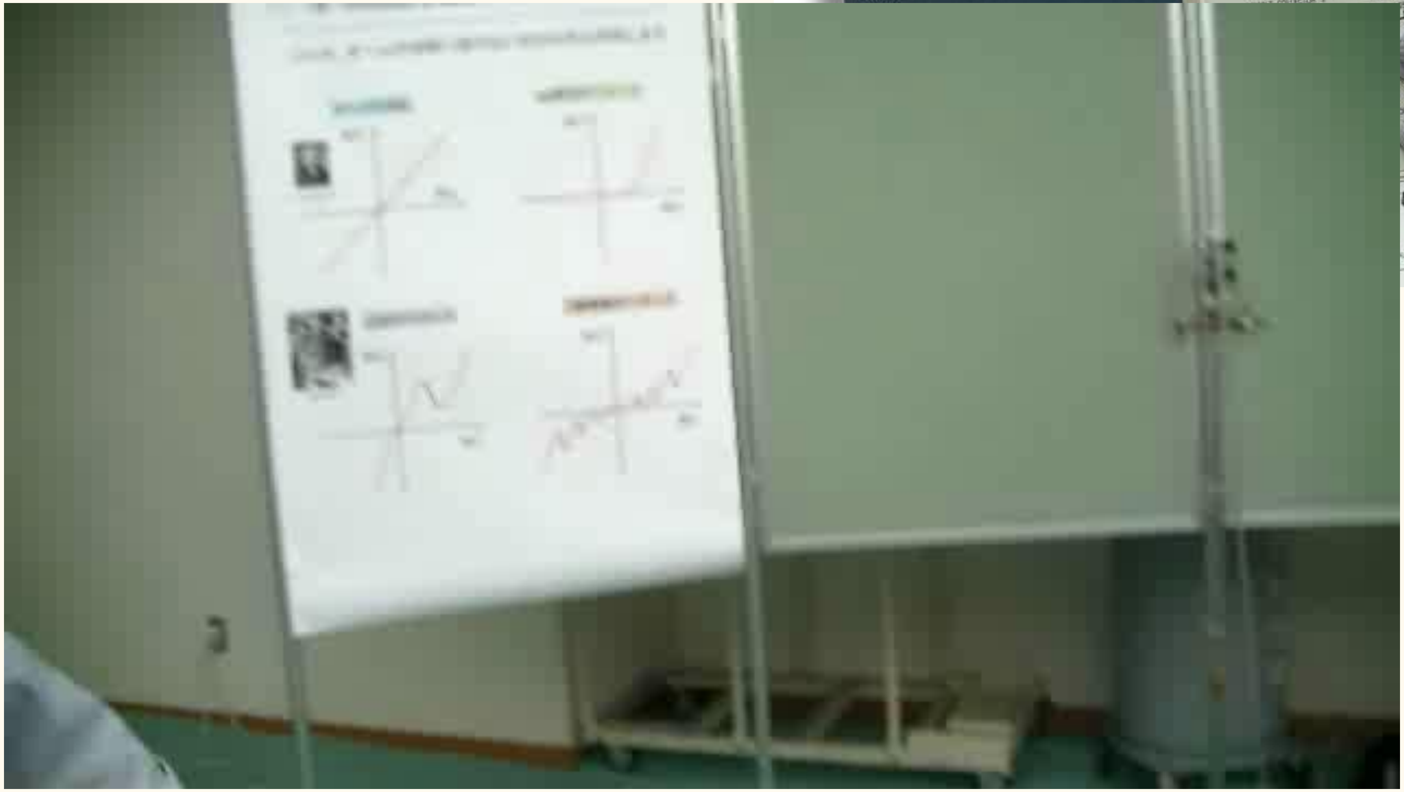
What is current-voltage characteristics?



Resistor

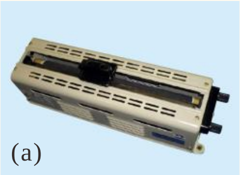
What is current-voltage characteristics?

Curve tracer



Variable resistors

Slide


(a)


(b)

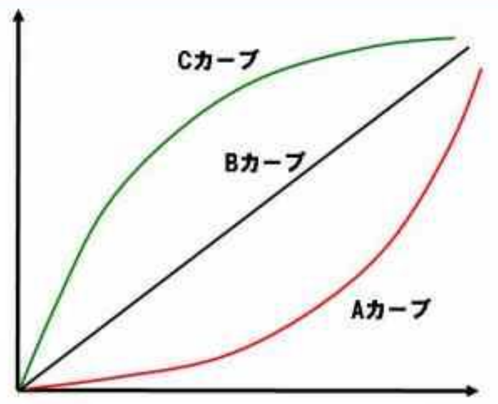

(c)


(d)

Carbon helical

Helical potentiometer

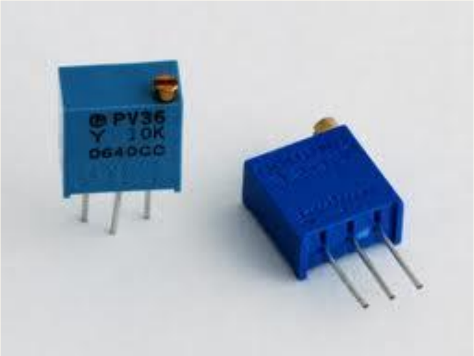
Rotary switch potentiometer



Resistance

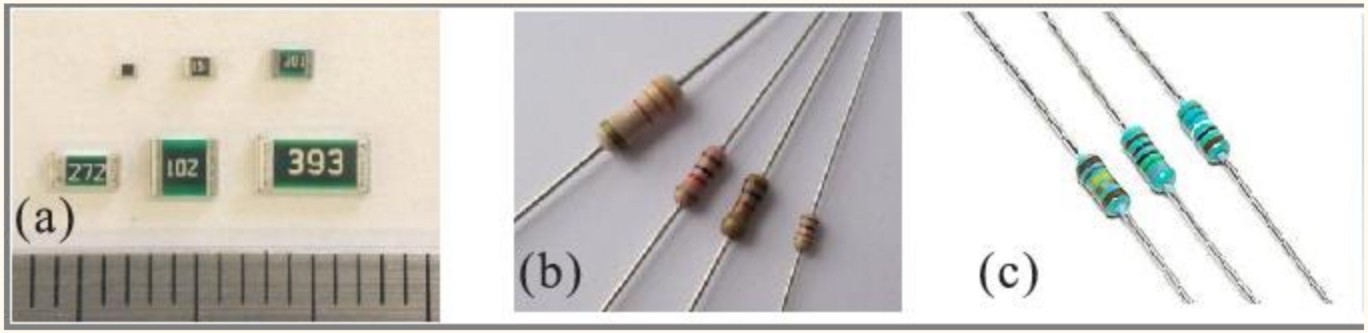
Rotation angle

trimmer



Cermet trimmer

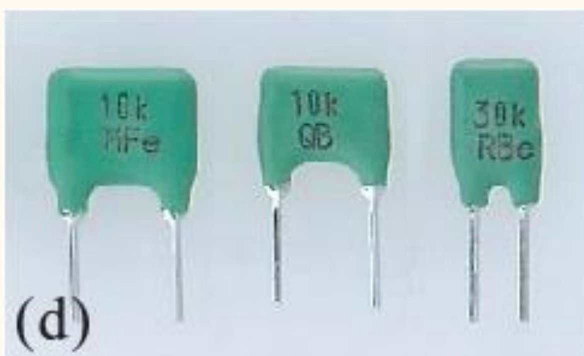
Fixed resistors



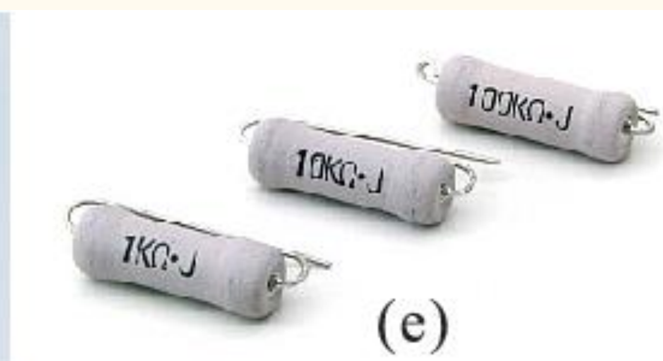
Chip resistors

Carbon film resistors

Metallic film resistors (spiral)

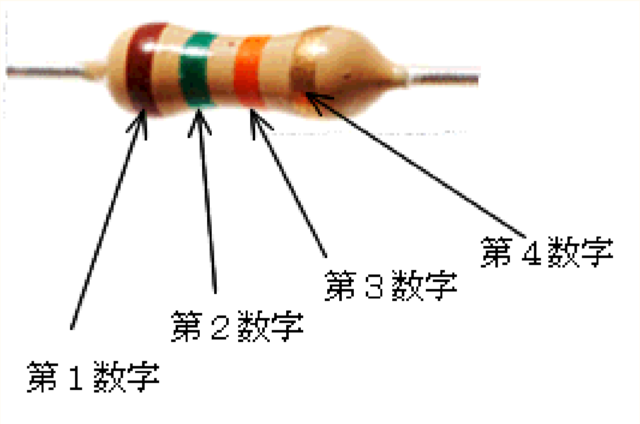


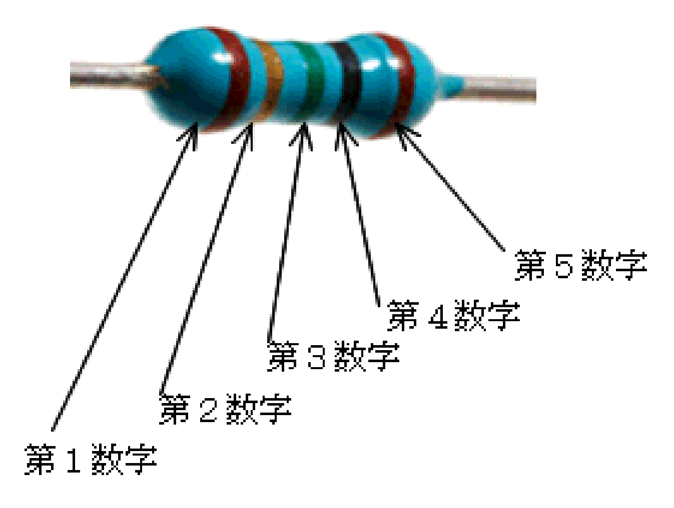
Metallic film resistors (meander)



High power type

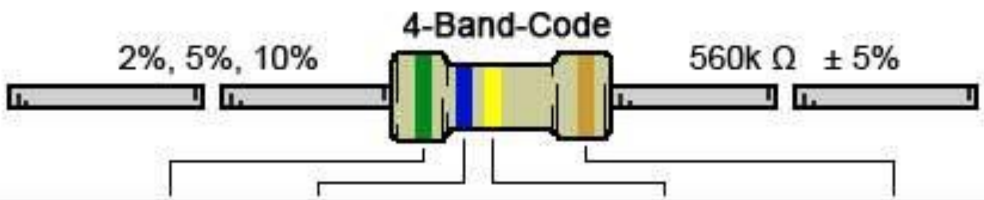
抵抗器のカラーコード



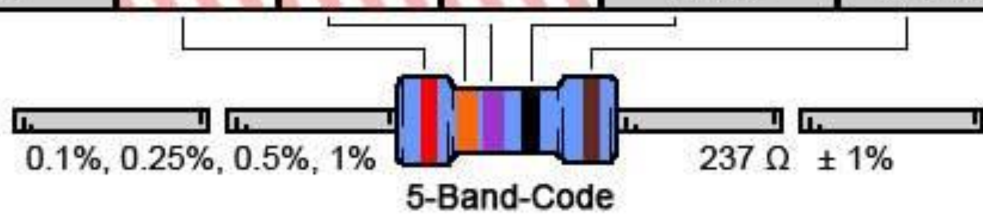


色	第1数字	第2数字	第3数字	第4数字	第5数字
黒	0	0	0	10^0	X
茶	1	1	1	10^1	±1%
赤	2	2	2	10^2	±2%
橙	3	3	3	10^3	X
黄	4	4	4	10^4	X
緑	5	5	5	10^5	X
青	6	6	6	10^6	X
紫	7	7	7	10^7	X
灰	8	8	8	10^8	X
白	9	9	9	10^9	X
金	X	X	X	10^{-1}	±5%
銀	X	X	X	10^{-2}	±10%
無色	X	X	X	X	±20%

Color code for resistors



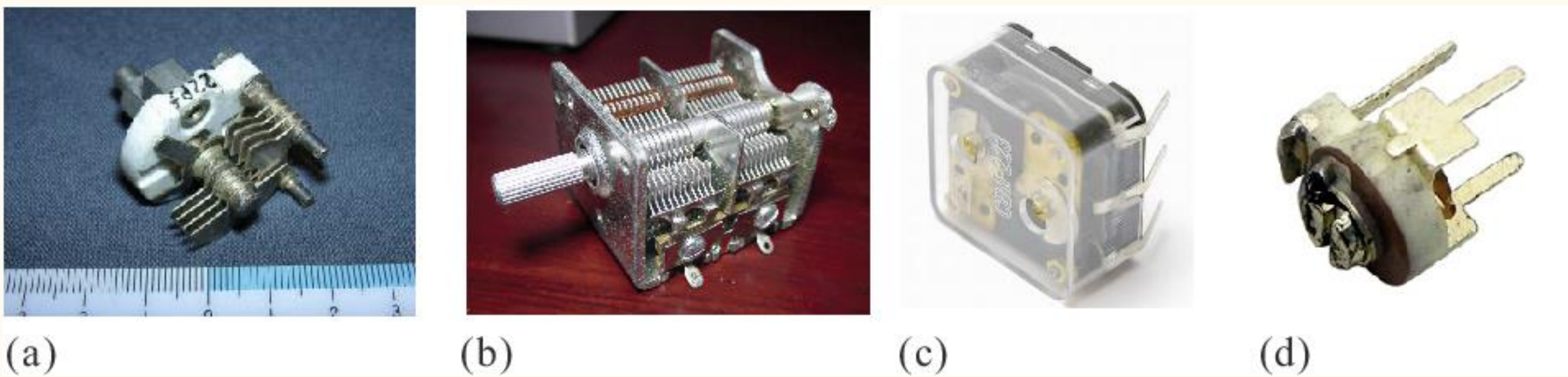
COLOR	1ST BAND	2ND BAND	3RD BAND	MULTIPLIER	TOLERANCE
Black	0	0	0	1Ω	
Brown	1	1	1	10Ω	± 1% (F)
Red	2	2	2	100Ω	± 2% (G)
Orange	3	3	3	1KΩ	
Yellow	4	4	4	10KΩ	
Green	5	5	5	100KΩ	± 0.5% (D)
Blue	6	6	6	1MΩ	± 0.25% (C)
Violet	7	7	7	10MΩ	± 0.10% (B)
Grey	8	8	8		± 0.05%
White	9	9	9		
Gold				0.1Ω	± 5% (J)
Silver				0.01Ω	± 10% (K)



Big
boys
race
our
young
girls
but
Violet
generally
wins.

Variable capacitors

$$C = \epsilon\epsilon_0 \frac{S}{d}$$



Steatite

Tandem

Poly-Ethylene

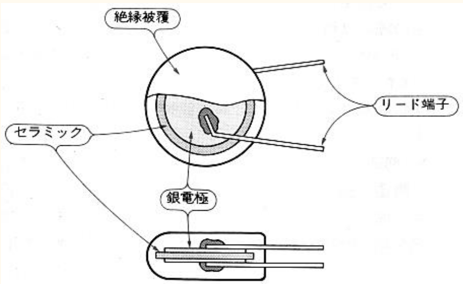
Ceramic

Air capacitor

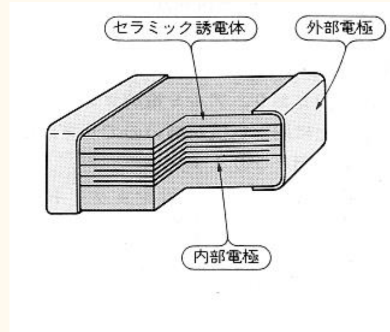
Fixed capacitors

Ceramic capacitors

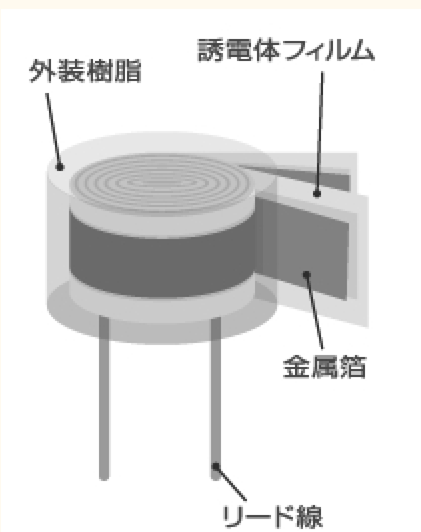
Single disk pair type



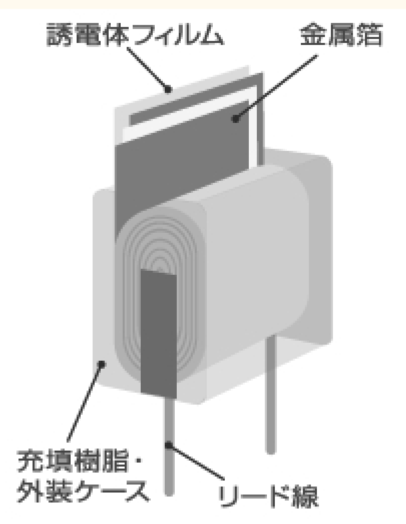
Stacked type



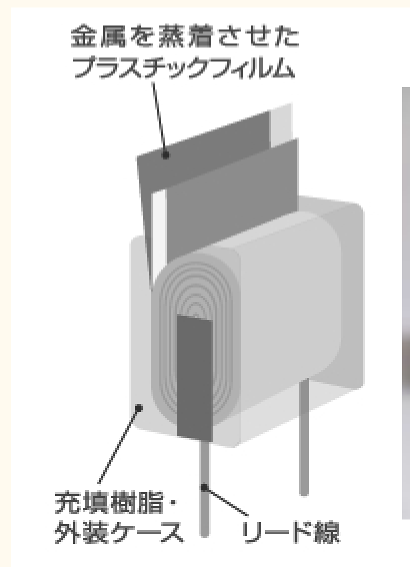
Film capacitors



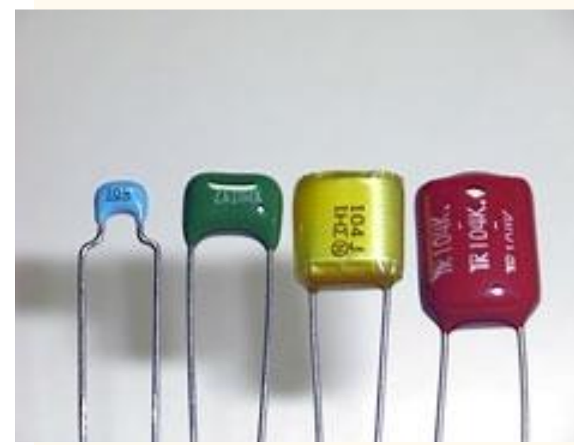
誘導



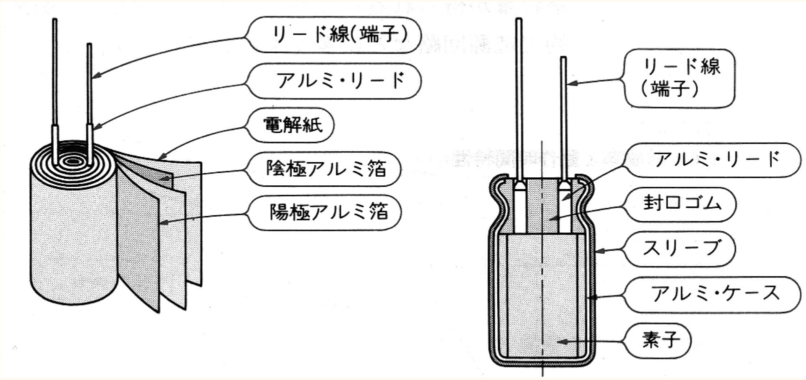
無誘導

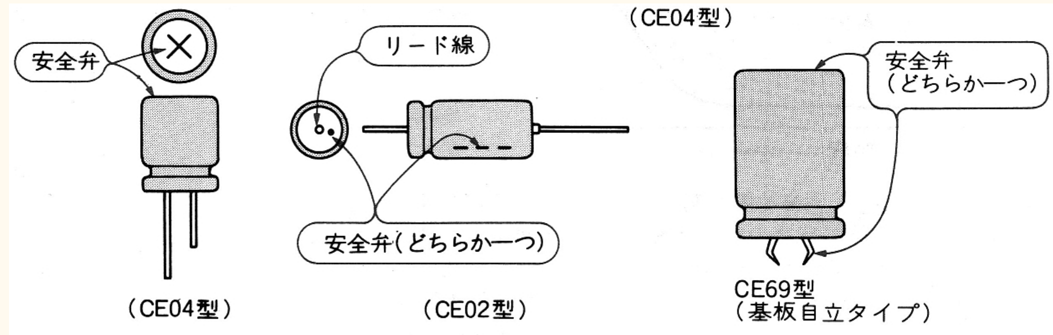


蒸着無誘導

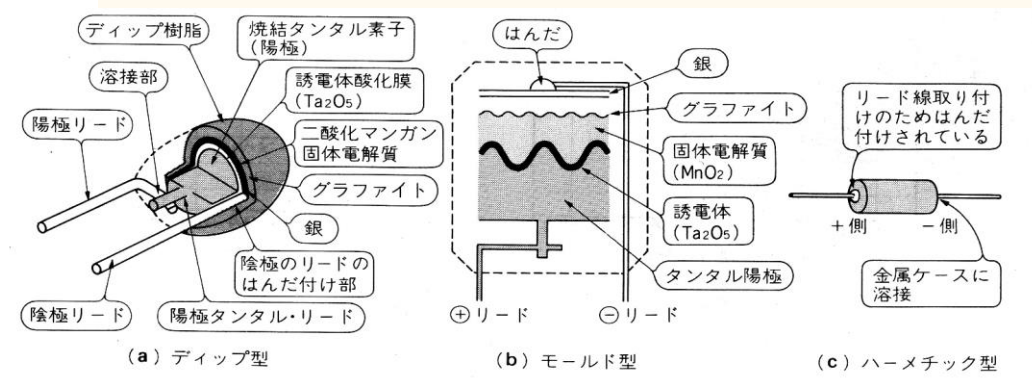


Chemical capacitors

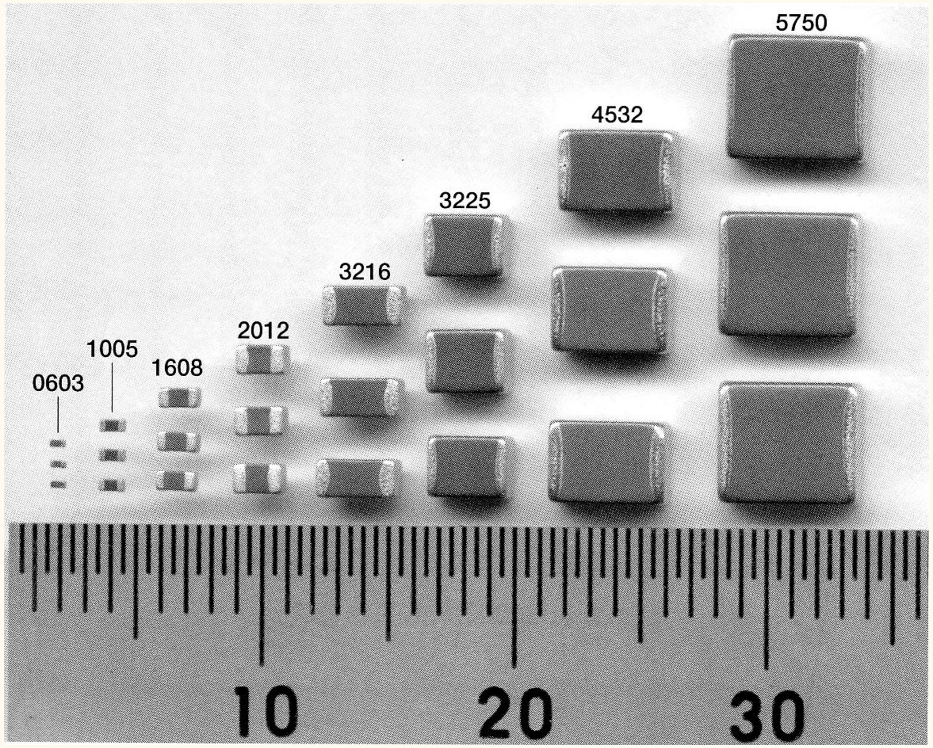


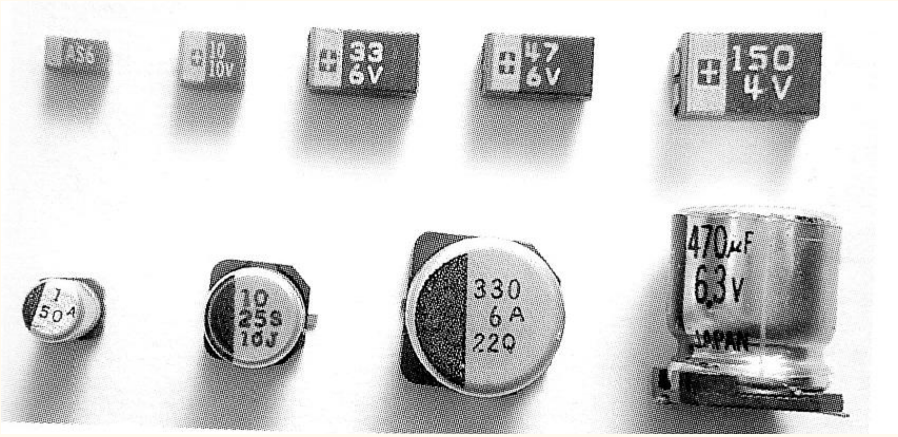


タンタル電解



Surface mount chip capacitors



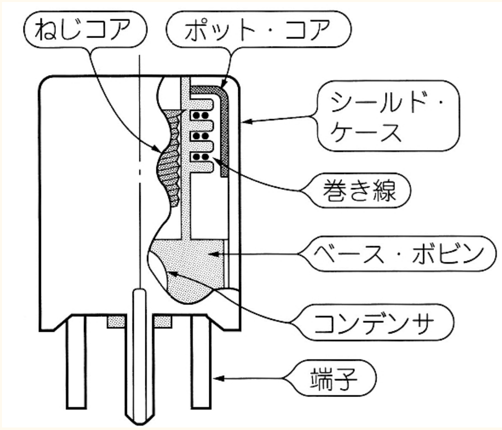


Variable inductors

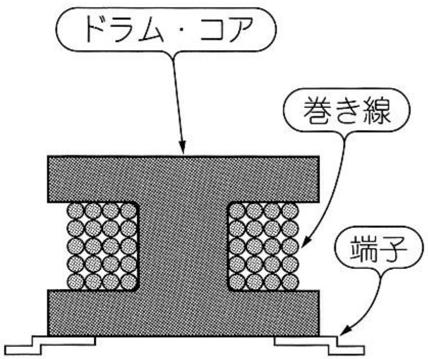
$$L = K \times \mu \frac{N^2}{l} S = K \times \mu n N S$$

$$K(\text{Nagaoka coef.}) = \frac{4}{3\pi\sqrt{1-k^2}} \left[\frac{1-k^2}{k^2} K(k) - \frac{1-2k^2}{k^2} E(k) - k \right]$$

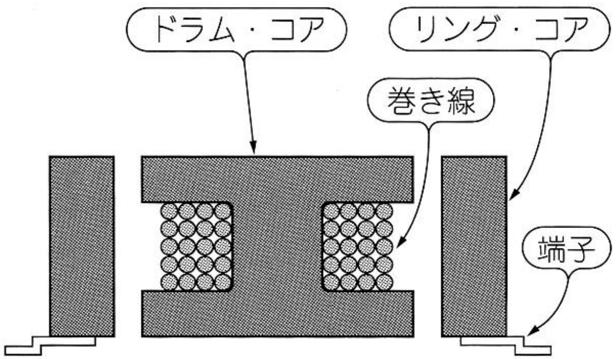
$$\frac{L}{2a} = \frac{\sqrt{1-k^2}}{k}$$



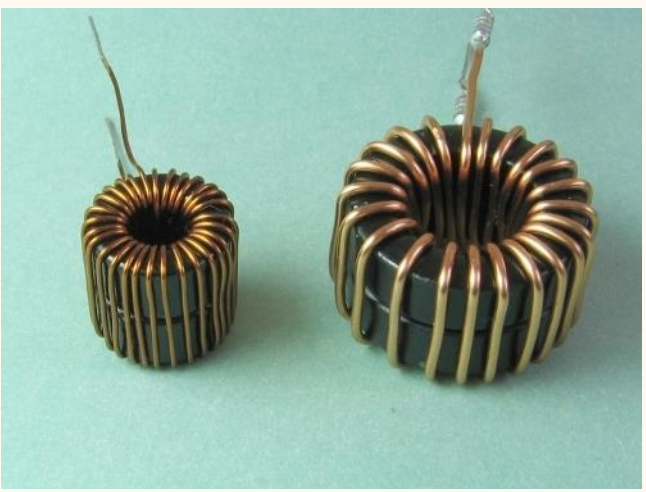
Fixed inductors



Open flux path



Closed flux path



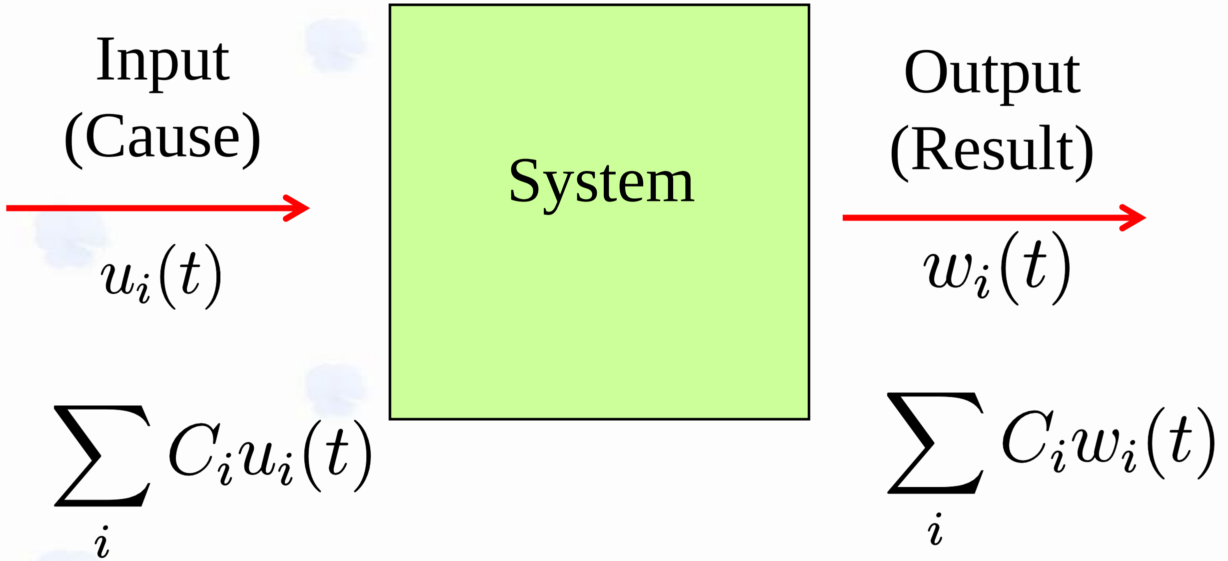
Toroidal coil

Chip inductor

Ch.2 Introduction to linear circuits

2.1.1 What is linear system?

$$f(C_1x_1 + C_2x_2) = C_1f(x_1) + C_2f(x_2)$$



Linear system: definition

$$w(t) = \mathscr{R}\{u(t)\}$$: Response

Requirements

Invariance: $\forall t_1 \quad w(t-t_1) = \mathscr{R}\{u(t-t_1)\}$

Causality: $u(t) = 0 \ (t < t_1) \ \rightarrow \ w(t) = 0 \ (t < t_1)$

Principle of superposition:

$$\forall C_1, C_2 \in C, \quad \mathscr{R}\{C_1 u_1(t) + C_2 u_2(t)\} = C_1 w_1(t) + C_2 w_2(t)$$

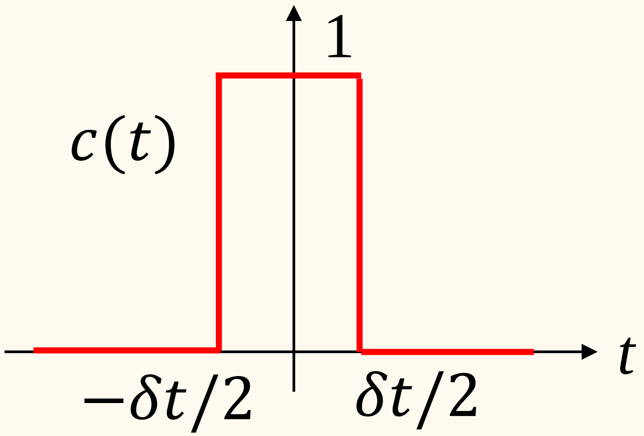
$$\mathscr{R}\left\{\sum_i C_i u_i(t)\right\} = \sum_i C_i w_i(t)$$

$$\mathscr{R}\left\{\int_{-\infty}^{\infty} c(q) u(q,t) dq\right\} = \int_{-\infty}^{\infty} c(q) \mathscr{R}\{u(q,t)\} dq$$

Transfer function

Variable q → time t

$$c(t) = \begin{cases} 1 & |t| \leq \delta t/2, \\ 0 & \text{others} \end{cases}$$



$$u(t) = \sum_i u(t_i)c(t-t_i) \quad \rightarrow \quad u(t) = \int_{-\infty}^{\infty} dt' u(t')c'(t-t')$$

$$c'(t-t') = \delta(t-t')$$

$$w(t) = \mathscr{R}\{u(t)\} = \mathscr{R}\left\{\int_{-\infty}^{\infty} u(t')\delta(t-t')dt'\right\}$$

$$= \int_{-\infty}^{\infty} u(t')\mathscr{R}\{\delta(t-t')\}dt' = \int_{-\infty}^{\infty} u(t')\xi(t,t')dt'$$

Transfer function (Impulse response)

$$\xi(t,t') \equiv \mathscr{R}\{\delta(t-t')\}$$: Impulse response, weight function

$$\text{Invariance} \to \xi(t,t_1) = \xi(t-t_1)$$

$$w(t) = \int_{-\infty}^{\infty} u(t')\xi(t-t')dt' = \int_{-\infty}^{\infty} u(t-t')\xi(t')dt'$$

Convolution

Fourier transform:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt, \quad x(t) = \int_{-\infty}^{\infty} X(\omega)e^{i\omega t}\frac{d\omega}{2\pi}$$

$$W(\omega) = U(\omega)\underline{\Xi(\omega)}$$

Transfer function

Summary

1. Electromagnetic field and electric circuits
   Metals: super screening material
   Circuit diagrams
   Local electromagnetic field
2. Introduction to linear circuits
   Transfer function

電子回路論　第2回

理学系研究科・物理専攻 (物性研究所) 勝本信吾

2016.10.5

Electric Circuits No.2

Shingo Katumoto

ノート・資料等の置き場

http://kats.issp.u-tokyo.ac.jp/kats/

2週に１回簡単な練習問題を出題　→　2週間以内に解答を提出

TA

試験は期末レポート．練習問題と合わせて採点します

Review

Ch.1 Electromagnetic field and electric circuits

Metals: super-screening material

(but not superconducting. The difference is important in designing superconducting circuits.)

Local electromagnetic field

↓

→ Lumped constant circuits (集中定数回路)

local magnetic fields (parts) are connected by metallic wires → Circuit diagrams

Resistors, Capacitors and Inductors

Ch.2 Introduction to linear response systems

Outline Today

1. Transfer function (伝達関数) (continued)
2. Representative passive devices in the linear treatment
3. Impedance, admittance and other parameters in the linear treatment
4. Power sources
5. Circuit networks
6. Four terminal (two terminal-pair) circuits
7. Circuit theorems

Linear response: Transfer function

response input

$$w(t) = \mathscr{R}\{u(t)\} = \mathscr{R}\left\{\int_{-\infty}^{\infty} u(t')\delta(t-t')dt'\right\} = \int_{-\infty}^{\infty} u(t')\mathscr{R}\{\delta(t-t')\}dt'$$

impulse response

$$= \int_{-\infty}^{\infty} u(t')\xi(t-t')dt' = \int_{-\infty}^{\infty} u(t-t')\xi(t')dt'$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \qquad W(\omega) = U(\omega)\Xi(\omega)$$

Transfer function

Laplace:

$$X(s) = \int_{0}^{\infty} e^{-st}x(t)dt \qquad W(s) = U(s)\Xi(s)$$

Expansion to the complex plane: $s \rightarrow \sigma + i\omega$

On the imaginary axis (the frequency space)

$$W(i\omega) = U(i\omega)\Xi(i\omega)$$

Impedance

Current to voltage

$$V_{12} = \hat{A} I_{12}$$

$$\begin{cases} V_{12} = R I_{12} & \text{resistor} \\ V_{12} = \dfrac{q(t)}{C} = \dfrac{1}{C}\displaystyle\int^{t} I_{12}(t')dt' & \text{capacitor} \\ V_{12} = L\dfrac{dI_{12}}{dt} & \text{inductor} \end{cases}$$

$$\Xi(i\omega) = \begin{cases} \displaystyle\int_{-\infty}^{\infty} e^{-st}\left[R\delta(t)\right] dt = R & \text{resistor} \\ \displaystyle\int_{-\infty}^{\infty} e^{-i\omega t}\left[\frac{1}{C}\int^{t}\delta(t')dt'\right] dt = \frac{1}{i\omega C} & \text{capacitor} \\ \displaystyle\int_{-\infty}^{\infty} e^{-st}\left[L\frac{d}{dt}\delta(t)\right] dt = i\omega L & \text{inductor} \end{cases}$$

↓

Impedance $Z(i\omega)$

Admittance

Voltage to current

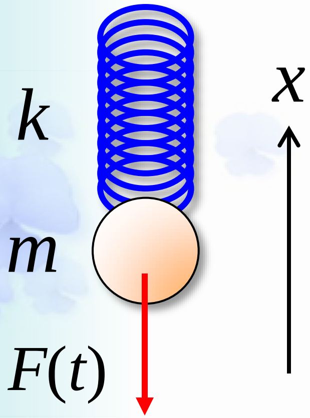
$$\mathcal{J}(i\omega) = Y(i\omega)\mathcal{V}(i\omega)$$

Admittance $Y(i\omega)$

$$Y(i\omega) = \frac{1}{Z(i\omega)}$$

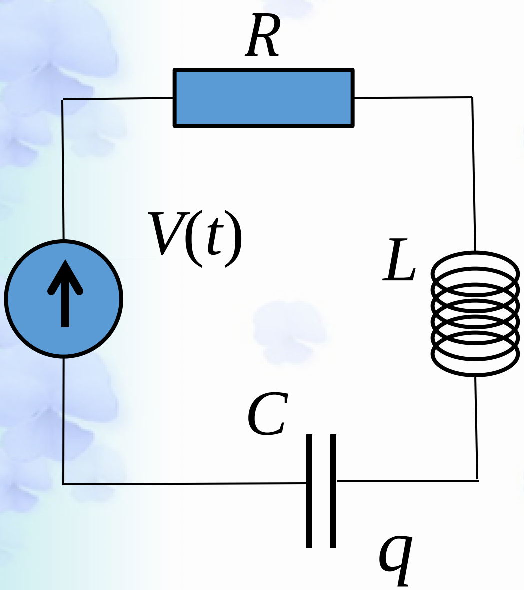
Example of equivalent circuit 「等価回路」の例

Spring pendulum (ばね振り子)



$$m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=F(t)$$

RLC circuit with electromotive force
(電源を接続したRLC回路)



$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=V(t)$$

Parallelism

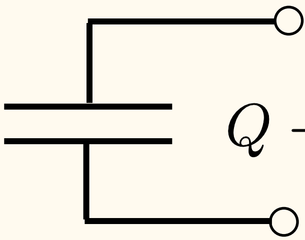
並行論

2.2 Power sources

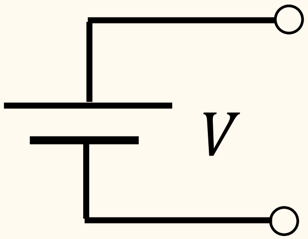
An active device: electric power source, electromotive force

Realistic power source: ideal power source + non-ideal factors

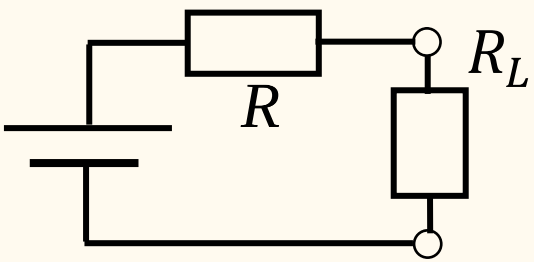
Ideal voltage source



$$Q \to \infty, \quad C \to \infty, \quad V = \frac{Q}{C} = \text{const.}$$



Voltage source
+resistor

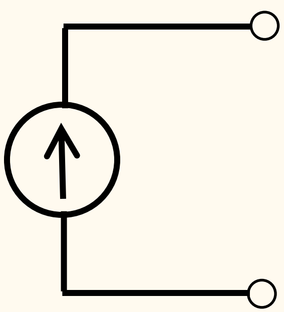


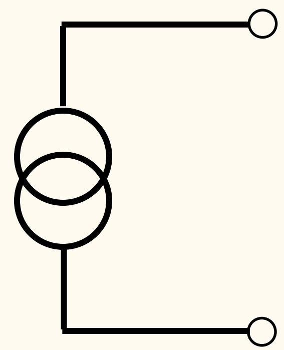
$$J = \frac{V}{R + R_L}$$

$$V_{out} = \frac{R_L}{R_L + R} V$$

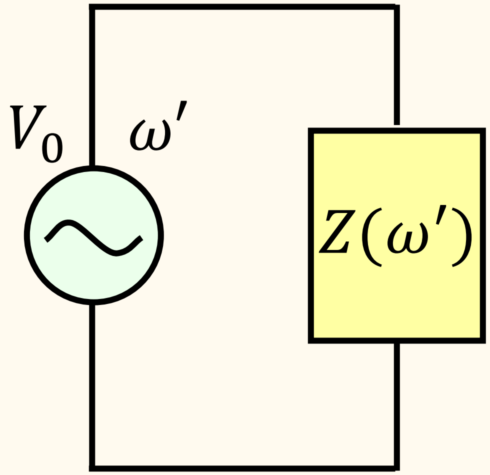
Ideal current source

$$R \to \infty, \ V \to \infty, \ J = \frac{V}{R} = \text{const.}$$





Power consumption



Energy dissipation per unit time: $P = VJ$

Electric power consumption

$$V(\omega', t) = V_0 e^{i\omega' t}$$

$$\mathcal{V}(i\omega) = \mathscr{F}\{V\} = 2\pi V_0 \delta(\omega - \omega')$$

$$J(\omega', t) = 2\pi \int_{-\infty}^{\infty} \frac{V_0}{Z(i\omega)} \delta(\omega - \omega') e^{i\omega t} \frac{d\omega}{2\pi} = \frac{V_0}{Z(i\omega')} e^{i\omega' t} = \frac{V(\omega', t)}{Z(i\omega')}$$

$$V = V_0 \cos \omega' t \qquad W(\omega', t) = V(\omega', t) J(\omega', t) = V_0^2 \cos^2 \omega t / Z(\omega')$$

Complex instantaneous power

$$\overline{W}(\omega') = \frac{V_0^2}{2Z(\omega')}$$

$$P(\omega') \equiv \mathrm{Re}[\overline{W}(\omega')]$$

Effective power (有効電力)

$$Q(\omega') \equiv \mathrm{Im}[\overline{W}(\omega')]$$

Reactive power (無効電力)

Power consumption (2)

$|\overline{W}(\omega')|$ Apparent power (皮相電力)

$$I_{\mathrm{M}} \equiv \frac{\mathrm{Re}[\overline{W}(\omega')]}{|\overline{W}(\omega')|} = \cos\left[\arg(\overline{W}(\omega'))\right] \equiv \cos\phi$$

Moment (力率)

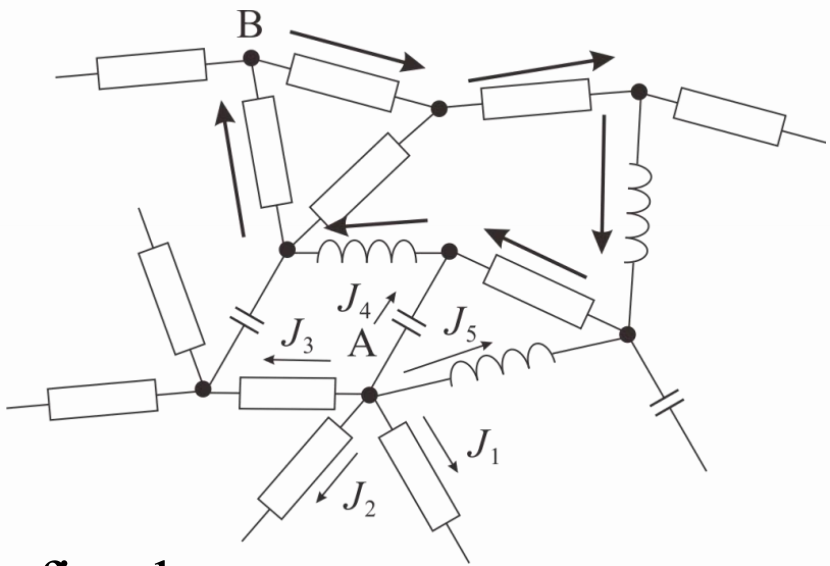
ϕ: Phase shift between voltage and current

$$\overline{W}(\omega') = |\overline{W}(\omega')|e^{i\phi}$$: generally holds

$$\overline{W}(\omega') = V^*(\omega')J(\omega')$$

$$W = P = \frac{V_0^2}{2R}$$

$$\frac{V_0}{\sqrt{2}}$$: effective value



2.3.1 Kirchhoff's law

At all nodes $\sum_i J_i = 0$ Kirchhoff's first law

↑ Charge conservation $\frac{\partial \rho}{\partial t} + \mathrm{div}\boldsymbol{J} = 0$

=0

For all looping paths $\sum_j V_j = 0$ Kichhoff's second law

↑ Single-valuedness of electric potential

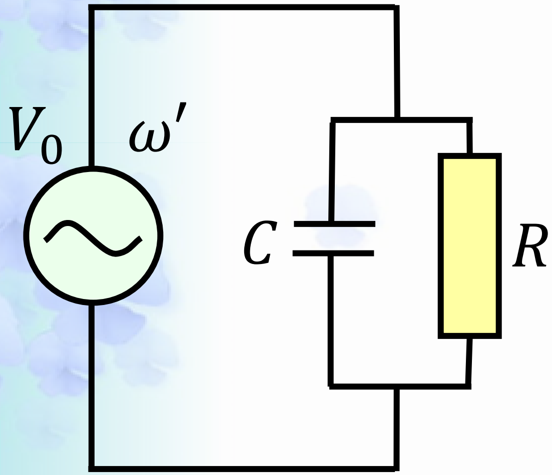
2.3 Circuit network (2)

From Kirchhoff's law, synthetic admittance and impedance are

for parallel connection: $$Y_{\text{tot}} = \sum_{i=1}^{n} Y_i, \qquad Z_{\text{tot}} = \left(\sum_{i=1}^{n} Z_i^{-1}\right)^{-1}$$

for series connection: $$Y_{\text{tot}} = \left(\sum_{i=1}^{n} Y_i^{-1}\right)^{-1}, \qquad Z_{\text{tot}} = \sum_{i=1}^{n} Z_i$$

Ex.)



$$Z(i\omega) = \left(\frac{1}{R} + i\omega C\right)^{-1}, \quad Y(i\omega) = \frac{1}{R} + i\omega C$$

$$P(\omega) = \frac{V_0^2}{R}\cos^2 \omega t, \quad Q(\omega) = \omega C V_0^2 \cos^2 \omega t$$

$$\frac{P(\omega)}{Q(\omega} = \frac{1}{\omega C R} = \tan\delta$$: Dissipation factor

2.3.3 Superposition theorem

Network: node, (directional) branch : directional graph (digraph)

All the branches: electromotive force E_i, resistance R_i

$$A\{(R)\}\begin{pmatrix} J_1 \\ \vdots \\ J_m \end{pmatrix} = \begin{pmatrix} E_1 \\ \vdots \\ E_m \end{pmatrix} \qquad \boldsymbol{R} = \begin{pmatrix} R_1 \\ \vdots \\ R_m \end{pmatrix}$$

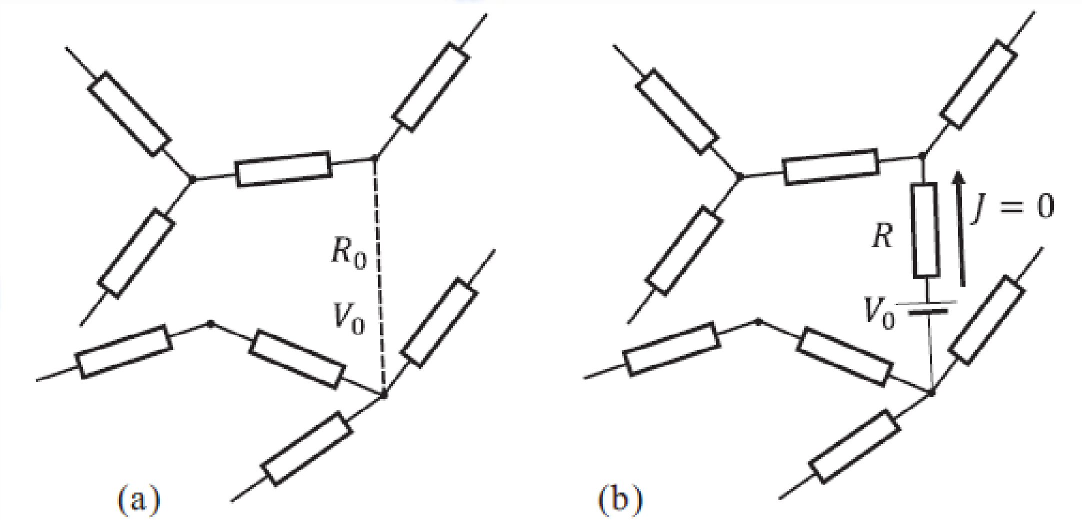
Superposition theorem:

The total current distribution is the superposition of those for single electromotive forces.

2.3.4 Ho (鳳) – Tevenin's theorem

Pick up two nodes in the network under consideration.
The voltage between these two nodes is V_0.
Set all the electromotive forces to zero and measure the resistance between the two nodes. The result is R_0.
Now connect the two nodes with resistance R and reset the electromotive forces to the original values. Then the current through resistance R is

$$J = V_0/(R + R_0)$$



2.3.5 Tellegen's theorem

$i = 1, \cdots, n$: index of nodes, $j = 1, \cdots, m$: index of branches

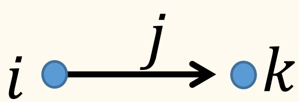
$$a_{ij} = \begin{cases} 1: & i \text{ is the start of } j, \\ -1: & i \text{ is the end of } j, \\ 0: & \text{others} \end{cases}$$ incidence matrix

$$\forall j: \quad \sum_{i=1}^{n} a_{ij} = 0$$: redundancy in $\{a_{ij}\}$

$\rightarrow (n-1) \times m$ matrix D : irreducible incidence matrix

Kirchhoff's first law: $D\boldsymbol{J} = 0$ $\quad J_j$: current along branch j

$\boldsymbol{W}$: W_i electrostatic potential of node i, $\boldsymbol{V}$: V_j voltage across branch j



$$V_j = W_i - W_k = a_{ij} W_i - a_{kj} W_k$$

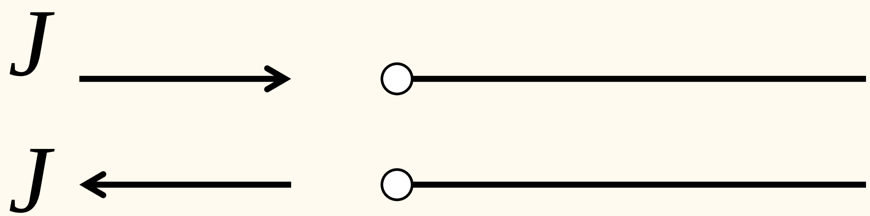
$$\boldsymbol{V} = {}^t\mathcal{D}\boldsymbol{W} \quad ({}^t\mathcal{D}\text{: transpose}) \qquad \text{(Kirchhoff's second law)}$$

$$\sum_{i=1}^{m} V_i J_i = ({}^t\mathcal{D}\boldsymbol{W}) \cdot \boldsymbol{J} = {}^t\boldsymbol{W}\mathcal{D}\boldsymbol{J} = 0 \qquad \boldsymbol{V} \perp \boldsymbol{J}$$

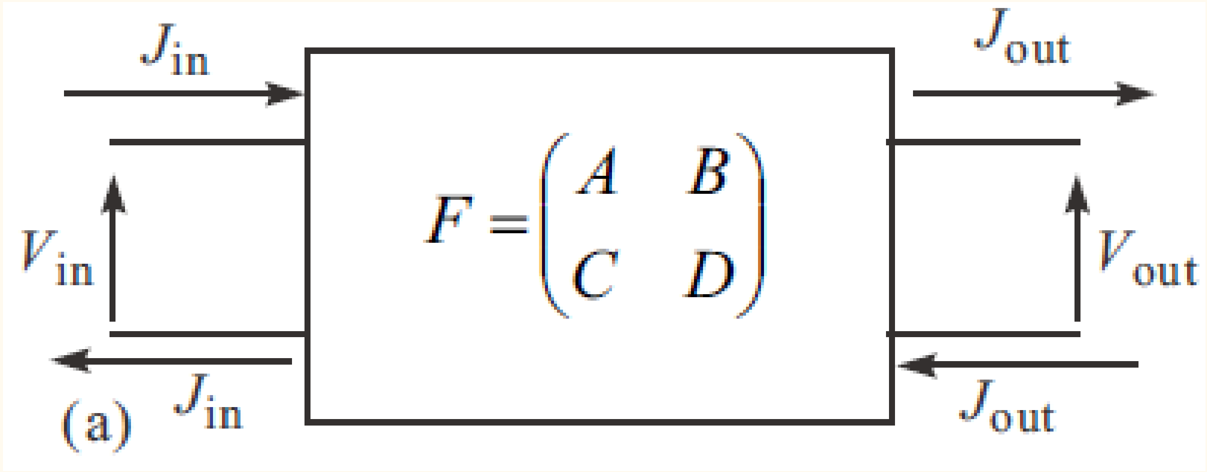
4-terminal (2-terminal pair) circuits 4端子回路

Terminal pair (端子対)

Current: circulation, no net current

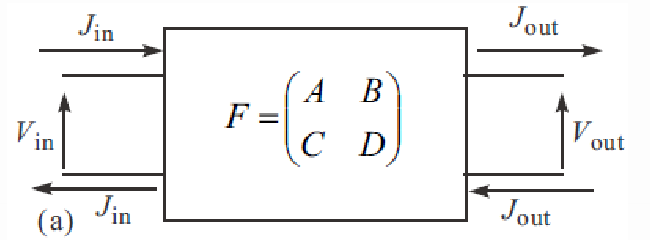


2-terminal pair (4-terminal) circuit



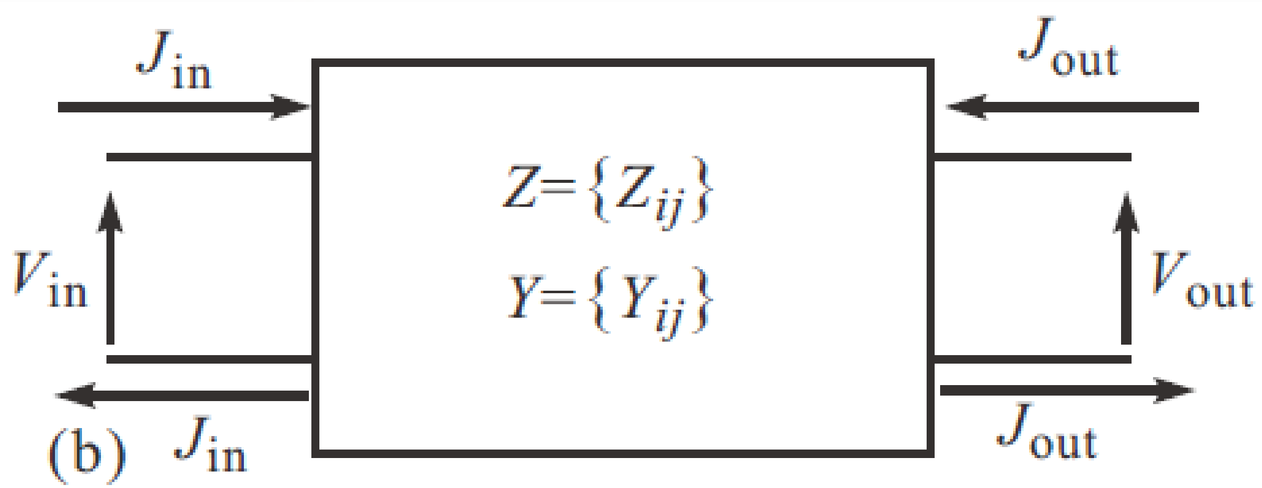
F-matrix of 4-terminal circuit

$$\begin{pmatrix} V_{\rm in} \\ J_{\rm in} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_{\rm out} \\ J_{\rm out} \end{pmatrix} \equiv F \begin{pmatrix} V_{\rm out} \\ J_{\rm out} \end{pmatrix}$$

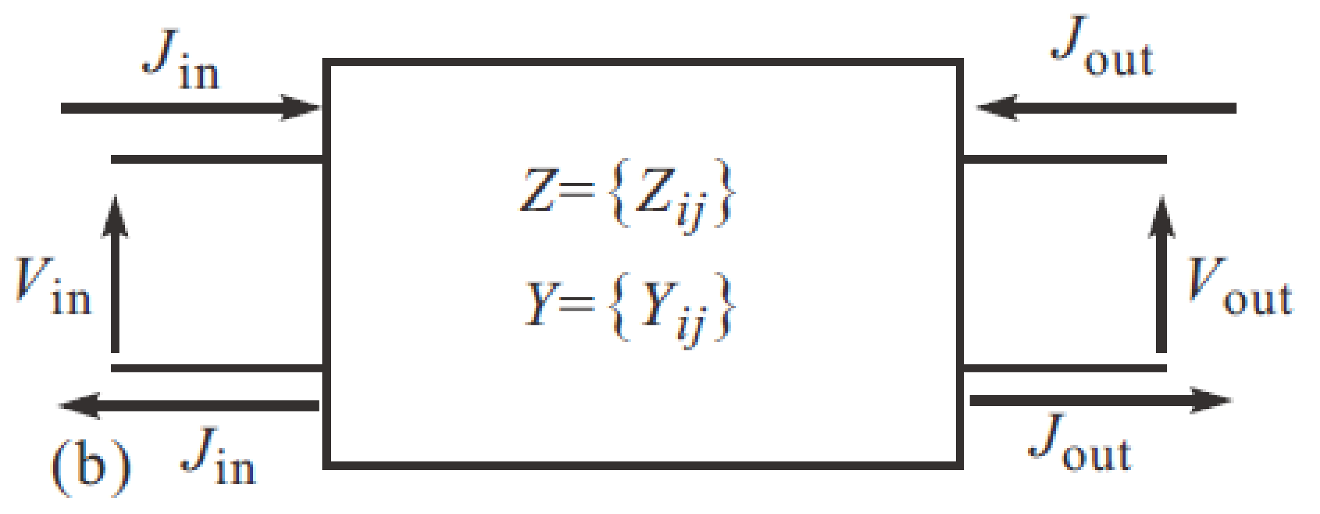


$$A = \left(\frac{V_{\rm in}}{V_{\rm out}}\right)_{J_{\rm out}=0}, \quad B = \left(\frac{V_{\rm in}}{J_{\rm out}}\right)_{V_{\rm out}=0}, \quad C = \left(\frac{J_{\rm in}}{V_{\rm out}}\right)_{J_{\rm out}=0}, \quad D = \left(\frac{J_{\rm in}}{J_{\rm out}}\right)_{V_{\rm out}=0}.$$

$$\begin{pmatrix} V_{\rm out} \\ J_{\rm out} \end{pmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} V_{\rm in} \\ J_{\rm in} \end{pmatrix} \equiv K \begin{pmatrix} V_{\rm in} \\ J_{\rm in} \end{pmatrix}$$



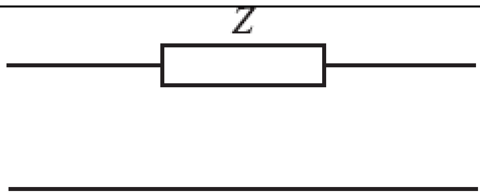
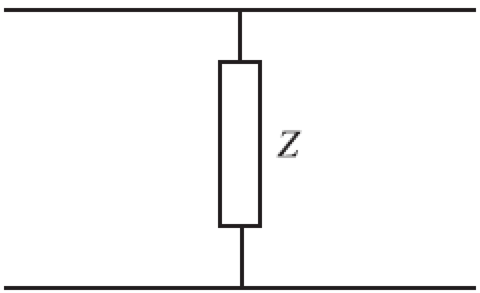
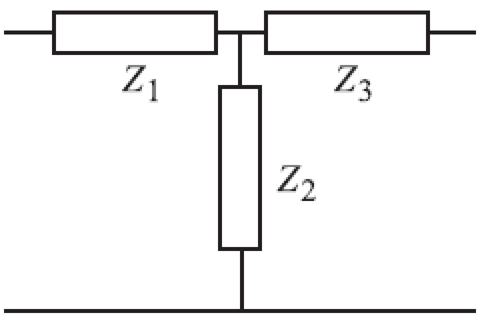
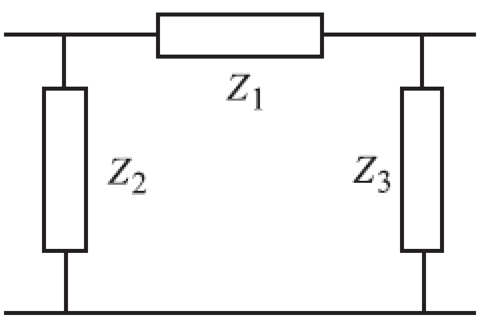
Impedance matrix, Admittance matrix



$$\begin{pmatrix} V_{in} \\ V_{out} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} J_{in} \\ J_{out} \end{pmatrix} \equiv Z \begin{pmatrix} J_{in} \\ J_{out} \end{pmatrix}$$

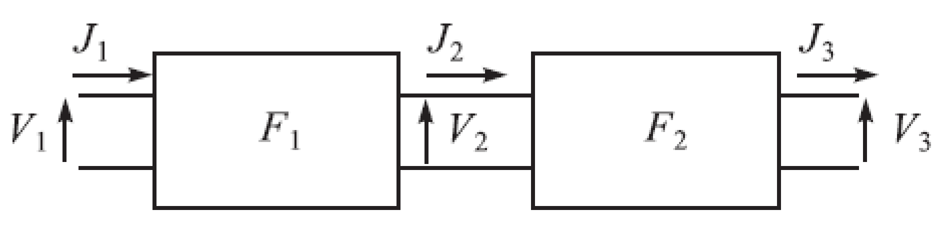
$$\begin{pmatrix} J_{in} \\ J_{out} \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_{in} \\ V_{out} \end{pmatrix} \equiv Y \begin{pmatrix} V_{in} \\ V_{out} \end{pmatrix}$$

Examples with impedances

	A	B	C	D
	1	Z	0	1
	1	0	$\dfrac{1}{Z}$	1
	$1+\dfrac{Z_1}{Z_2}$	$\dfrac{Z_1Z_2+Z_2Z_3+Z_3Z_1}{Z_2}$	$\dfrac{1}{Z_2}$	$1+\dfrac{Z_3}{Z_2}$
	$1+\dfrac{Z_1}{Z_3}$	Z_1	$\dfrac{Z_1+Z_2+Z_3}{Z_2Z_3}$	$1+\dfrac{Z_1}{Z_2}$

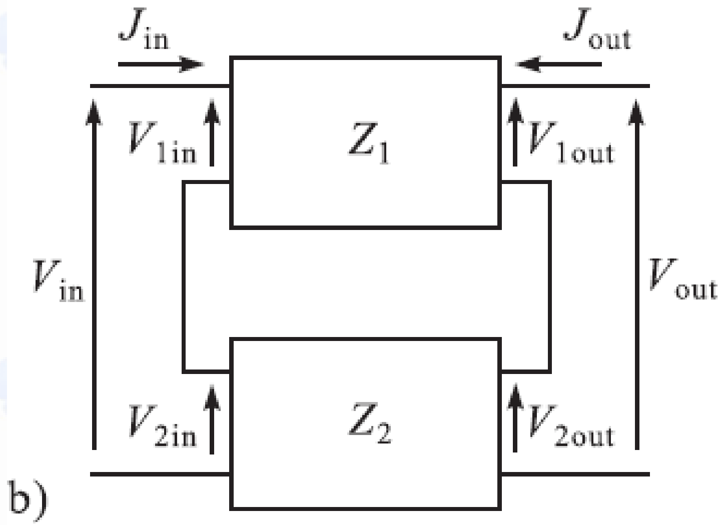
Connections of 4-terminal circuits

Cascade



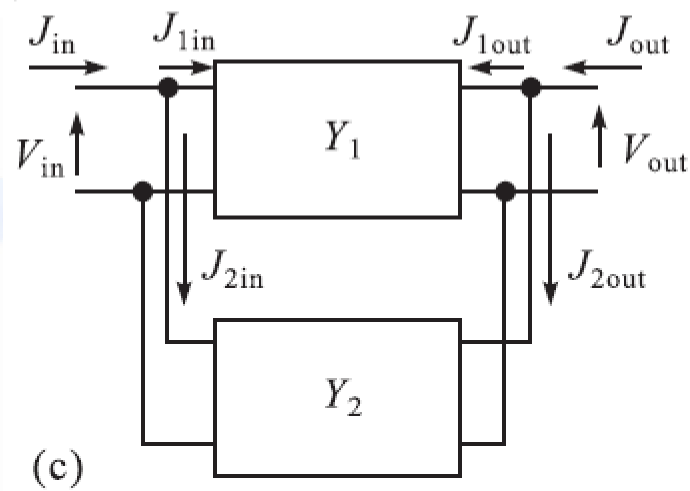
$$F_{\mathrm{tot}} = \prod_{i=1}^{N} F_i$$

Series



$$Z_{\mathrm{tot}} = \sum_{i=1}^{N} Z_i$$

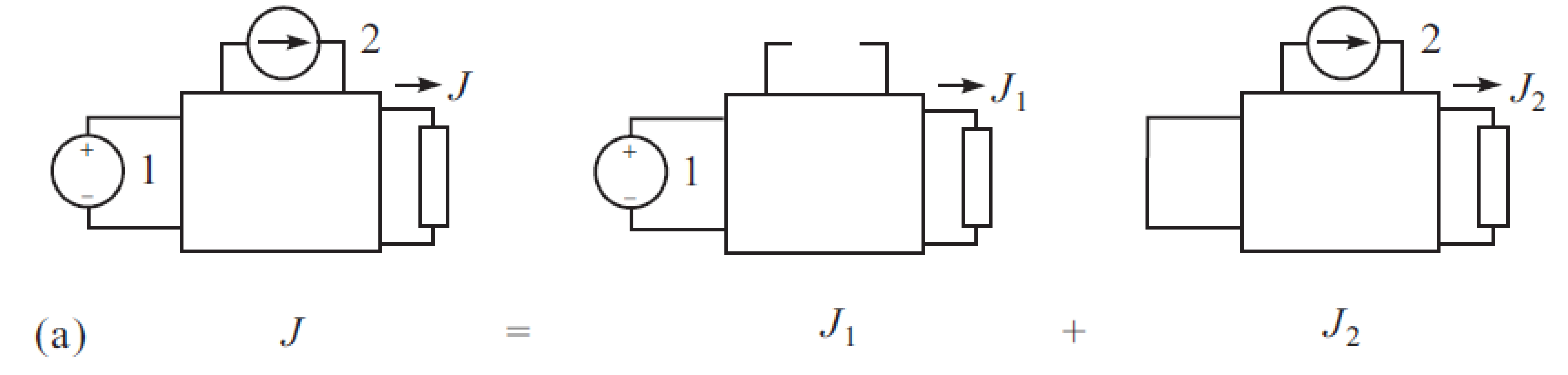
Parallel



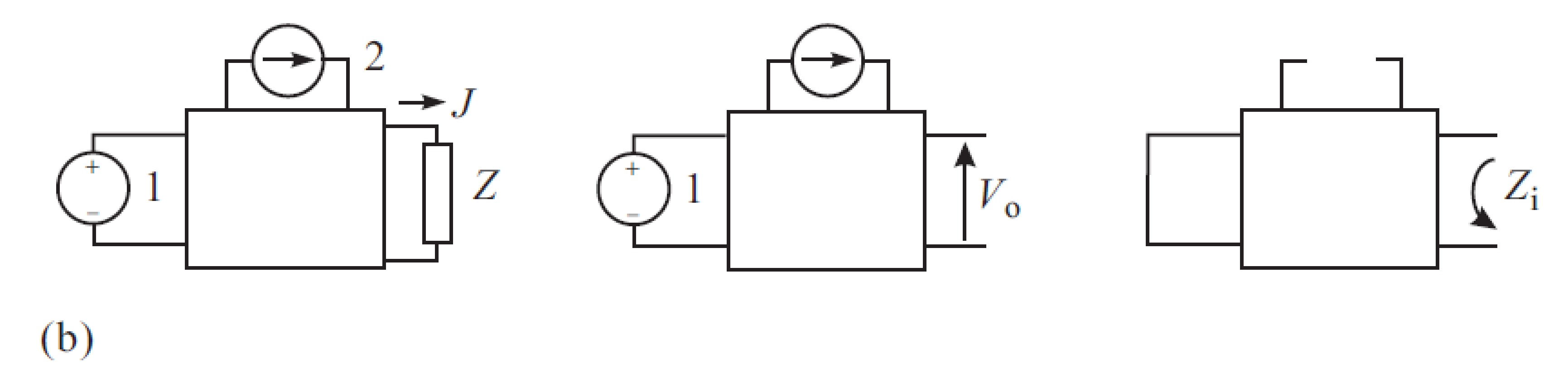
$$Y_{\mathrm{tot}} = \sum_{i=1}^{N} Y_i$$

Theorems for terminal-pair circuits

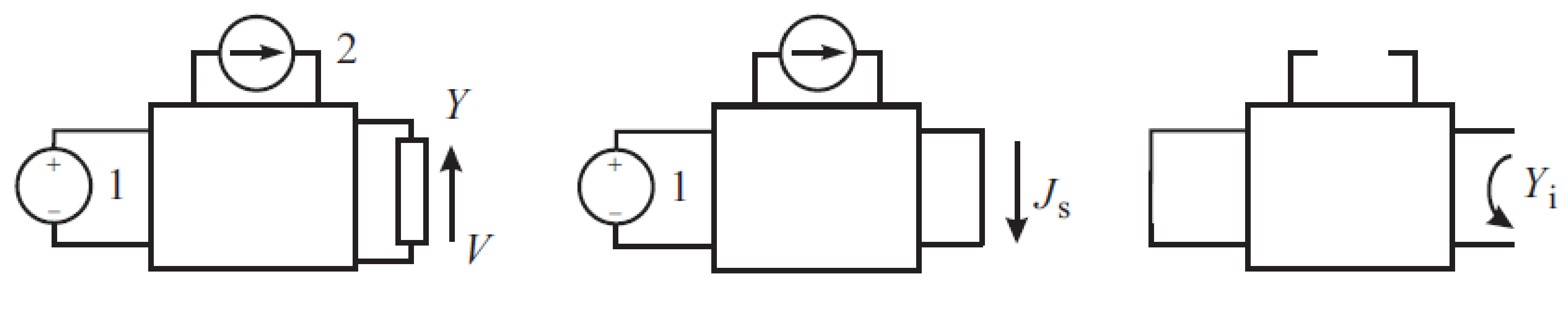
Superposition theorem



Ho-Thevenin's theorem



Norton's theorem



$$V = \frac{J_s}{Y + Y_i}$$

Duality 双対性

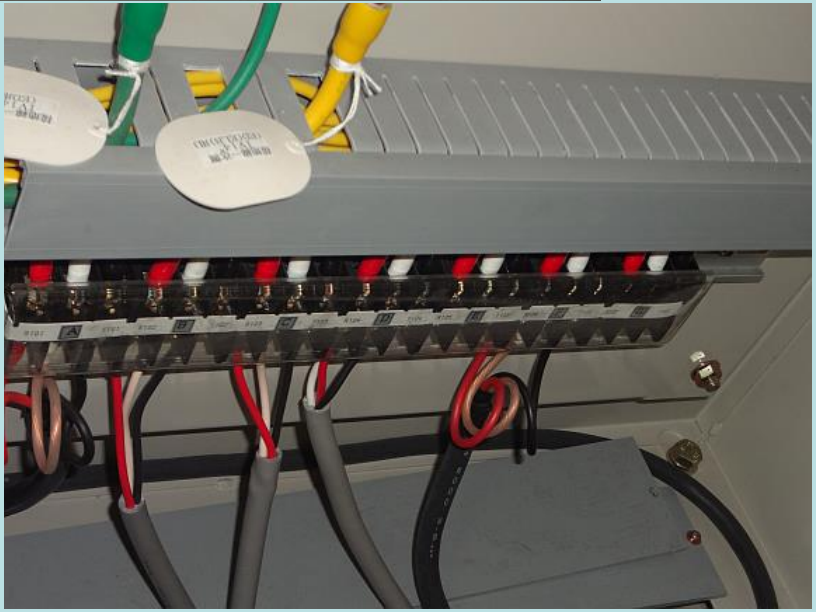
直列接続	並列接続
開放	短絡
電場	磁場
キルヒホッフの第 2 法則	キルヒホッフの第 1 法則
電圧	電流
インピーダンス	アドミッタンス
抵抗	コンダクタンス
静電容量	インダクタンス
鳳-テブナンの定理	ノートンの定理

Duality

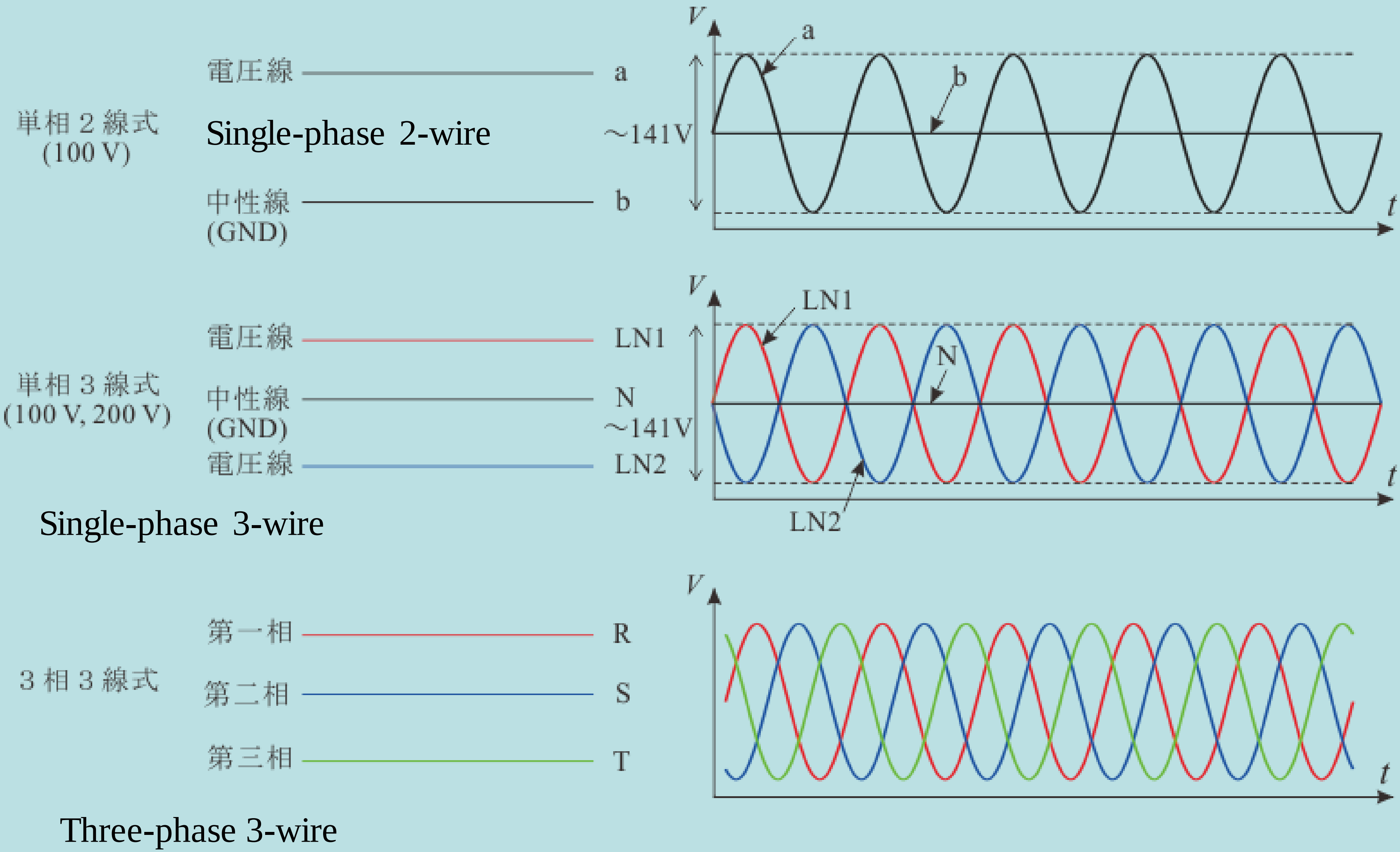
Series	**Parallel**
Open	Short
Voltage	Current
Impedance	Admittance
Capacitance	Inductance
Electric field	Magnetic field
Resistance	Conductance
Ho-Thevenin	Norton
Kirchhoff's 2^{nd} law	Kirchhoff's 1^{st} law

Power Sources in Lab. 電源の雑知識

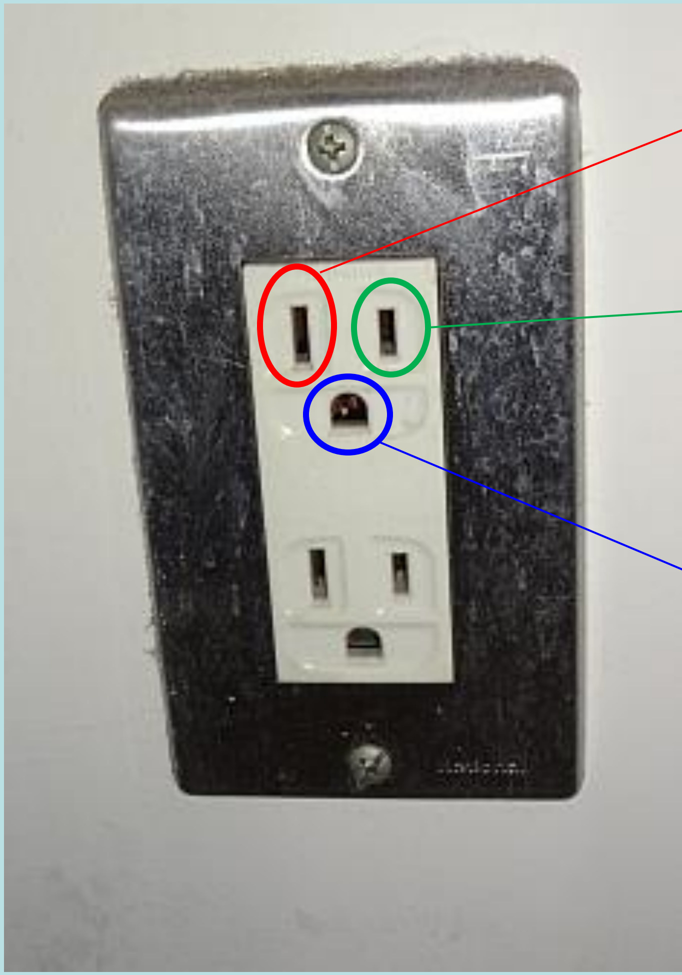
AC Power from distribution board 配電盤からの電力供給



AC Power from distribution board 配電盤からの電力供給



Japanese outlet tap definition 日本式コンセント



Cold line 中性線

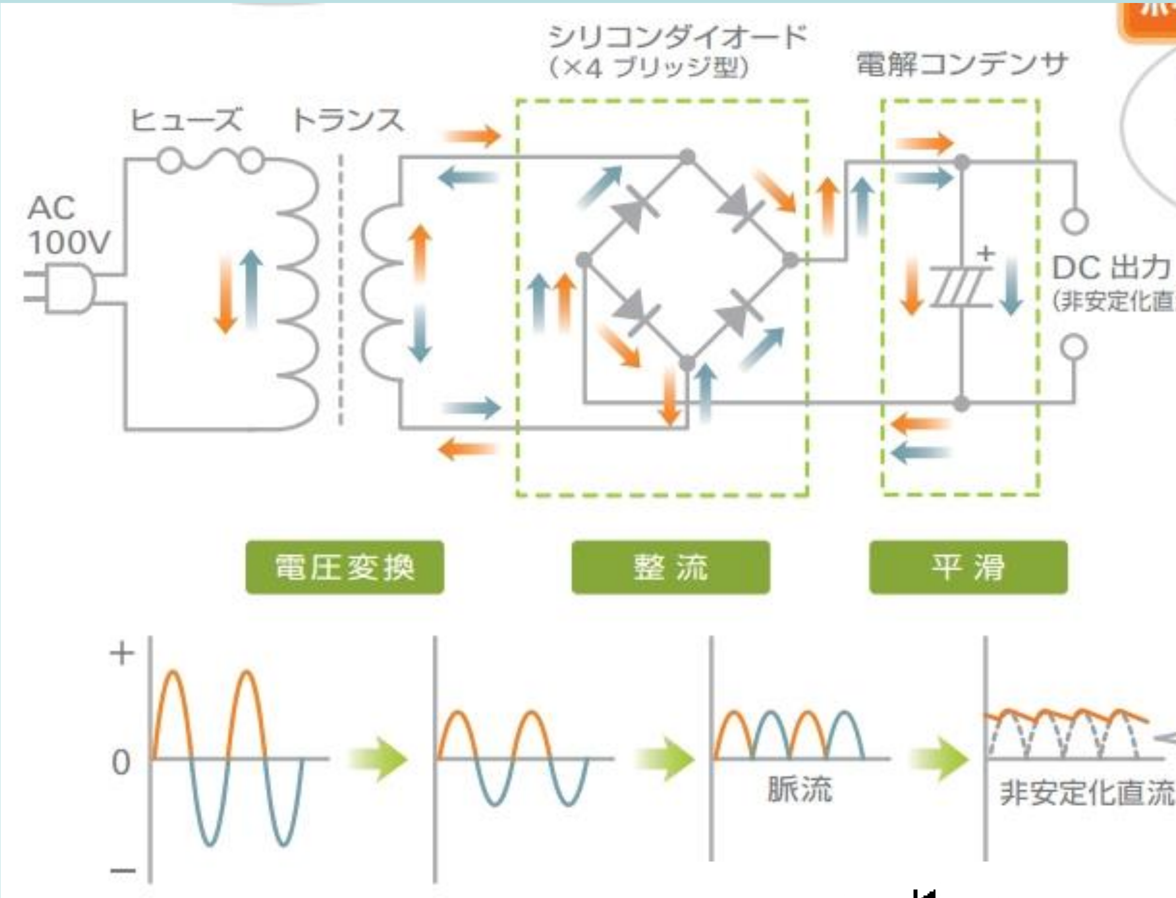
Hot line 電圧線

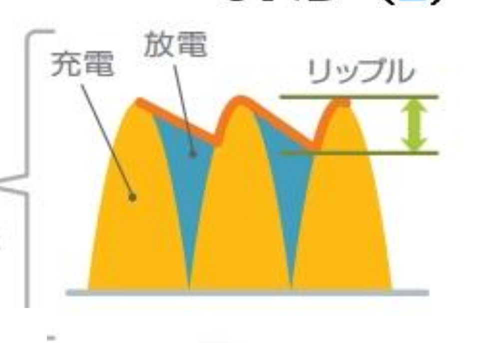
Ground 接地線

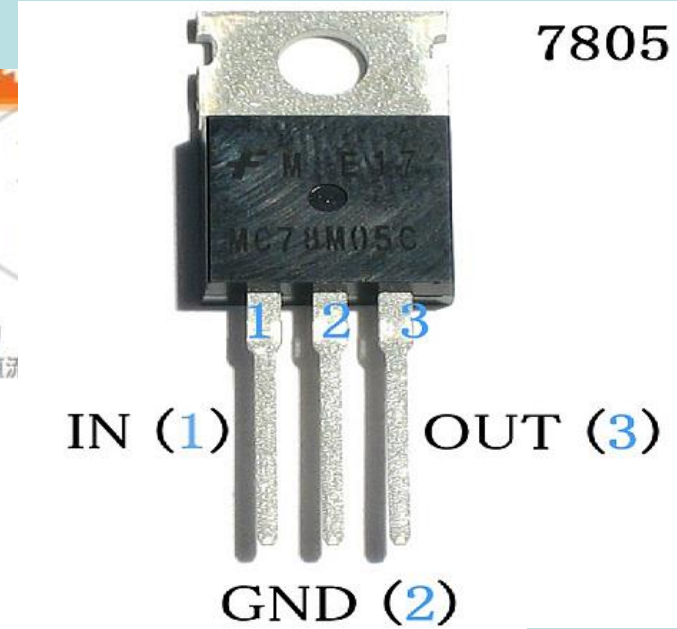
検電ドライバー
Electroscopic
Screwdriver

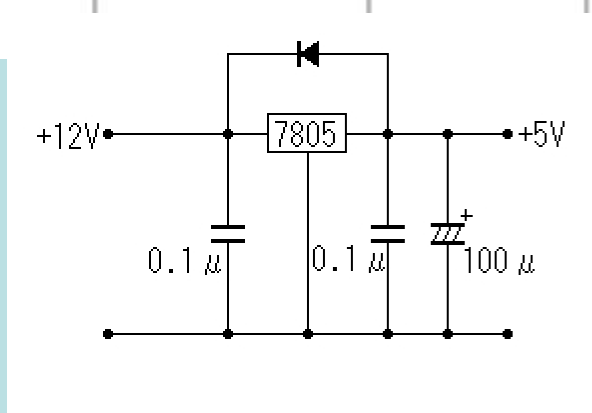
DC Stabilized Power Supply 直流安定化電源

Series (Dropper) regulation









From TDK web page

Series regulator power supply

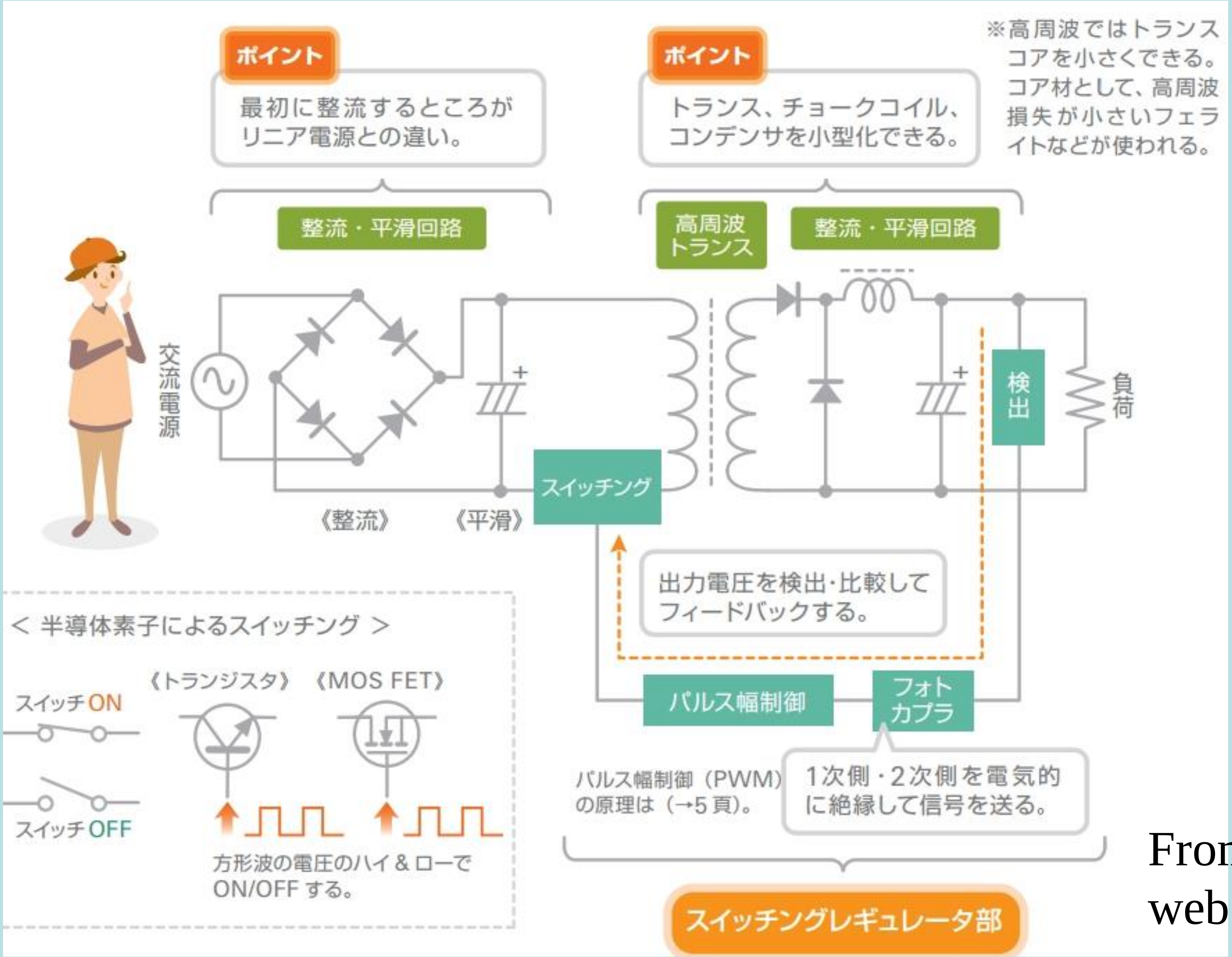
Uni-polar

Dual tracking

High precision

Bi-polar current source

Switching regulation

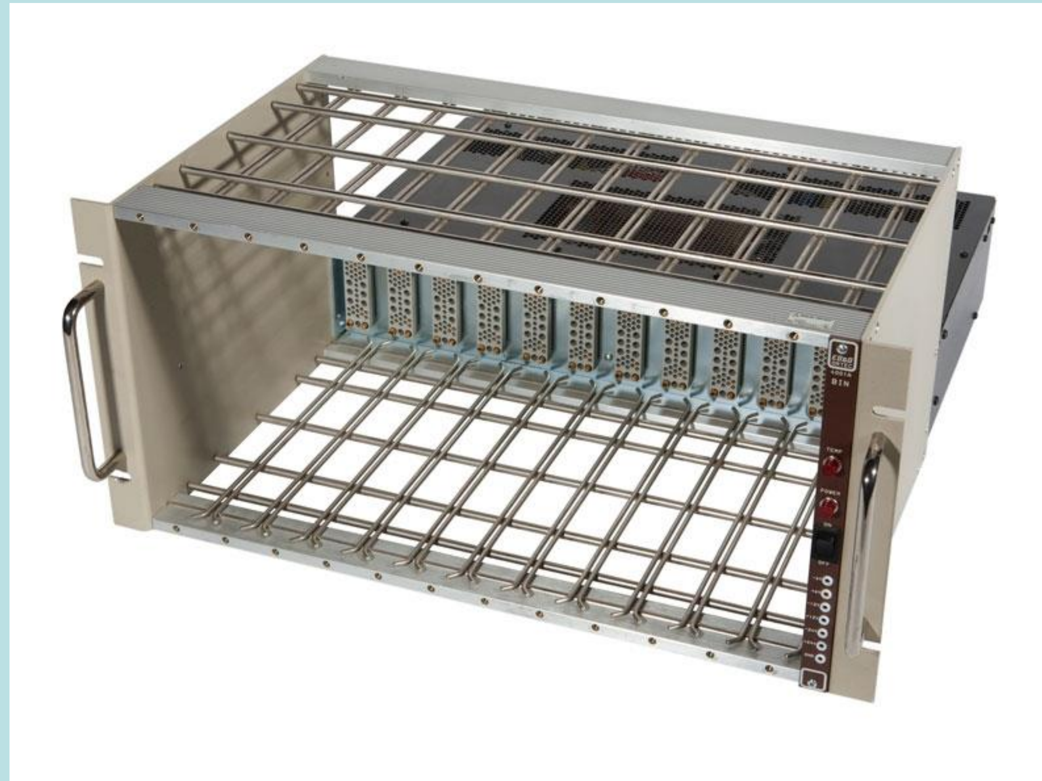


From TDK
web page

Switching regulator power supply

Molecular beam epitaxy
Control panel

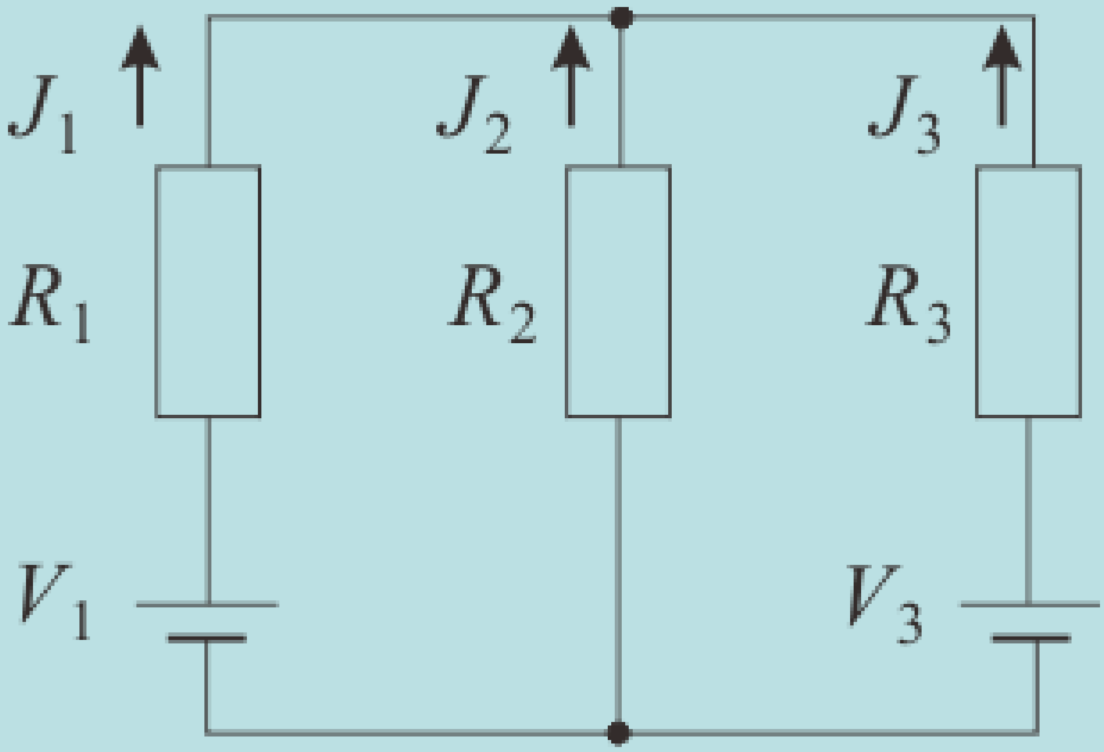
Bin 電源ビン



Complicated power lines ⟶ Bin

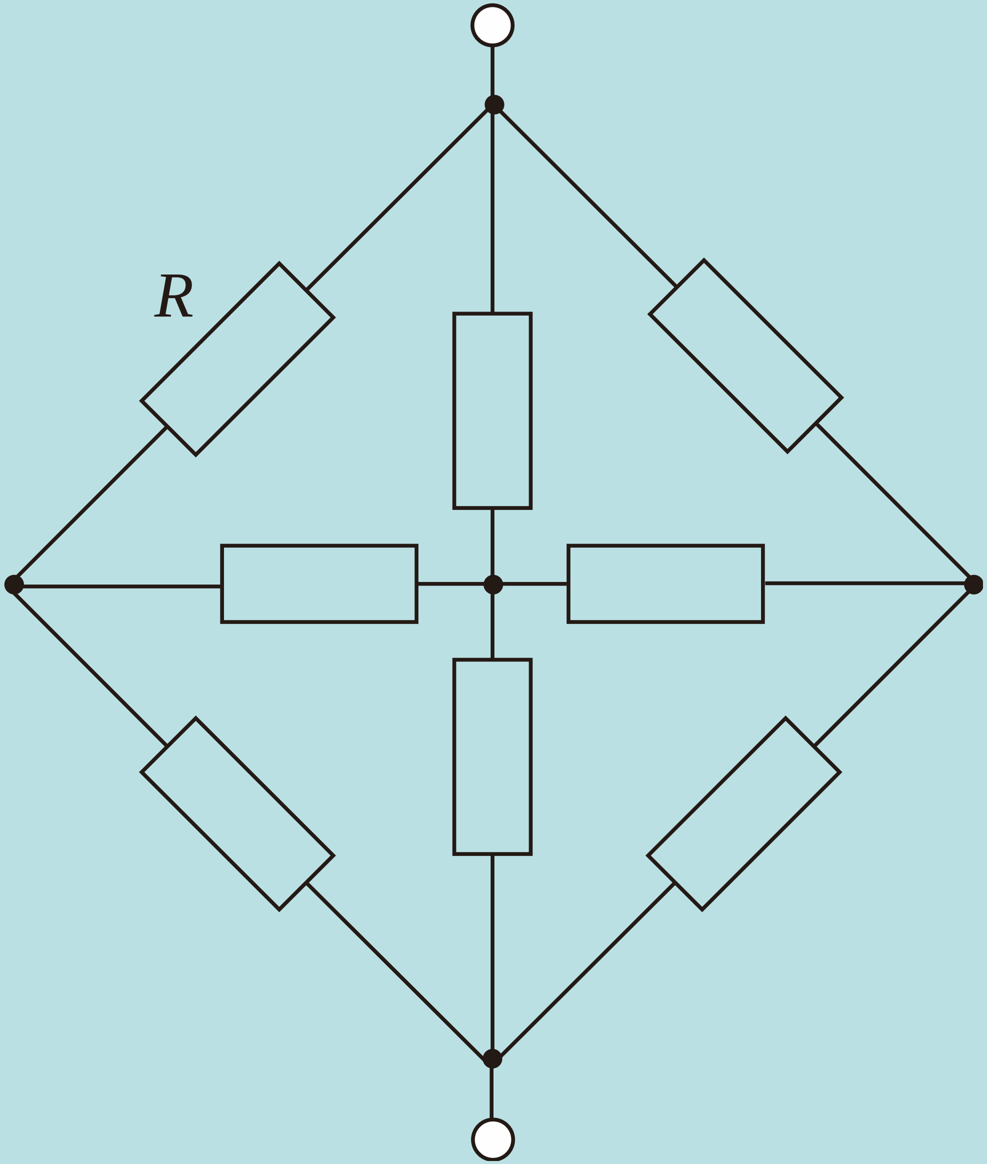
Exercise A-1

Express J_1, J_2, J_3 with other parameters.



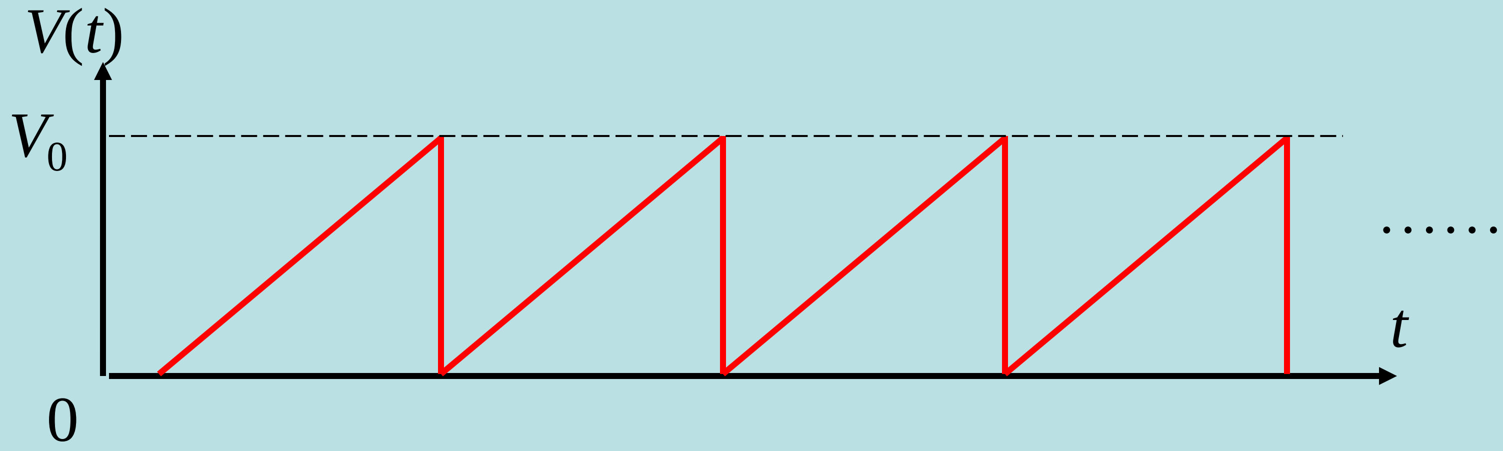
Exercise A-2

All the resistors have the same resistance R. Obtain the combined resistance.



Exercise A-3

Obtain the effective value of voltage for the saw tooth wave.



電子回路論第3回
Electric Circuits for Physicists

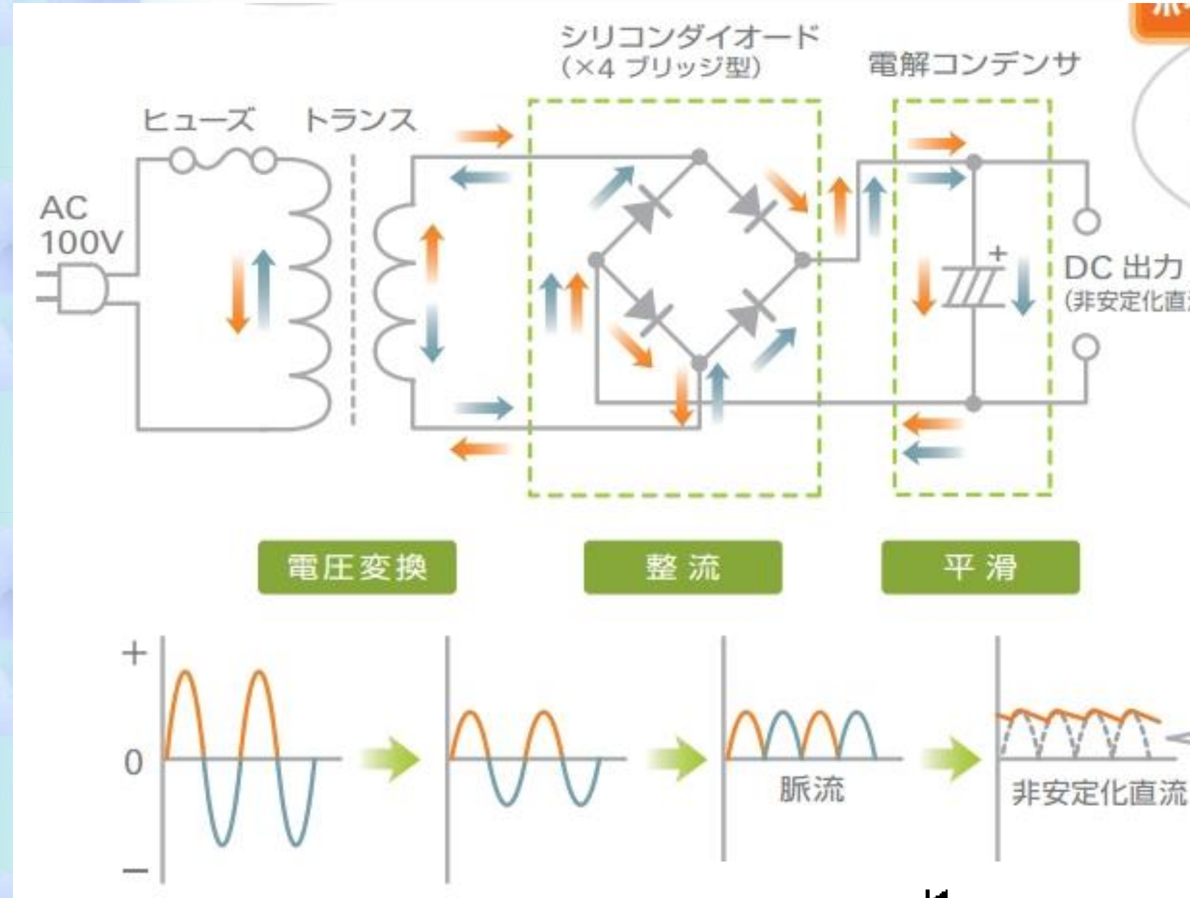
東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

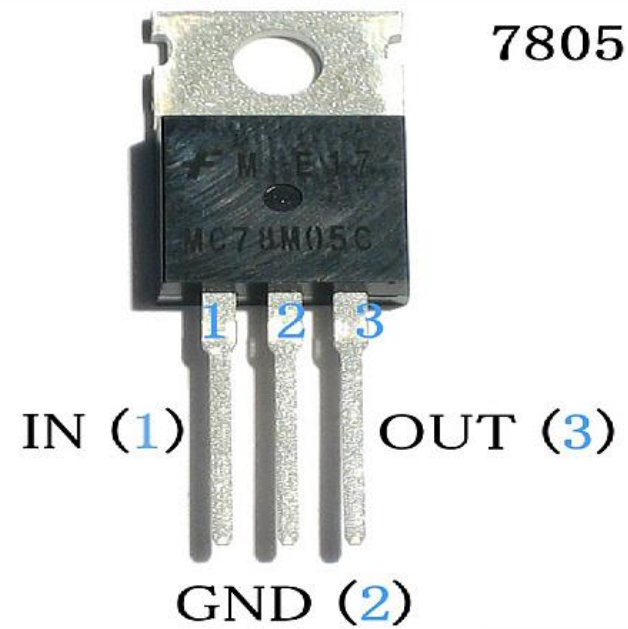
電源の雑知識 (続き）

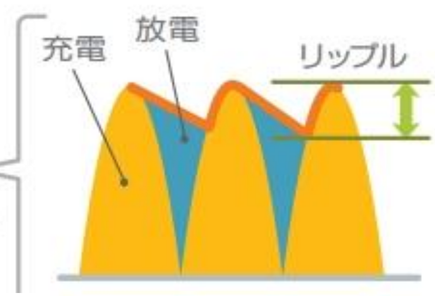
Miscellaneous knowledge on power supplies (continued)

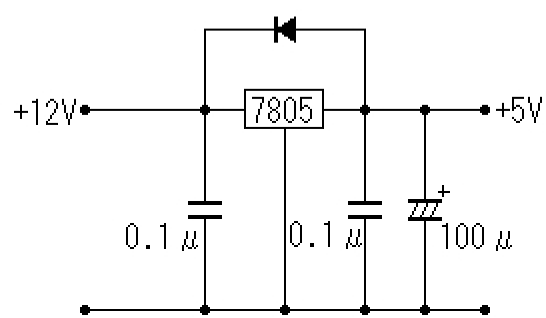
DC Stabilized Power Supply 直流安定化電源

Series (Dropper) regulation









From TDK web page

Series regulator power supply

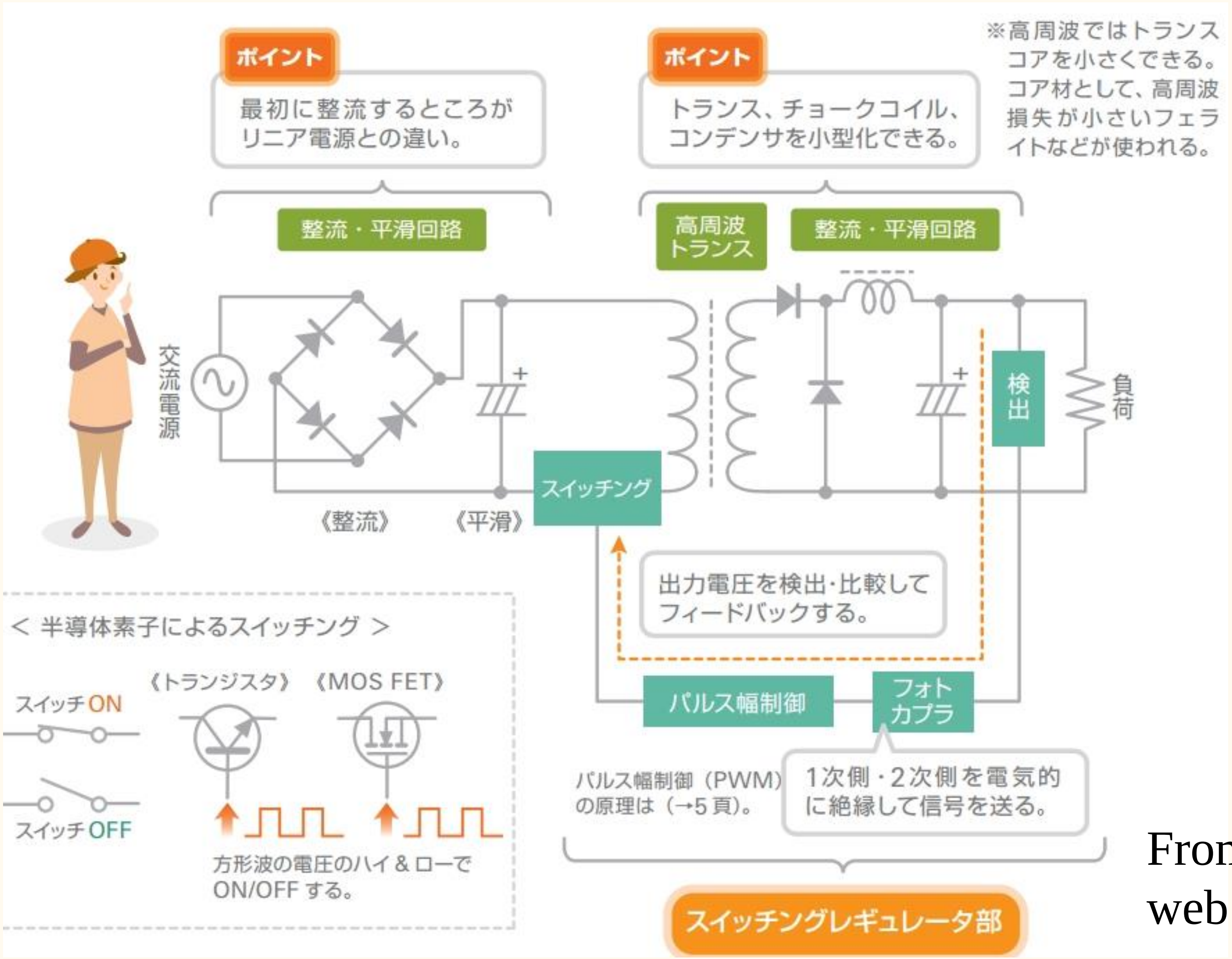
Uni-polar

Dual tracking

High precision

Bi-polar current source

Switching regulation

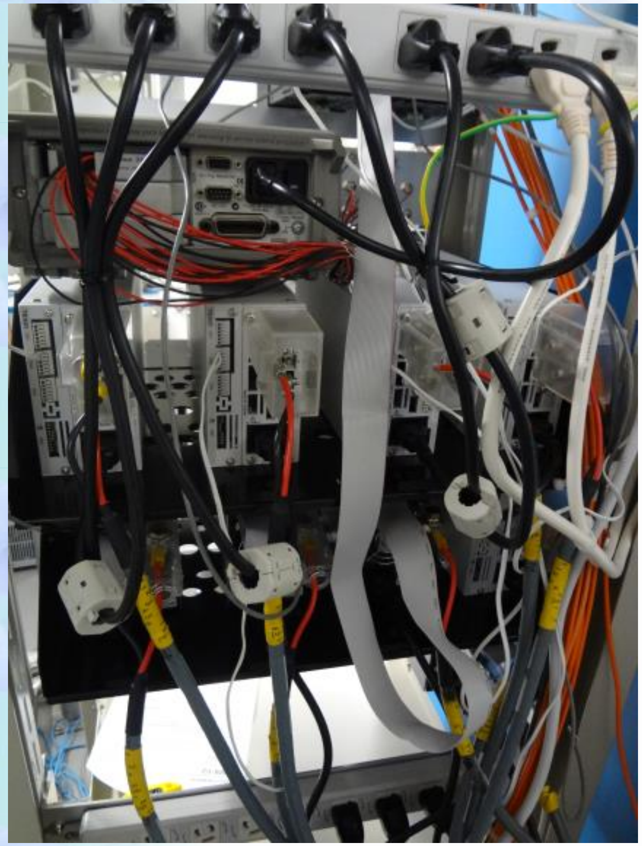


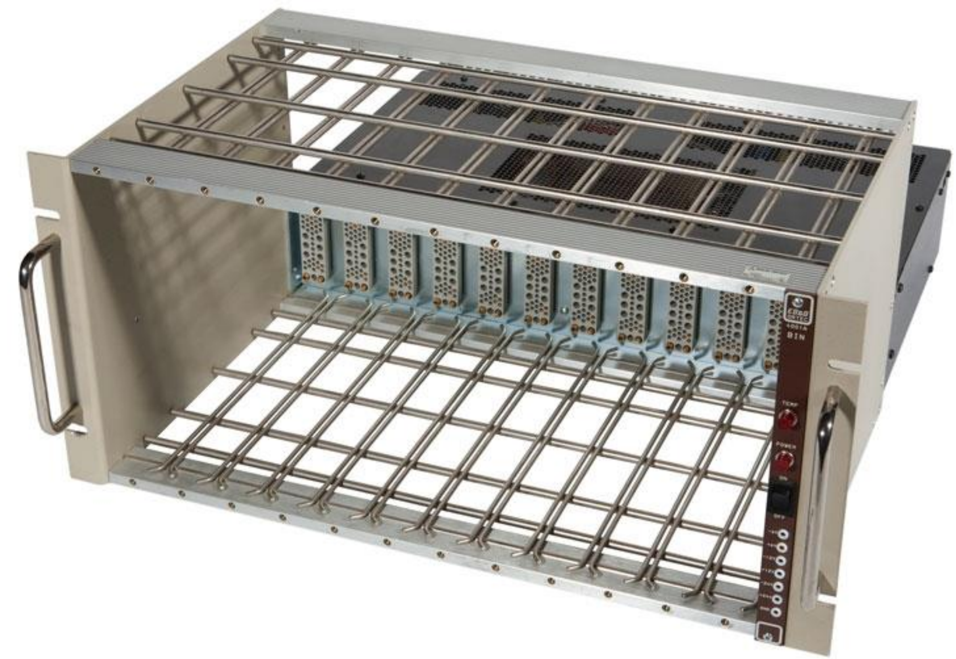
From TDK web page

Switching regulator power supply

Molecular beam epitaxy
Control panel

Bin 電源ビン





Complicated power lines ⟶ Bin

Outline Today

2.5 Theorems for paired terminal circuits

Superposition, Ho-Tevenin, Reciprocity

2.6 Duality

2.7 Passive devices (elements) and active devices

Ch.3 Transfer function and transient response

3.1 Transfer function of single-pair terminal circuits

Resonance circuit

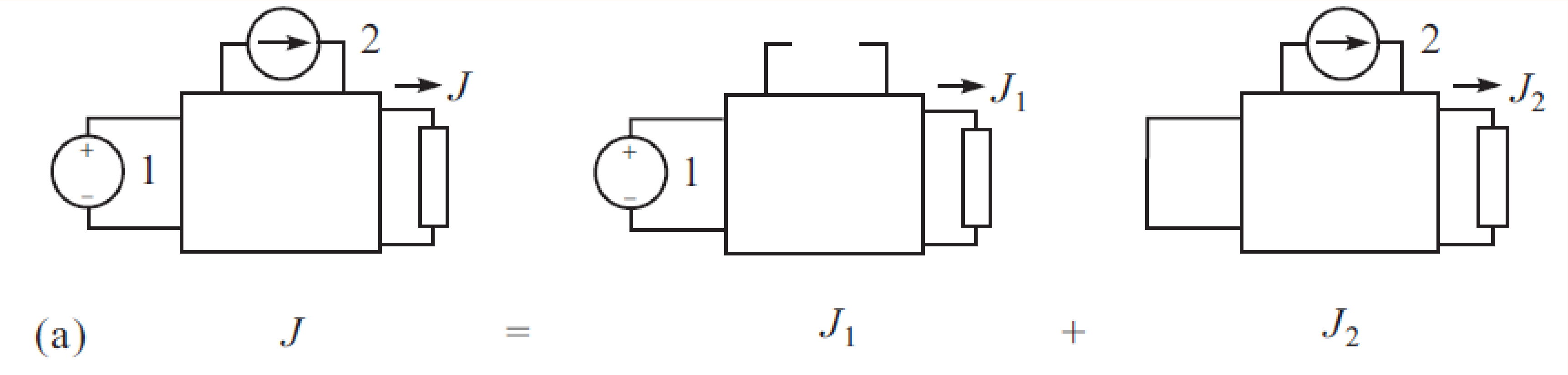
Bode plot

General properties

Appendix B Bridges and balance circuits

Appendix C General properties of resonance circuits

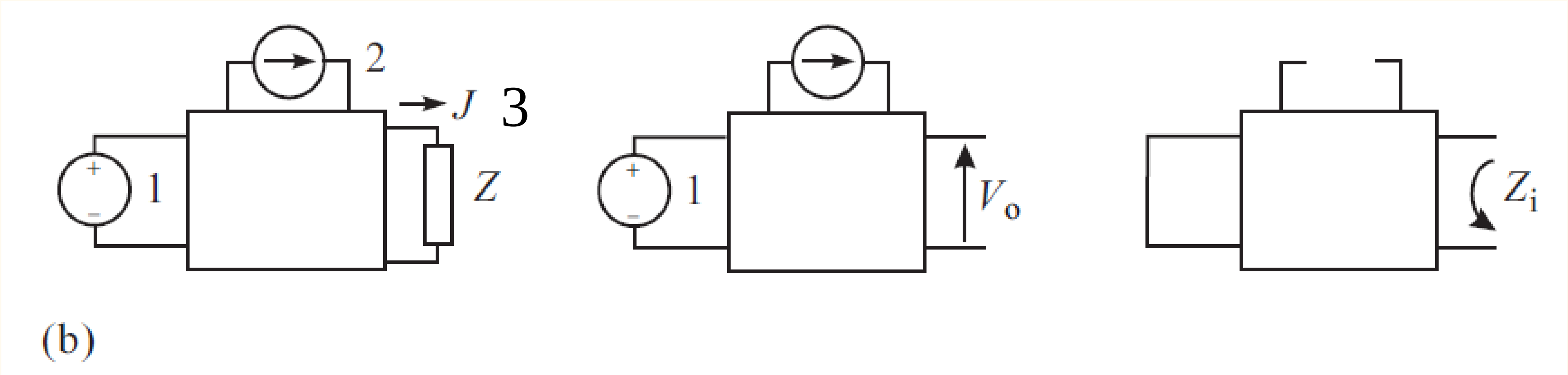
Superposition theorem



$$J = \sum_i J_i$$

J_i: The current caused by *i*-th power source.

Ho-Thevenin's theorem



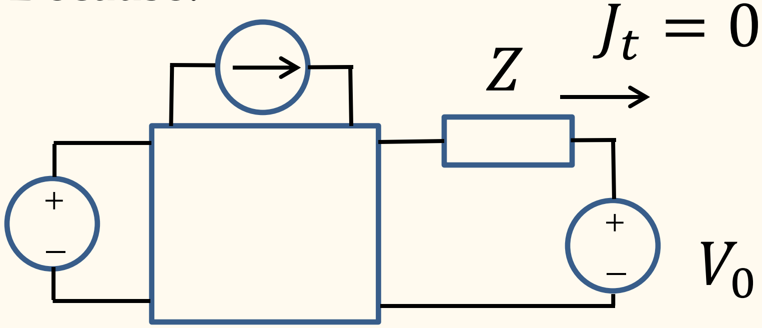
Consider a circuit with an open terminal pair (No.3). Obtain current J when the open pair is connected with impedance Z.

1. Measure the open terminal voltage V_0.
2. Turn off all the power sources (voltage sources: short, current sources: open). Measure the open circuit impedance Z_i.

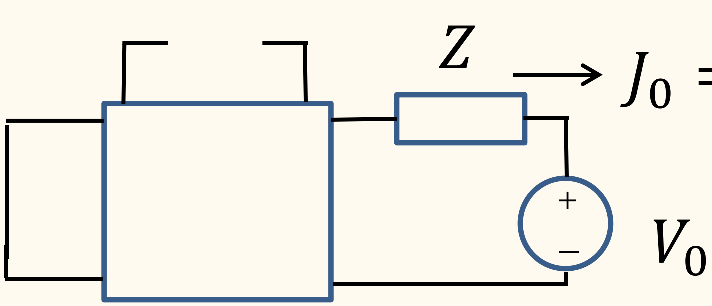
Then

$$J = \frac{V_0}{Z + Z_i}$$

Because:



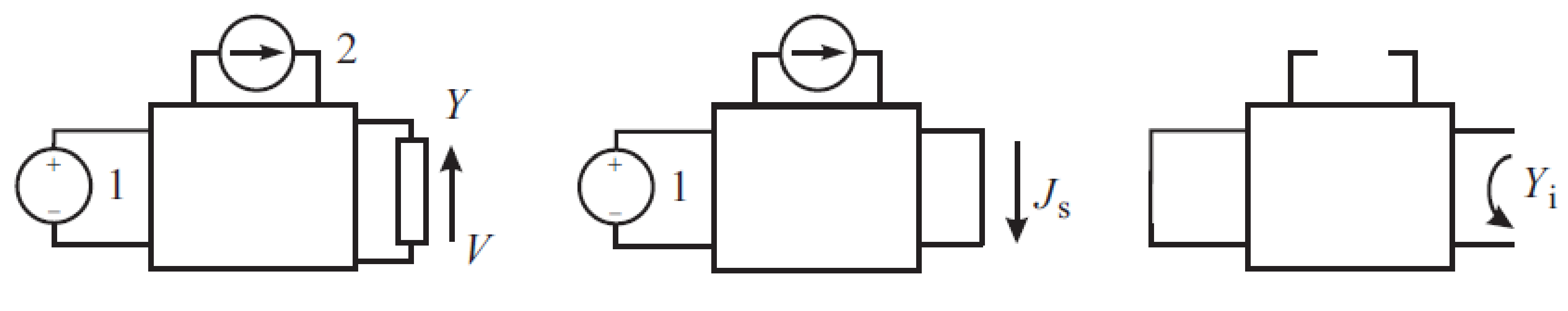
$$J_t = 0$$



$$J_0 = -\frac{V_0}{Z + Z_i}$$

$$J = J_t - J_0 = \frac{V_0}{Z + Z_i}$$

Norton's theorem



$$V = \frac{J_s}{Y + Y_i}$$

Dual theorem for Ho-Tevenin.

Comments on Tellegen's theorem

$i = 1, \cdots, n$: index of nodes, $j = 1, \cdots, m$: index of branches

$$a_{ij} = \begin{cases} 1 : & i \text{ is the start of } j, \\ -1 : & i \text{ is the end of } j, \\ 0 : & \text{others} \end{cases}$$

incidence matrix

redundancy $\rightarrow (n-1) \times m$ matrix D : irreducible incidence matrix

J_j, V_j : current and voltage along branch j, W_i : potential of node i.

Kirchhoff's first law: $D\boldsymbol{J} = 0$ Second law: $\boldsymbol{V} = {}^t\mathcal{D}\boldsymbol{W}$

$$\sum_{i=1}^{m} V_i J_i = ({}^t\mathcal{D}\boldsymbol{W}) \cdot \boldsymbol{J} = {}^t\boldsymbol{W}\mathcal{D}\boldsymbol{J} = 0 \qquad \boldsymbol{V} \perp \boldsymbol{J}$$

($V_i J_i$:) Power of i-th branch

Comments

1. Power conservation law
2. Holds for any kind of circuit (irrespective of linear, or non-linear)
3. Holds for two independent circuit conditions (as long as D is the same)

Reciprocity theorem

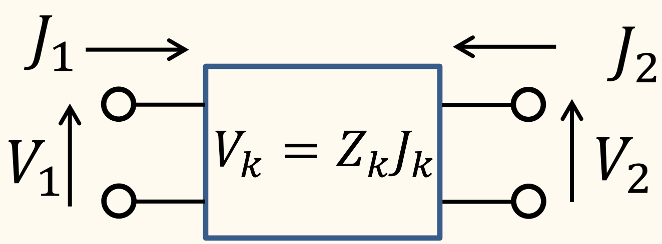
An n-terminal pair linear circuit

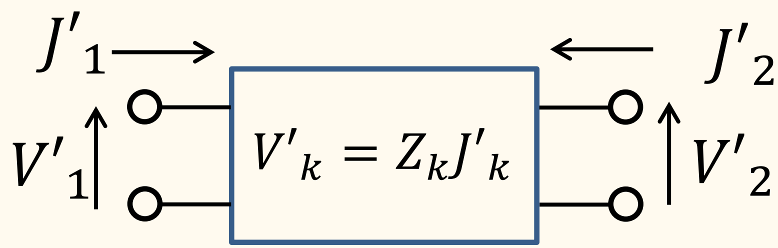
At one state $(V_1, J_1), (V_2, J_2), \cdots, (V_n, J_n)$,

at another state $(V'_1, J'_1), (V'_2, J'_2), \cdots, (V'_n, J'_n)$

$$\sum_{i=1}^{n} V_i J'_i = \sum_{i=1}^{n} V'_i J_i$$

Proof: Consider a two terminal-pair circuit with m branches.





Tellegen's theorem
(and comment no.3)

$$-V_1 J'_1 - V_2 J'_2 + \sum_k V_k J'_k = 0$$

$$-V'_1 J_1 - V'_2 J_2 + \sum_k V'_k J_k = 0$$

$$V_k J'_k = V'_k J_k = Z_k J_k J'_k$$

$$\therefore V_1 J'_1 + V_2 J'_2 = V'_1 J_1 + V'_2 J_2 \quad //$$

(This also holds for circuits with mutual inductances.)

2.6 Duality 双対性

直列接続	並列接続
開放	短絡
電場	磁場
キルヒホッフの第 2 法則	キルヒホッフの第 1 法則
電圧	電流
インピーダンス	アドミッタンス
抵抗	コンダクタンス
静電容量	インダクタンス
鳳-テブナンの定理	ノートンの定理

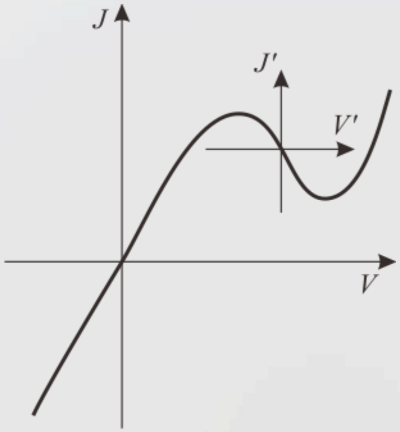
2.6 Duality

Series	**Parallel**
Open	Short
Voltage	Current
Impedance	Admittance
Capacitance	Inductance
Electric field	Magnetic field
Resistance	Conductance
Ho-Thevenin	Norton
Kirchhoff's 2nd law	Kirchhoff's 1st law

2.7 Definition: Passive elements and active elements

Two terminal: current J, voltage V

$JV \geq 0$: passive element
$JV < 0$: active element



Locally active two-terminal element

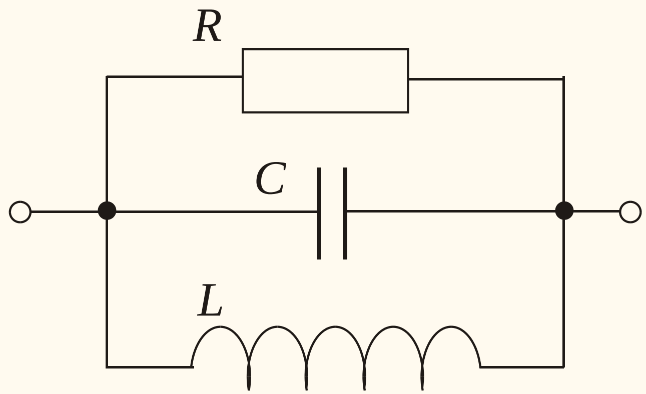
More than three-terminal: treat as a terminal pair circuit

$$P = J_{\text{in}}V_{\text{in}} + J_{\text{out}}V_{\text{out}}$$

$P \geq 0$: passive element
$P < 0$: active element

Ch.3 Transfer function and transient response

3.1.1 Resonance Phenomena



Harmonic oscillator: $\dfrac{d^2q}{dt^2} = -\omega_0^2 q$

Kirchhoff's law

$$L\frac{dJ_{\rm L}}{dt} = -L\frac{d^2q_{\rm L}}{dt^2} = \frac{q}{C} = RJ_{\rm R} = R\frac{dq_{\rm R}}{dt}$$

$$dq_{\rm L} + dq_{\rm R} + dq = 0$$

$$\frac{d^2q}{dt^2} + \frac{1}{CR}\frac{dq}{dt} + \frac{1}{LC}q = \frac{d^2q}{dt^2} + \frac{1}{\tau}\frac{dq}{dt} + \omega_0^2 q = 0$$

$$q = \exp(\lambda t) \qquad \lambda = \frac{1}{2\tau}\left[-1 \pm \sqrt{1 - 4(\omega_0\tau)^2}\right] \approx -\frac{1}{2\tau} \pm i\omega_0 \quad (\omega_0\tau \gg 1)$$

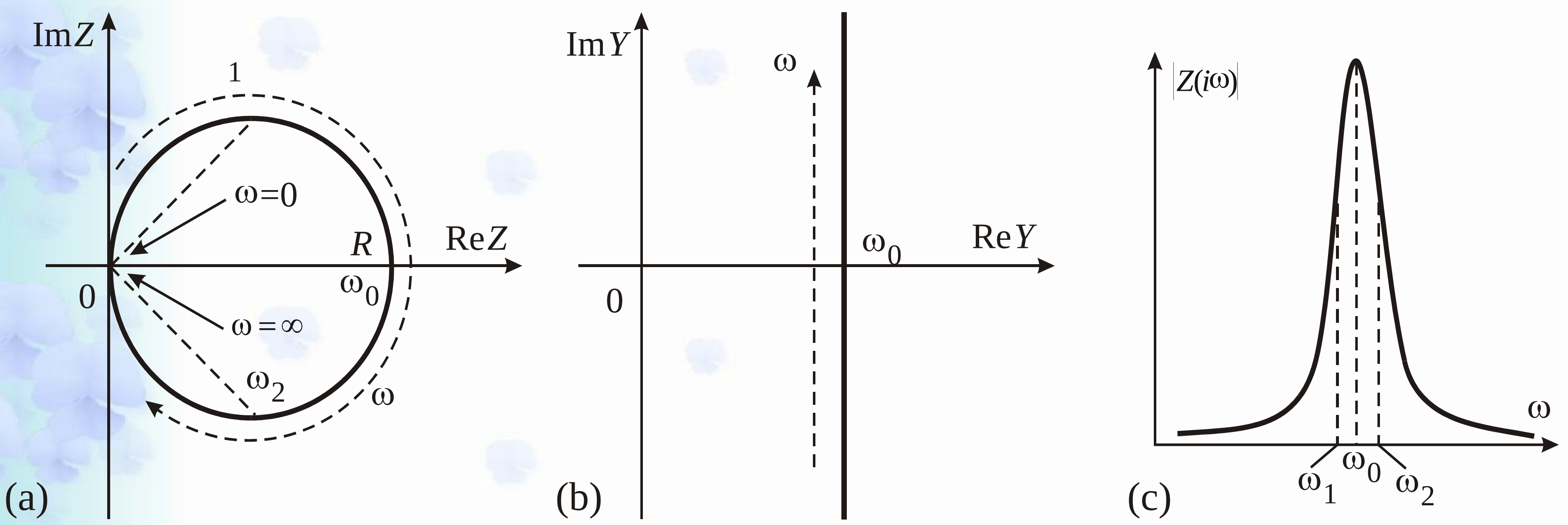
Resonant (angular) frequency $\omega_0 \equiv \dfrac{1}{\sqrt{LC}}$

Transfer function, resonance and phase shift

$$Z_{\text{tot}}(i\omega) = \left[\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)\right]^{-1} \qquad \omega_0 \equiv \frac{1}{\sqrt{LC}}$$

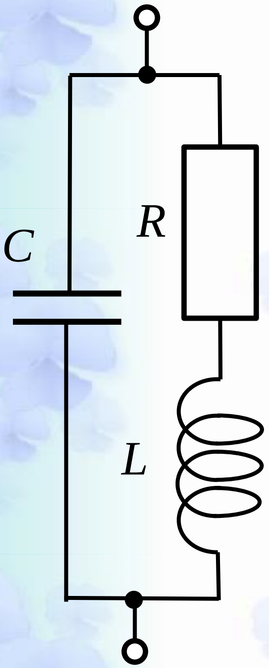
Resonance: Reactance =0

Total Phase Shift Change: π

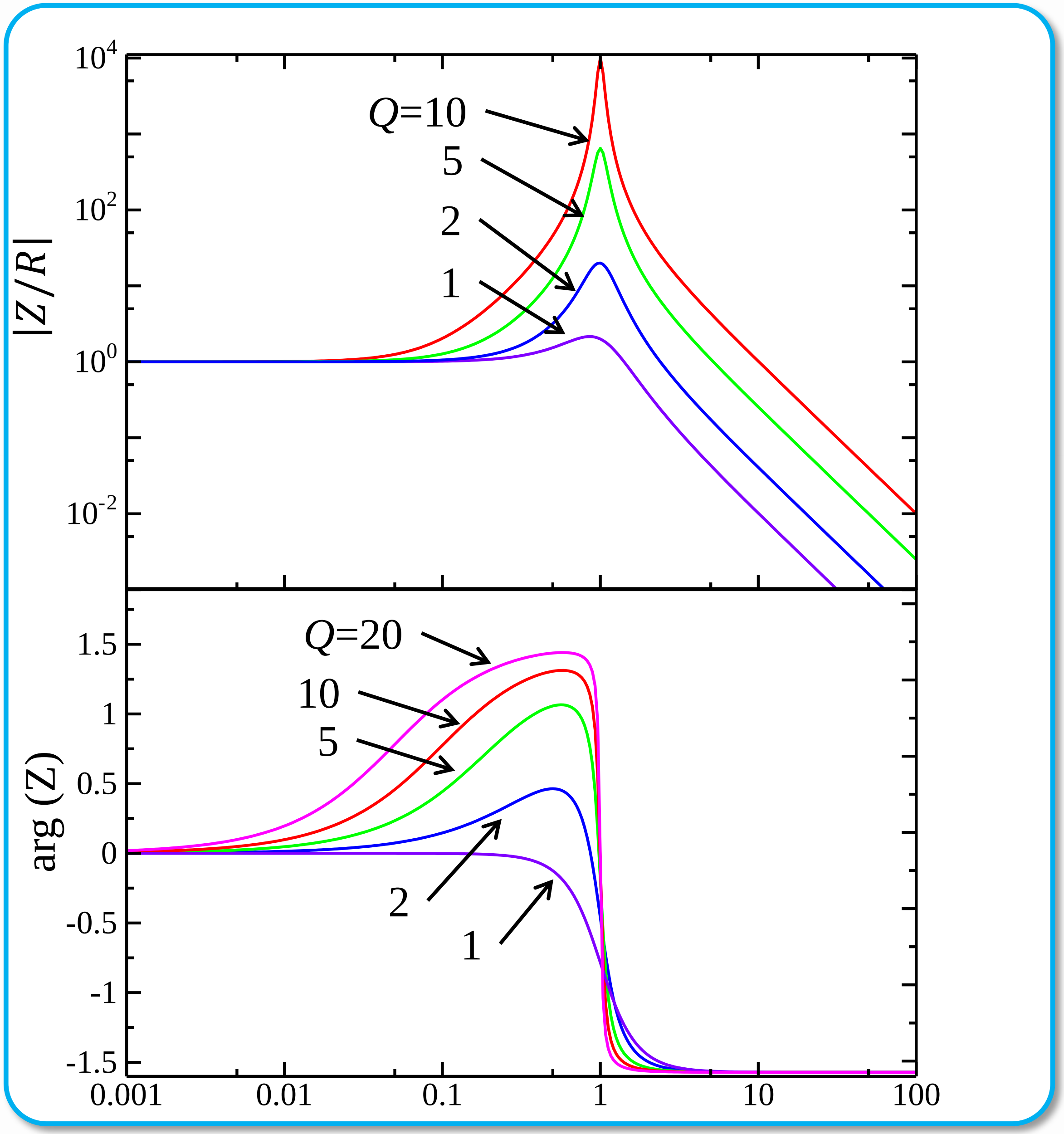


Bode diagram

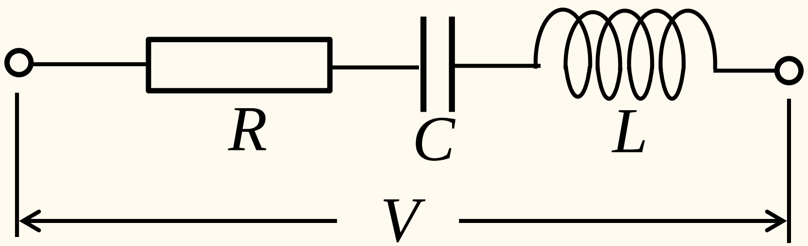
$$Z(i\omega) = \frac{R + i\omega L}{1 - \omega^2 LC + i\omega CR} = \frac{R + i\omega L}{1 - \frac{\omega^2}{\omega_0^2} + i\omega CR}$$

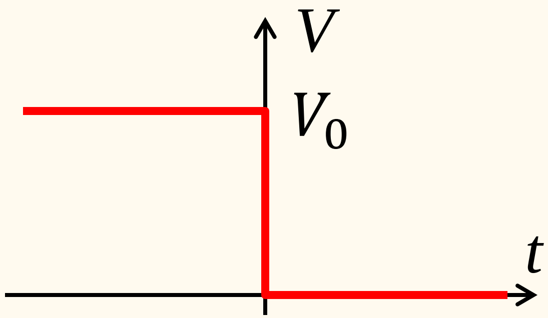


$$Q \approx \omega_0 \frac{L}{R}$$



Transient response of resonant circuit





$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0\quad(t>0),\quad q(0)=CV_0$$

$$q(t)=CV_0e^{st}\ \rightarrow\ Ls^2+Rs+C^{-1}=0$$

$$s=(-\omega_0\pm\sqrt{\omega_0^2-4\alpha^2}\,)(\omega_0/2\alpha)\quad \alpha\equiv(CR)^{-1}$$

$$(\omega_0/2)<\alpha\ \rightarrow\ \text{imaginary part}$$

$$q(t)=CV_0\exp[(-\gamma\pm i\omega_s)t],\quad \gamma\equiv\frac{\omega_0^2}{2\alpha},\quad \omega_s\equiv\omega_0\left(1-\frac{\omega_0^2}{4\alpha^2}\right)^{1/2}$$

Damped oscillation with time constant γ^{-1}, frequency ω_s

Transient response of resonance circuit (transfer function)

Synthesized impedance, admittance

$$Z_{\rm tot}(s) = sL + R + \frac{1}{sC}, \quad Y_{\rm tot}(s) = Z_{\rm tot}(s)^{-1}$$

Zero (pole) of $Z_{\rm tot}(s)$ ($Y_{\rm tot}(s)$) $s = \left(-\omega_0 \pm \sqrt{\omega_0^2 - 4\alpha^2}\,\right)(\omega_0/2\alpha)$

$Z_{\rm tot}(s_0) = 0$ Time constant: Re(s_0) Frequency: Im(s_0)

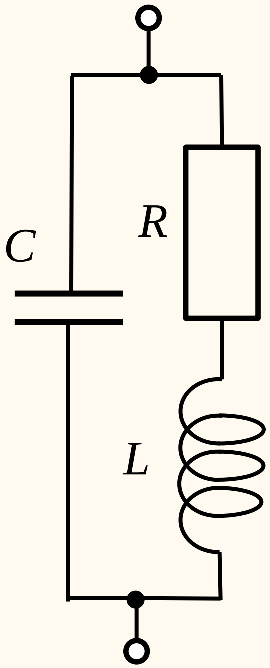
Laplace transformation of voltage: $V(s)$

$$J(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} Y(s)V(s)e^{st}ds = \sum_i R(s_i)V(s_i)e^{s_i t} \quad (c > 0)$$

Natural current

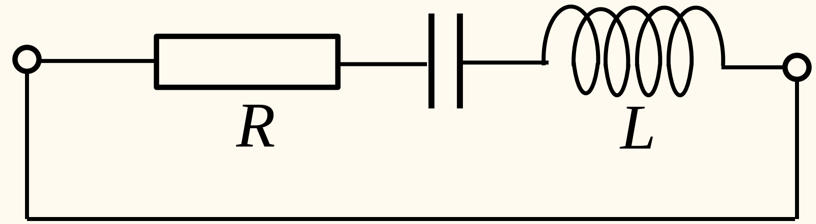
s_i: poles of $Y(s)$ $R(s_i) = Y(s)(s - s_i)|_{s=s_i}$

Driving point impedance



Open

=



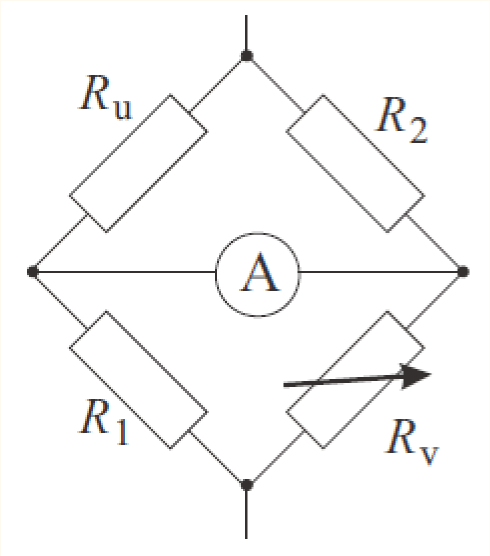
Short

$$Z_{\text{tot}}(s) = sL + R + \frac{1}{sC}$$

$$Z^{(2)}_{\text{tot}}(s) = \left(\frac{1}{R + sL} + sC\right)^{-1} = \frac{sL + R}{s^2LC + sRC + 1}$$

$$Z_{\text{tot}}(s) \text{ zero is pole for } Z^{(2)}_{\text{tot}}(s)$$

Resistance bridge 抵抗ブリッジ

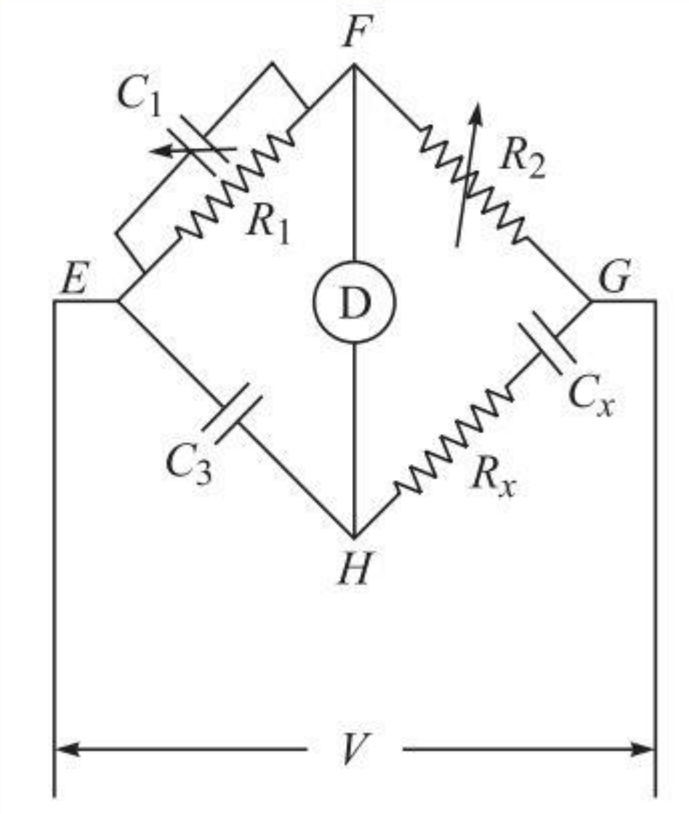


Wheatstone bridge

AVS-47 Resistance bridge

Not a "bridge" circuit!

Schering Bridge



$$Z_1 Z_x = Z_2 Z_3, \quad Z_x = Z_2 Z_3 Y_1$$

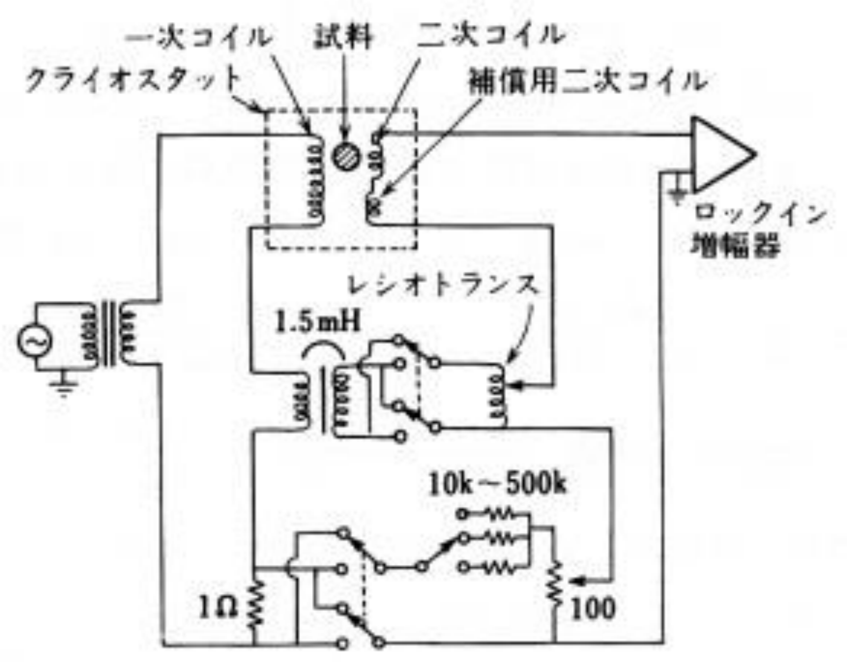
$$Z_x = R_x + \frac{1}{i\omega C_x}, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{i\omega C_3}, \quad Y_1 = \frac{1}{R_1} + i\omega C_1$$

$$R_x + \frac{1}{i\omega C_x} = R_2 \frac{1}{i\omega C_3}\left(\frac{1}{R_1} + i\omega C_1\right)$$

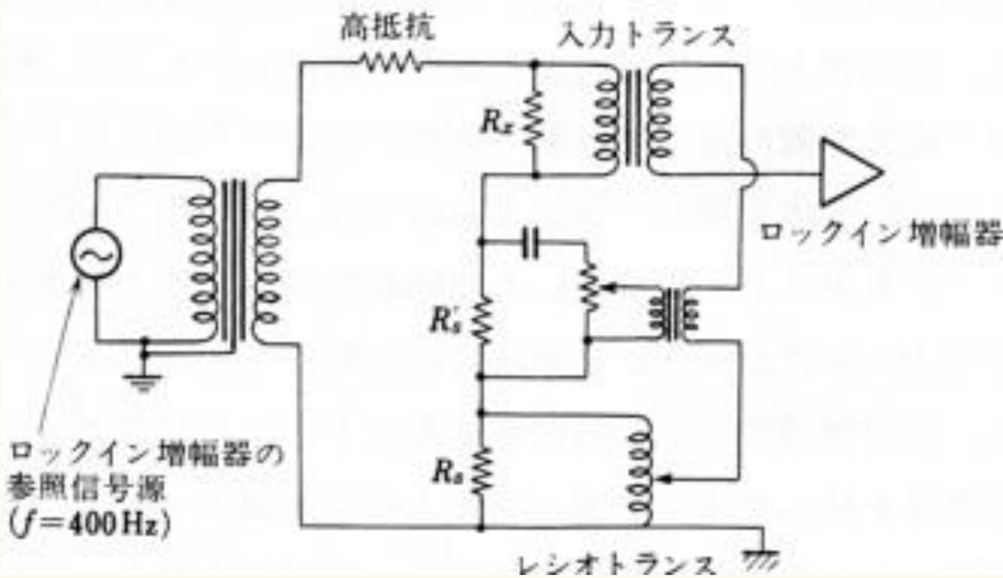
$$R_x = \frac{R_2 C_1}{C_3}, \quad C_x = \frac{R_1}{R_2} C_3$$

Hartshorn bridge

Magnetic moment measurement



Resistance measurement



Capacitance bridge キャパシタンス ブリッジ

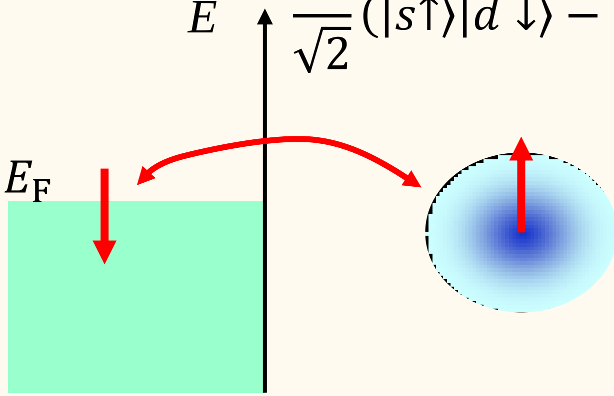
General Radio
3-terminal
Capacitance bridge

Agilent E4981A

Kondo Resonance and Phase shift

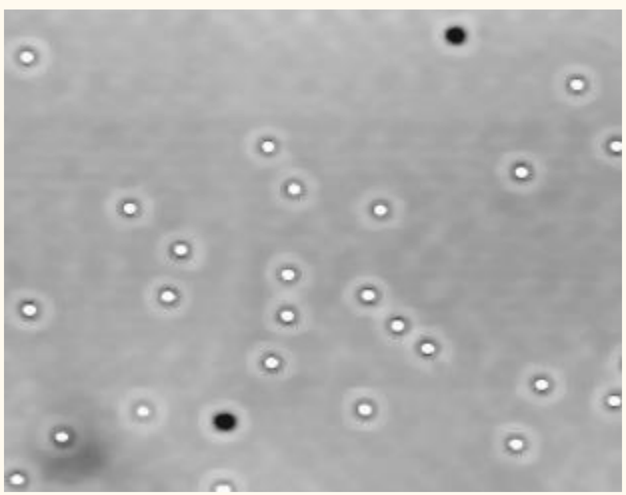
Jun Kondo

$$\frac{1}{\sqrt{2}}\left(|s\uparrow\rangle|d\downarrow\rangle - |s\downarrow\rangle|d\uparrow\rangle\right)$$

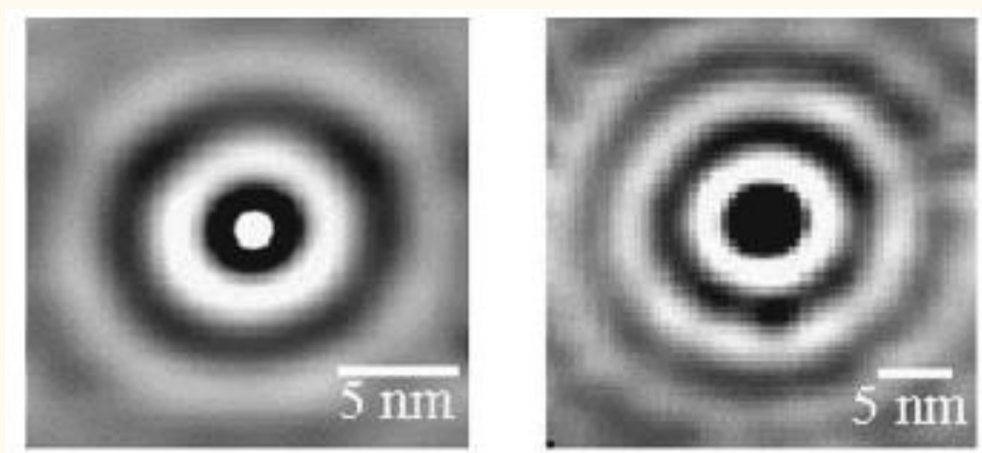


Many body resonance.
But still has the phase shift of $\pi/2$!

Co atoms on Ag (111) surface



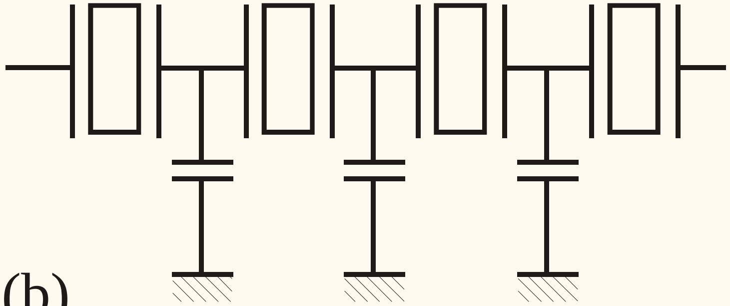
Co (magnetic) Defect (non-magnetic)



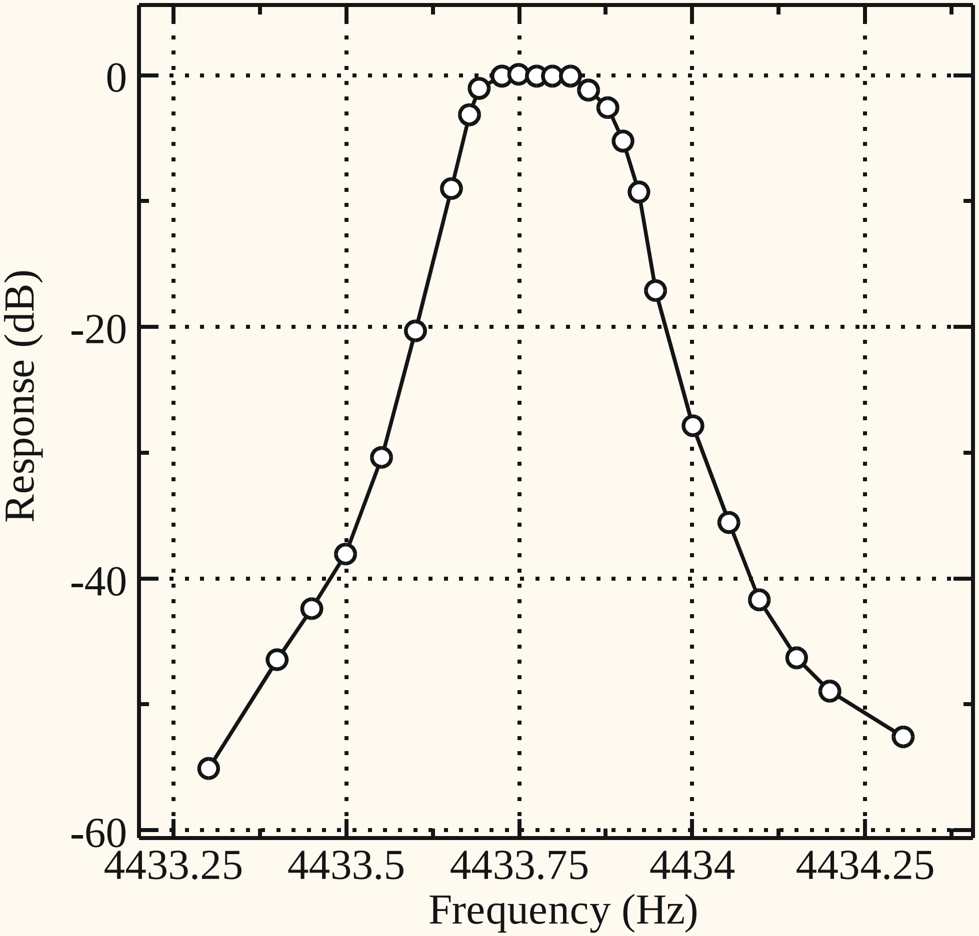
Schneider et al., Phys. Rev. B65, 121406 (2002).

Quartz crystal filter

(a)



(b)



(c)

Circuit Simulator

Download LTSpice from the web site of Linear Technology

What is Spice?

SPICE: Simulation Program with Integrated Circuit Emphasis

A language which describes electronic circuits (corresponding to circuit diagrams).

ex) a CR circuit and a dc power source

```
* 0---R1---1---C1---2---V1---0
R1 0 1 10
C1 1 2 20
V1 2 0 5
.END
```

Graphical user interface: Circuit diagram

Linear Technology
web site

Operation example

TECHNOLOGY

国内ニュースサイト ENGLISH 中文网站 品質 採用 問い合わせ

製品 | ソリューション | デザインサポート | 購入 | 会社

Home > デザインサポート > ソフトウェア

Design Simulation and Device Models

リニアテクノロジーは高性能なスイッチング・レギュレータやアンプ、データ・コンバータ、フィルターなどを使用した回路を、初めての設計者でも短時間に容易に評価できるよ
ムデザイン・シミュレーション・ツールを提供しています。

- LTspice IV
- LTpowerCAD
- LTpowerPlay
- Amplifier Simulation & Design
- Filter Simulation & Design
- Timing Simulation & Design
- Data Converter Evaluation Software
- Dust Networks Starter Kits

LTSPICE IV

LTspice IV

LTspice IVは高性能なSpice IIIシミュレータと回路図入力、波形ビューワに改善を加え、スイッチング・レギュレータのシミュレーションを容易にするためのモデルを搭載しています。Spiceの改善により、スイッチング・レギュレータのシミュレーションは、通常のSpiceシミュレータ使用時に比べて著しく高速化され、ほとんどのスイッチング・レギュレータにおいて波形表示をほんの数分で行なうことができます。Spiceとリニアテクノロジーのスイッチング・レギュレータの80%に対応するMacro Model、200を超えるオペアンプ用モデルならびに抵抗、トランジスタ、MOSFETモデルをここからダウンロードできます。

- **LTspice IV（Windows用）をダウンロード（2014年5月5日更新）**
- **・LTspice IV（Mac OS X 10.7+用）をダウンロード**
- 関連情報 & ショートカット
- Mac OS X用ショートカット
- スタート・ガイド
- ユーザ・ガイド（ヘルプ・ファイル参照）
- トランスの使用
- デモ回路集
- セミナーの開催予定を見る

LTspiceのツイッターをフォロー

LTspiceに関するビデオを見る

MYLINEAR ログイン

LTPOWERCAD

Summary

- Theorems for paired terminal circuits
  - Superposition, Ho-Tevenin, Reciprocity
- Duality
- Passive devices (elements) and active devices

Transfer function and transient response

- Transfer function of single-pair terminal circuits
  - Resonance circuit
  - Bode plot
  - General properties

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

電子回路論第４回
Electric Circuits for Physicists

Introduction of useful free software

Circuit Simulator

LTSpice (Linear Technology)

・・とはならない

By トランジスタ技術

Circuit Simulator

Download LTSpice from the web site of Linear Technology

What is Spice?

SPICE: Simulation Program with Integrated Circuit Emphasis

A language which describes electronic circuits (corresponding to circuit diagrams).

ex) a CR circuit and a dc power source

```
* 0---R1---1---C1---2---V1---0
R1 0 1 10
C1 1 2 20
V1 2 0 5
.END
```

Graphical user interface: Circuit diagram

Linear Technology
web site

Newest version
LTSpice XVII !!

Operation example

TECHNOLOGY

国内ニュースサイト　ENGLISH　中文网站　品質　採用　問い合わせ

製品 | ソリューション | デザインサポート | 購入 | 会社

Home > デザインサポート > ソフトウェア

Design Simulation and Device Models

リニアテクノロジーは高性能なスイッチング･レギュレータやアンプ、データ･コンバータ、フィルターなどを使用した回路を、初めての設計者でも短時間に容易に評価できるよう
ムデザイン･シミュレーション･ツールを提供しています。

- LTspice IV
- LTpowerCAD
- LTpowerPlay
- Amplifier Simulation & Design
- Filter Simulation & Design
- Timing Simulation & Design
- Data Converter Evaluation Software
- Dust Networks Starter Kits

LTSPICE IV

LTspice IV

LTspice IVは高性能なSpice IIIシミュレータと回路図入力、波形ビューワに改善を加え、スイッチング･レギュレータのシミュレーションを容易にするためのモデルを搭載しています。Spiceの改善により、スイッチング･レギュレータのシミュレーションは、通常のSpiceシミュレータ使用時に比べて著しく高速化され、ほとんどのスイッチング･レギュレータにおいて波形表示をほんの数分で行なうことができます。Spiceとリニアテクノロジーのスイッチング･レギュレータの80%に対応するMacro Model、200を超えるオペアンプ用モデルならびに抵抗、トランジスタ、MOSFETモデルをここからダウンロードできます。

- **LTspice IV（Windows用）をダウンロード（2014年5月5日更新）**
- **･LTspice IV（Mac OS X 10.7+用）をダウンロード**
- 関連情報 & ショートカット
- Mac OS X用ショートカット
- スタート･ガイド
- ユーザ･ガイド（ヘルプ･ファイル参照）
- トランスの使用
- デモ回路集
- セミナーの開催予定を見る

LTspiceのツイッターをフォロー

LTspiceに関するビデオを見る

MYLINEAR ログイン

LTPOWERCAD

References

For circuit basics:

"Foundations of Analog and Digital Electronic Circuits"

by A. Agarwal and J. H. Lang (Elsevier, 2005)
1000 printed pages!
Ch.1 The Circuit Abstraction

"Teach Yourself Electricity and Electronics" 3rd Ed.

by Stan Gibilisco (McGraw-Hill, 2002) 748 printed pages

"Schaum's outlines: Electric Circuits" 6th ed.

by M. Nahvi, J. A. Edminister (McGraw-Hill, 2014) 504pages

For measurement circuits:

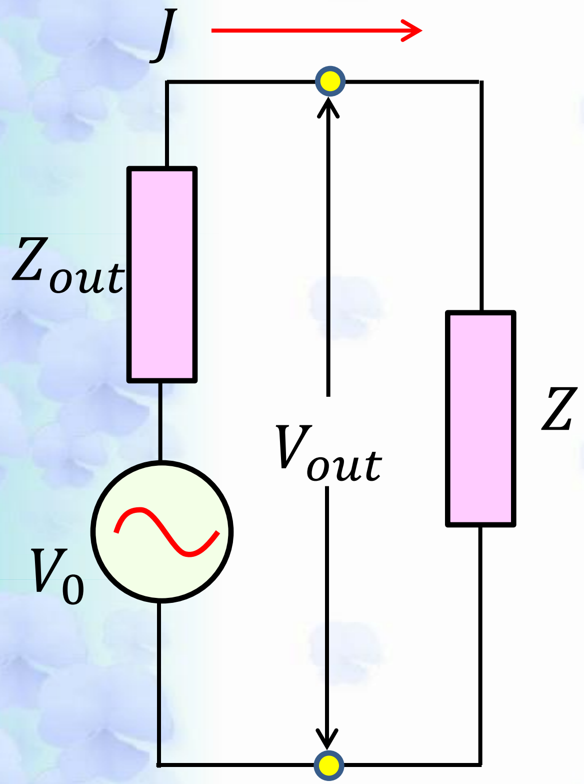
"Electrical and Electronics Measurements"

by G. K. Banerjee (PHI Learning Private, 2012) 835 pages

Outline today

3.2 Two terminal-pair passive circuits
3.2.1 Impedance matching (concept)
3.2.2 Poles and zeros of transfer function and Bode diagram
3.2.3 Image impedance
3.2.4 Impedance matching with terminal-pair circuits
3.2.5 Fidelity and distortion
3.2.6 Filter circuits

Impedance matching



$$V_{\text{out}}(i\omega) = V_0(i\omega) - Z_{\text{out}}(i\omega)J(i\omega)$$

$$P = \text{Re}(V_{\text{out}}^* J) = \text{Re}\left(\frac{Z^* V_0^*}{Z^* + Z_{\text{out}}^*}\frac{V_0}{Z + Z_{\text{out}}}\right)$$

$$= \frac{|V_0|^2}{|Z + Z_{\text{out}}|^2}\text{Re}(Z)$$

Maximum power: $P_{\text{max}} = \dfrac{|V_0|^2}{4\text{Re}(Z_{\text{out}})^2}$

Impedance matching condition: $Z = Z_{\text{out}}^*$

Zeros and Poles of Transfer Functions

$$W(s) = B\frac{(s-\beta_1)\cdots(s-\beta_m)}{(s-\alpha_1)\cdots(s-\alpha_n)}$$

$\{\alpha_j\}$: Poles

$\{\beta_j\}$: Zeros

Bode diagram

$$\log|W(i\omega)| = \log|B| + \sum_{j=1}^{m}\log|(i\omega-\beta_j)| - \sum_{j=1}^{n}\log|(i\omega-\alpha_j)|,$$

$$\arg(W(i\omega)) = \arg(B) + \sum_{j=1}^{m}\arg(i\omega-\beta_j) - \sum_{j=1}^{n}\arg(i\omega-\alpha_j)$$

$$W(s) = \frac{1}{s+1}$$

$$\frac{d\theta}{d(\log\omega)} = -\frac{e^x}{e^{2x}+1}, \quad \frac{d^2\theta}{dx^2} = -\frac{e^x(1-e^{2x})}{(e^{2x}+1)^2}$$

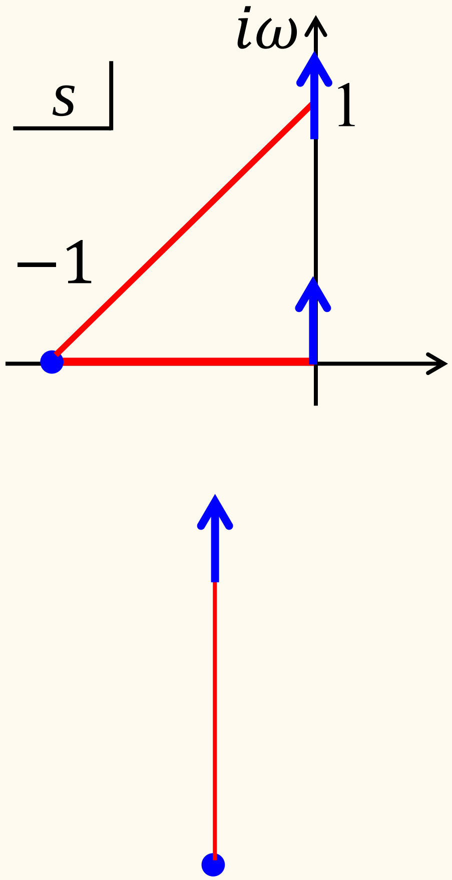
$$\arg[W] = \theta$$

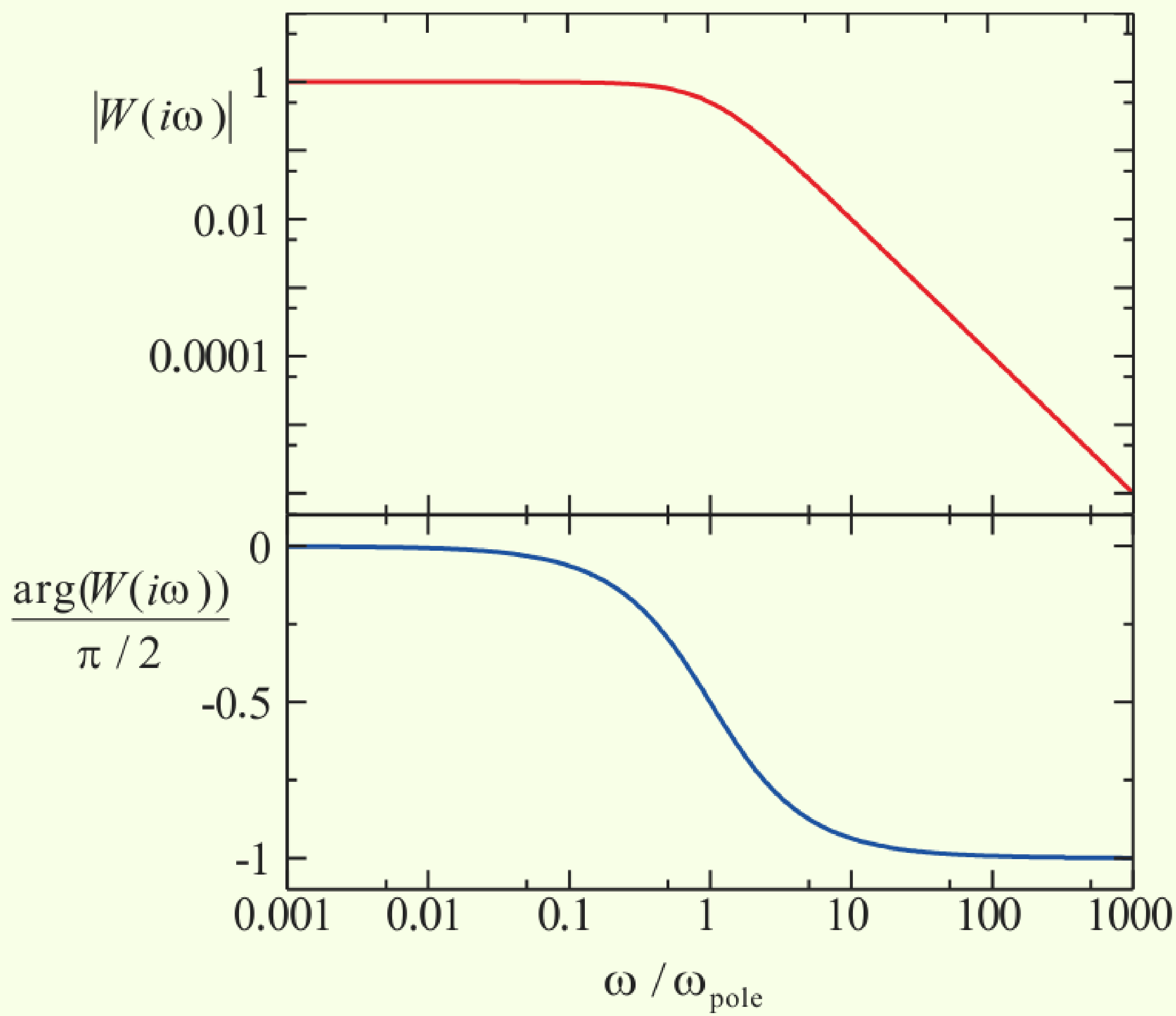
$$\log\omega = x$$

$$\frac{d(\log|W(i\omega)|)}{d(\log\omega)} = -\frac{e^2x}{1+e^{2x}}, \quad \frac{d^2(\log|W|)}{dx^2} = -\frac{2e^{2x}}{(1+e^{2x})^2}$$

Effect of a Pole on the Real Axis for Bode Diagram

$$W(s) = \frac{1}{s+1}$$



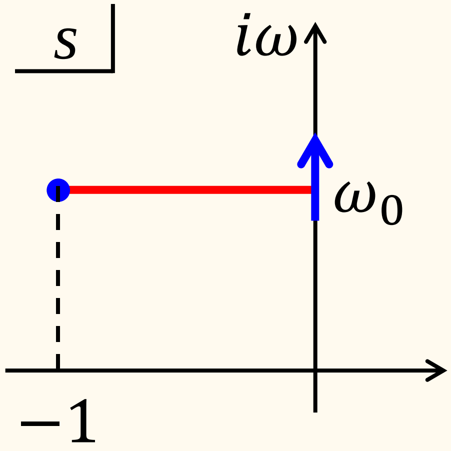


Effect of a Resonance Pole (Finite Imaginary Part)

$$W(s) = \frac{1}{s + 1 - i\omega_0}$$

$(\omega_0 > 0)$

(fake example)



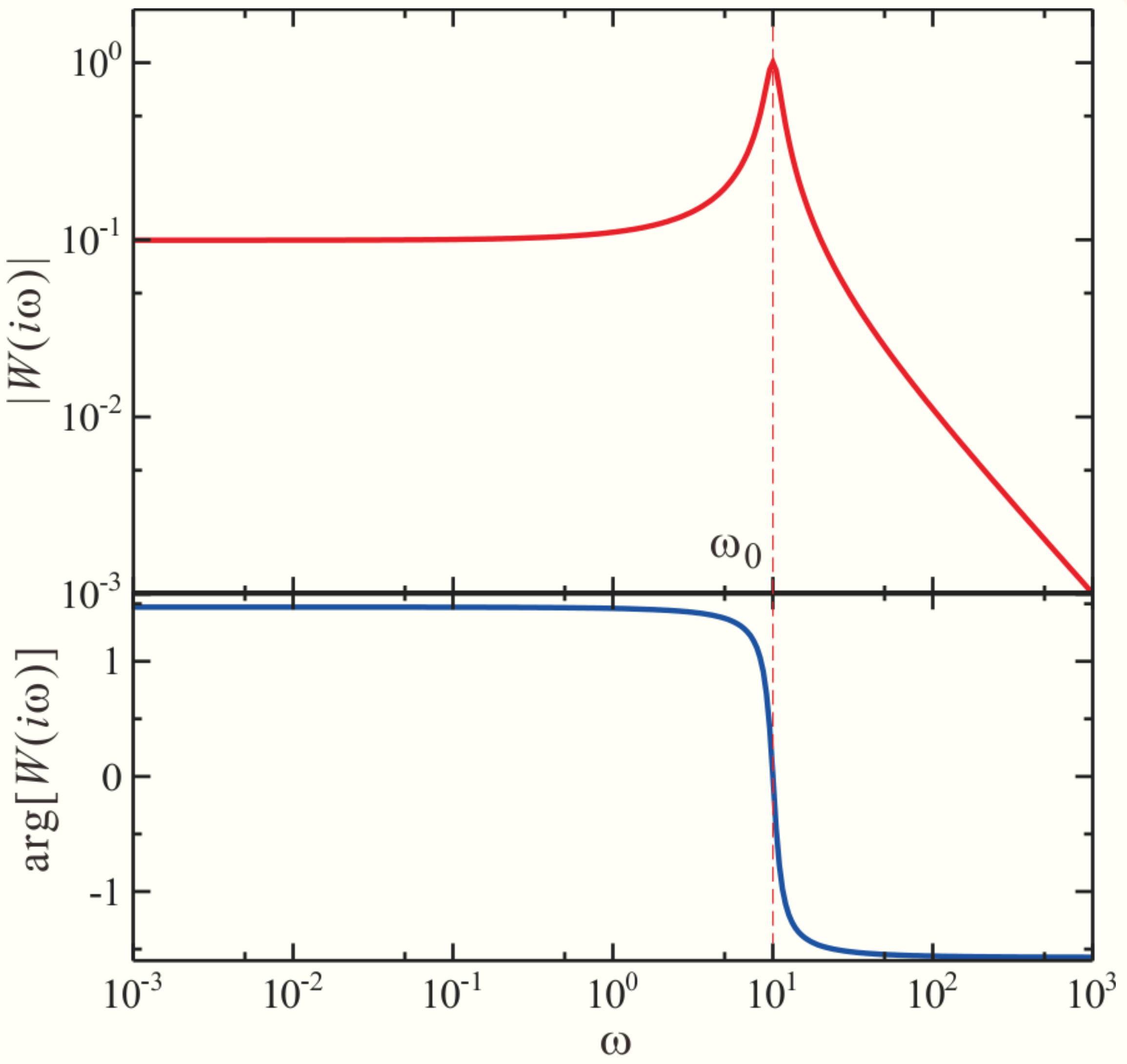
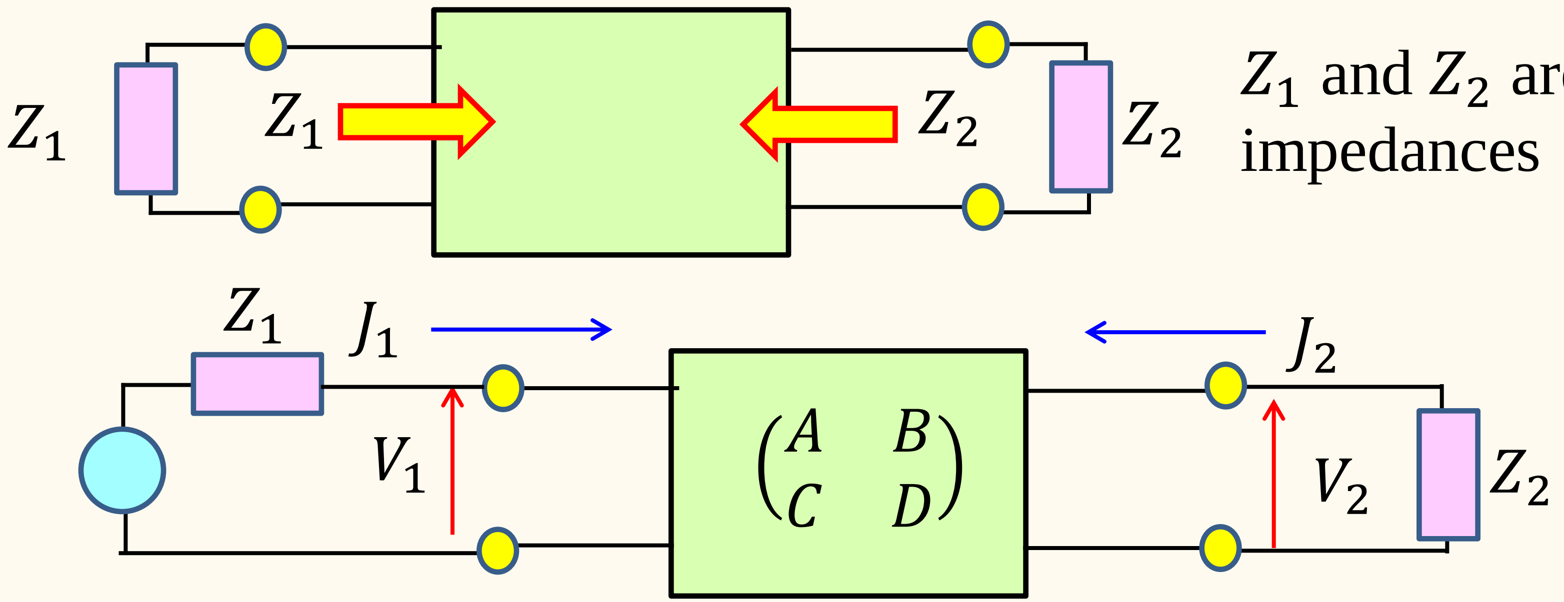


Image parameters



Z_1 and Z_2 are image impedances

$$\begin{cases} V_1 = AV_2 - BJ_2, \\ J_1 = CV_2 - DJ_2 \end{cases}$$

$$V_2 = -J_2 Z_2$$

$$\begin{cases} Z_1 = \dfrac{V_1}{J_1} = \dfrac{AZ_2 + B}{CZ_2 + D} \\ Z_2 = \dfrac{DZ_1 + B}{CZ_1 + A} \end{cases}$$

Image parameters

$$Z_1 = \sqrt{\frac{AB}{CD}}, \quad Z_2 = \sqrt{\frac{DB}{CA}}$$

$$\frac{V_1}{V_2} = \sqrt{\frac{A}{D}}(\sqrt{AD} + \sqrt{BC}), \quad \frac{J_1}{-J_2} = \sqrt{\frac{D}{A}}(\sqrt{AD} + \sqrt{BC})$$

$$e^\theta \equiv \sqrt{\frac{V_1 J_1}{-V_2 J_2}} = \sqrt{\frac{Z_1}{Z_2}} \frac{J_1}{-J_2} = \sqrt{\frac{Z_2}{Z_1}} \frac{V_1}{V_2} = \sqrt{AD} + \sqrt{BC}$$

θ: Image propagation constant

$$\theta = \alpha + i\beta \ \ (\alpha, \beta \in \mathbb{R})$$

$$\alpha = \frac{1}{2} \ln \left| \frac{V_1 J_1}{V_2 J_2} \right|, \quad \beta = \frac{1}{2} \arg \left[\frac{V_1 J_1}{-V_2 J_2} \right]$$

Image attenuation constant   Image phase shift

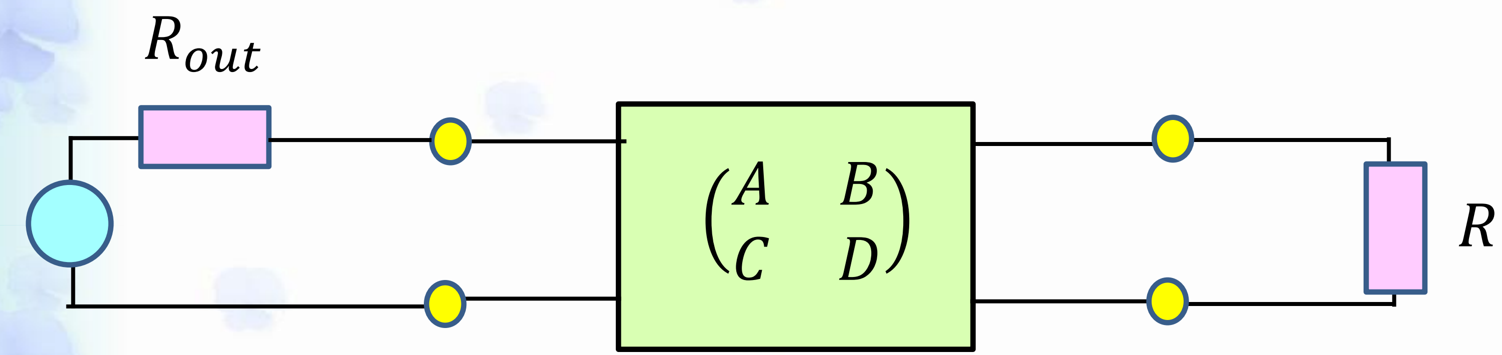
Image parameters

$$A = \sqrt{\frac{Z_1}{Z_2}} \cosh\theta, \quad B = \sqrt{Z_1 Z_2} \sinh\theta,$$

$$C = \frac{1}{\sqrt{Z_1 Z_2}} \sinh\theta, \quad D = \sqrt{\frac{Z_2}{Z_1}} \cosh\theta$$

Z_1, Z_2, θ: Image parameters

Impedance matching with two terminal-pair circuits



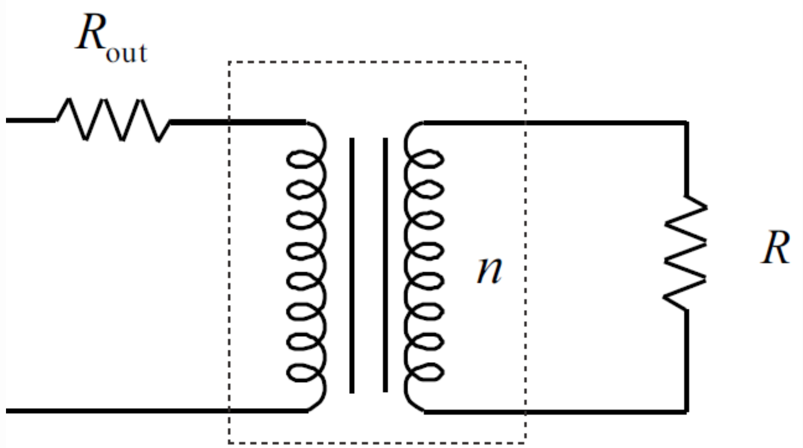
$ABCD \neq 0$ $\quad R_{\text{out}} = \sqrt{\frac{AB}{CD}}, \quad R = \sqrt{\frac{BD}{AC}}$

$A = D = 0$ $\quad R_{\text{out}} = \frac{AR + B}{CR + D}, \quad R = \frac{DR_{\text{out}} + B}{CR_{\text{out}} + A} \to RR_{\text{out}} = B/C$

$B = C = 0$ $\quad R_{\text{out}}/R = A/D$

Matching transformer

$$n = \sqrt{R/R_{\text{out}}}$$



Fidelity and distortion in wave transformation

Linear response: $w(t) = \mathscr{L}\{u(t)\}$

$$w(t) = A_0 u(t - \tau_0) \qquad \therefore W(i\omega) = A_0 e^{-i\omega\tau_0} U(i\omega)$$

$$\Xi(i\omega) = A_0 e^{-i\omega\tau_0}$$

No distortion condition:

$$|\Xi(i\omega)| = A_0, \ \arg[\Xi(i\omega)] = -\omega\tau_0$$

(1) No filter effect

$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega} \qquad (\phi(\omega) = \arg[\Xi(i\omega)])$$

(2) No dispersion in group delay

Breaks (1): amplitude distortion, (2): delay distortion

Effect of distortion

Sinusoidal amplitude distortion (amplitude modulation)

$$A(\omega) = a_1 \cos(\tau_1 \omega) + a_0, \;\; \phi(\omega) = -\tau_0 \omega$$

$$\begin{aligned}
w(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega) U(i\omega) e^{i(\omega t + \phi(\omega))} d\omega \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega U(i\omega) \{a_1 \cos(\tau_1 \omega) + a_0\} e^{i\omega(t-\tau_0)} \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega U(i\omega) \left[a_0 + \frac{a_1}{2} (e^{i\tau_1\omega} + e^{-i\tau_1\omega}) \right] e^{i\omega(t-\tau_0)} \\
&= a_0 u(t-\tau_0) + \frac{a_1}{2} [u(t-\tau_0+\tau_1) + u(t-\tau_0-\tau_1)]
\end{aligned}$$

Paired echo

Effect of distortion

Sinusoidal group delay distortion

$$A(\omega) = A_0,\ \phi(\omega) = -\tau_0\omega + b_1 \sin(\tau_1\omega)$$

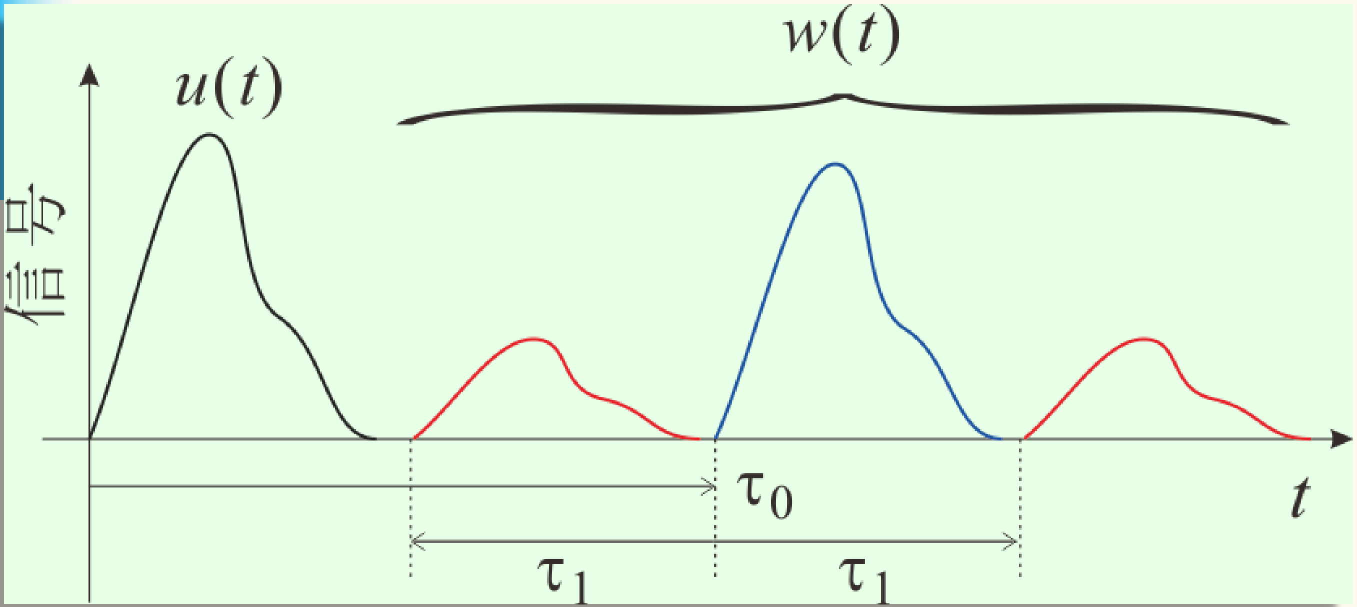
$$\exp[ib_1 \sin(\tau_1\omega)] \approx 1 + \frac{ib_1}{2i}(e^{i\tau_1\omega} - e^{-i\tau_1\omega})$$

$$w(t) = A_0[u(t-\tau_0) + \frac{b_1}{2}\{u(t-\tau_0+\tau_1) - u(t-\tau_0-\tau_1)\}]$$

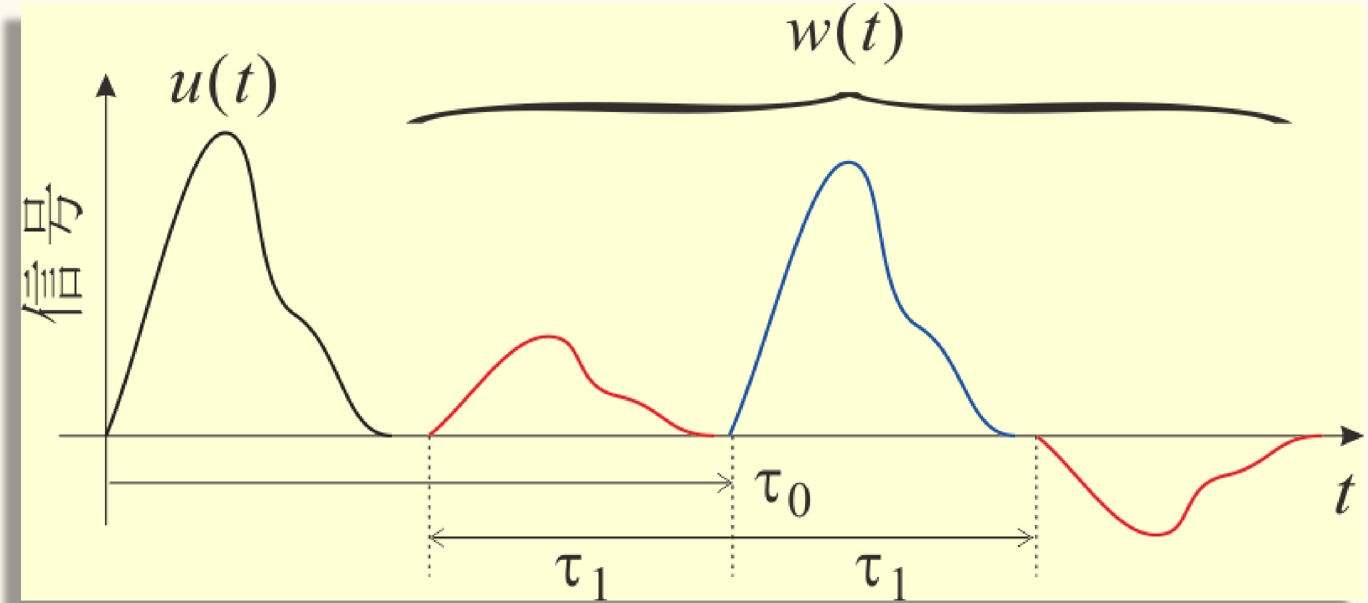
Paired echo

Distortion (paired echo)

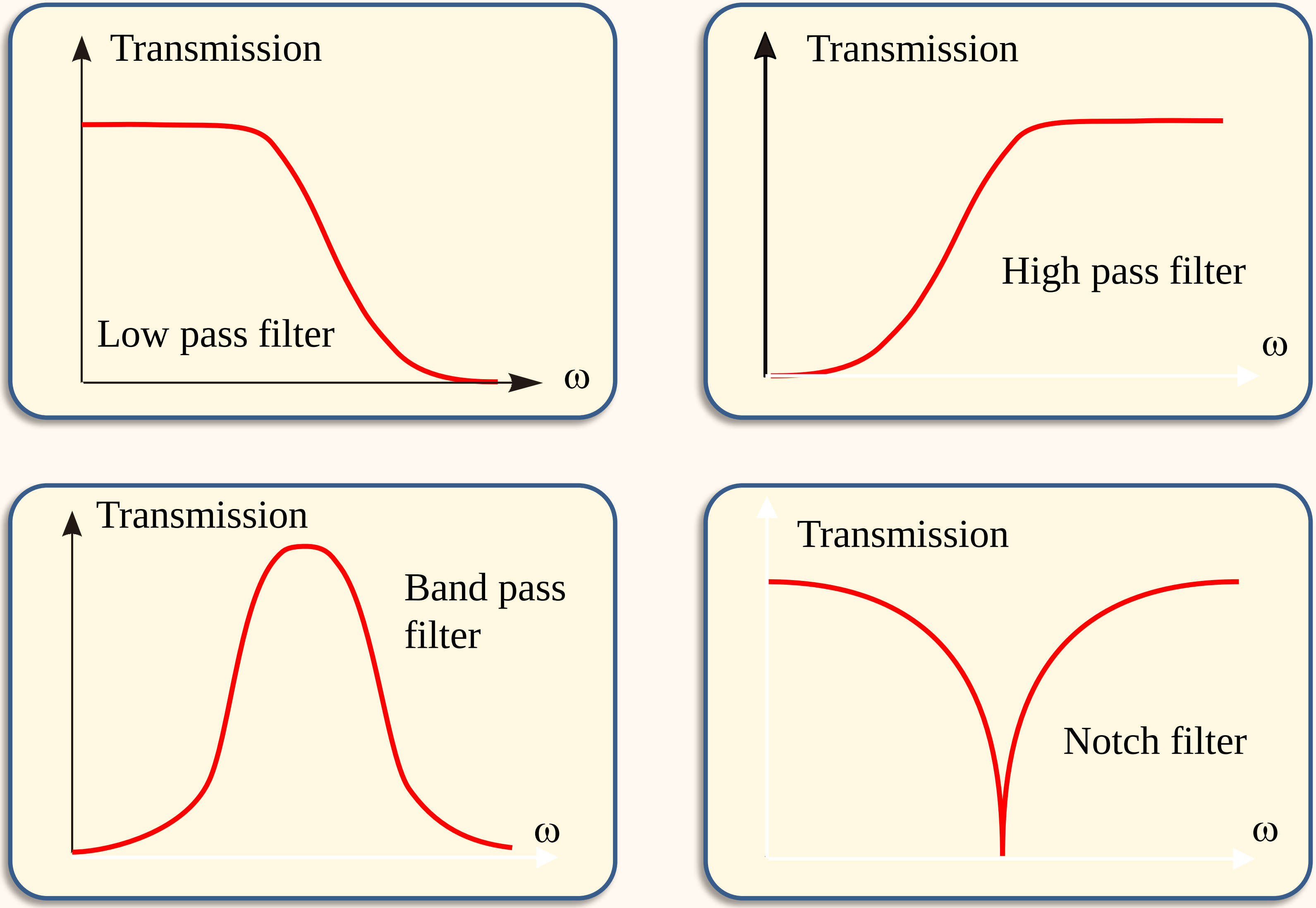
Cosine Amplitude Distortion



Sine Delay Distortion



Filter Circuit

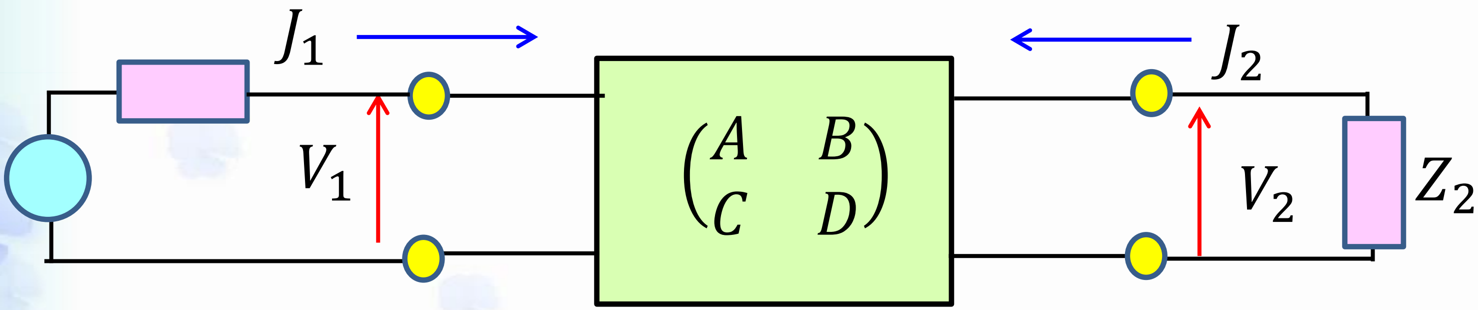


Transmission

Voltage transmission coefficient: $T(i\omega) \equiv \dfrac{V_2(i\omega)}{V_1(i\omega)}$

$$\log T = \log|T| + i\arg T = -\alpha - i\beta$$

attenuation Phase shift

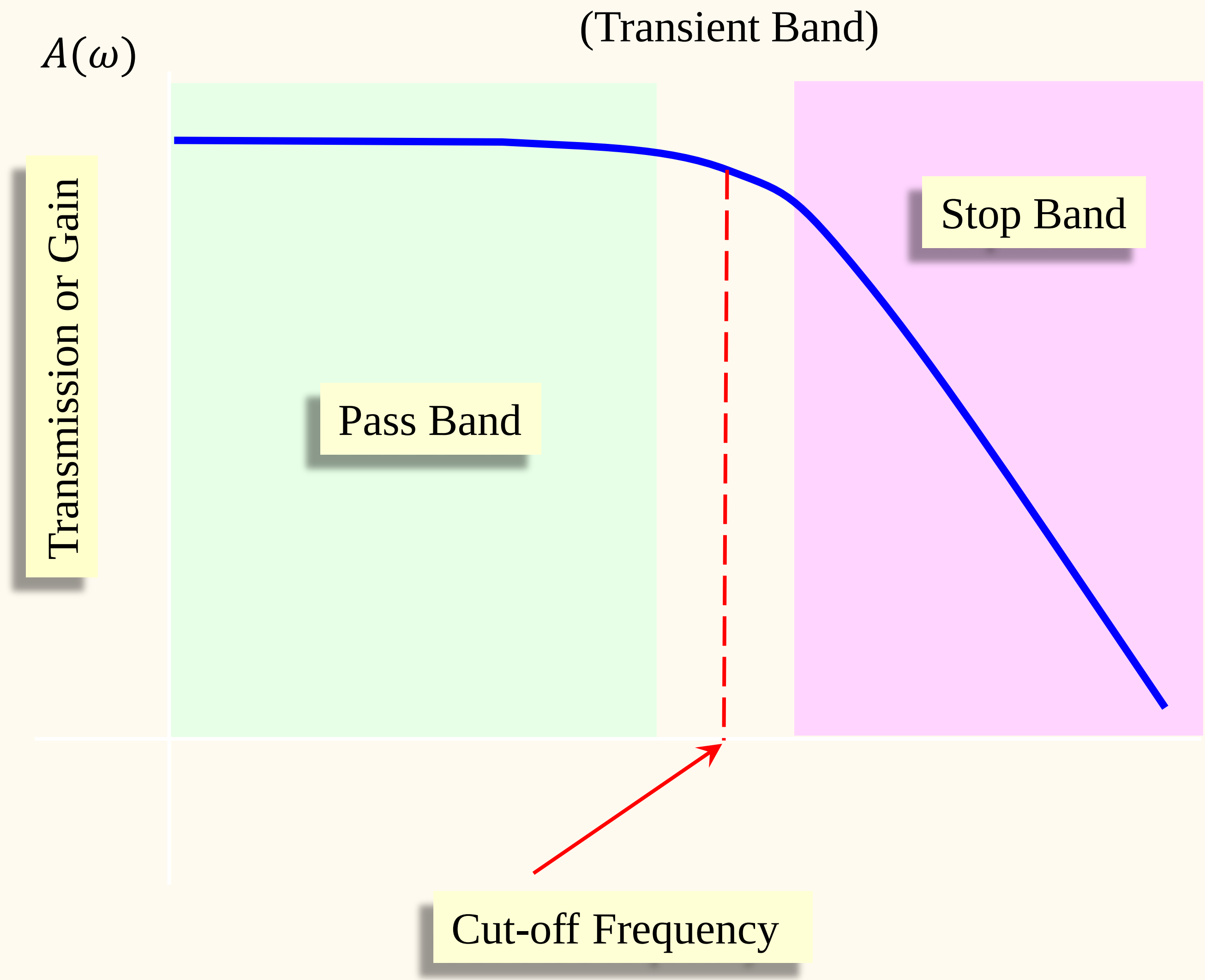


Square root power transmission coefficient

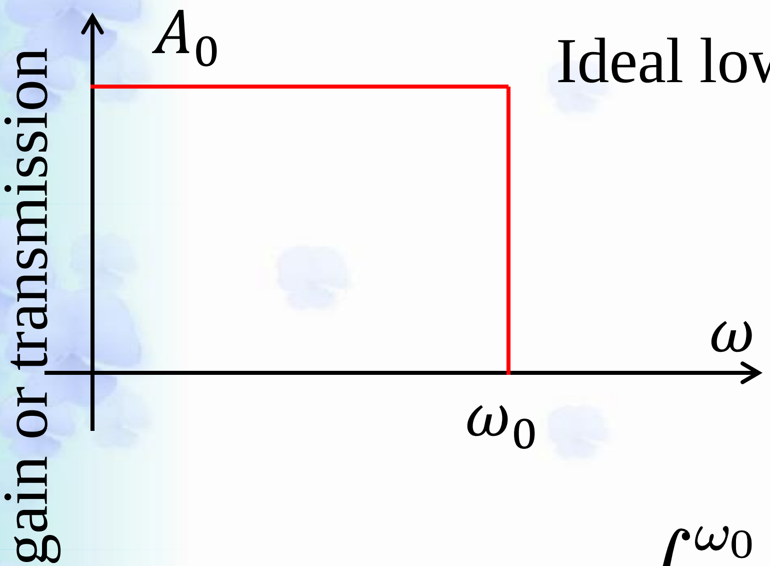
$$S_\mathrm{B} \equiv \sqrt{\frac{P_0}{P_2}} = \frac{R_2 A + B + C R_1 R_2 + D R_1}{2\sqrt{R_1 R_2}}$$

Terms for Filters

$$\Xi(i\omega) = A(\omega)e^{i\phi(\omega)}$$



Ideal filter (not exist)



Ideal low pass filter

$$\Xi(i\omega) = A_0 H(\omega_0 - \omega)$$

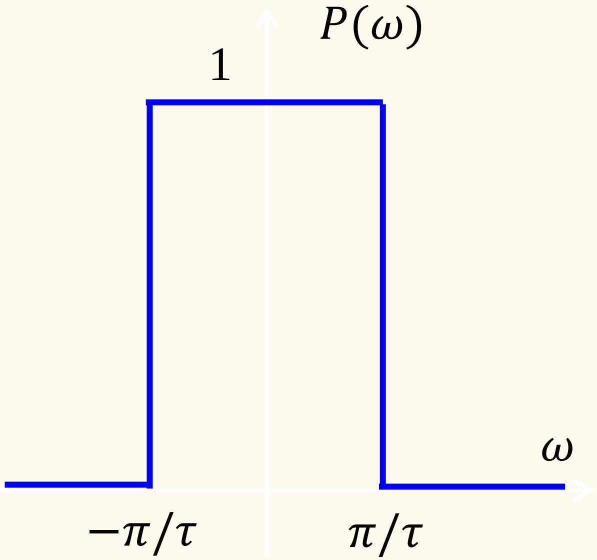
Heaviside function

$$w(t) = \int_{-\omega_0}^{\omega_0} A_0 e^{i\omega t} \frac{d\omega}{2\pi}$$

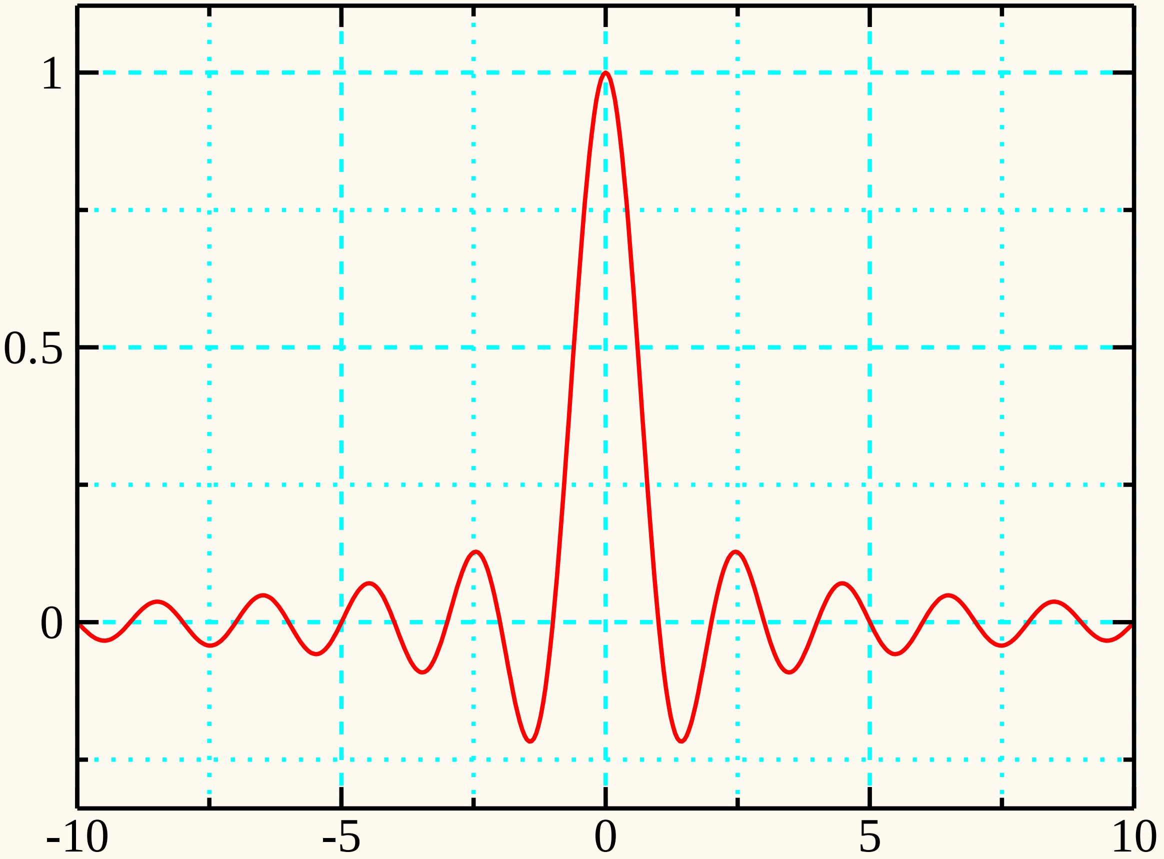
$$= A_0 \int_{-\omega_0}^{\omega_0} \frac{d\omega}{2\pi} \cos \omega t$$

$$= 2A_0 f_0 \frac{\sin \omega_0 t}{\omega_0 t} = 2A_0 f_0 \mathrm{sinc}(2f_0 t)$$

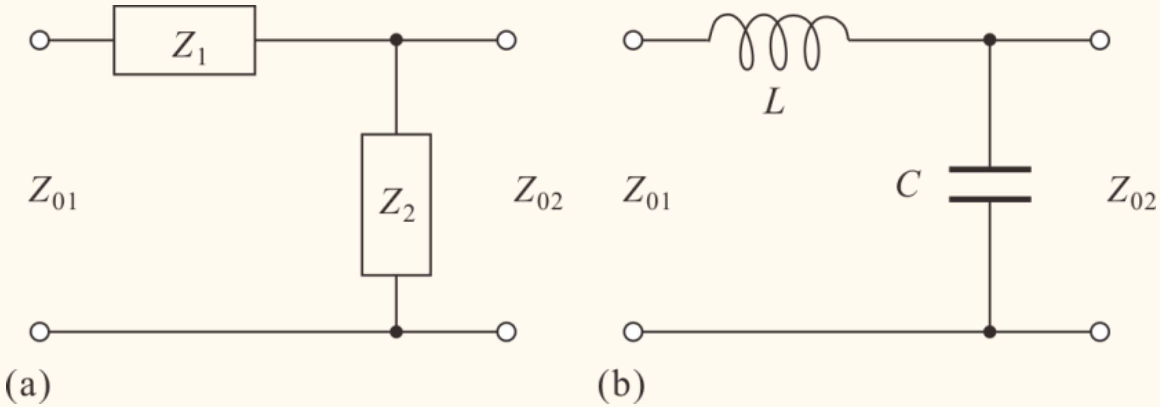
Sinc function

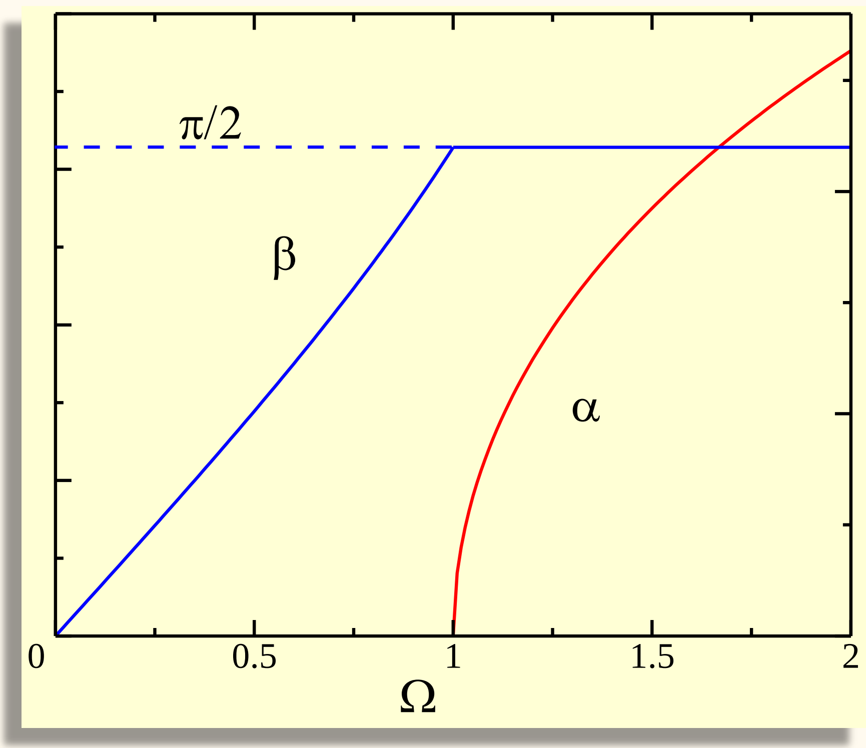


$$\operatorname{sinc}(x) = \frac{\sin \pi x}{\pi x}$$



Constant K type filter

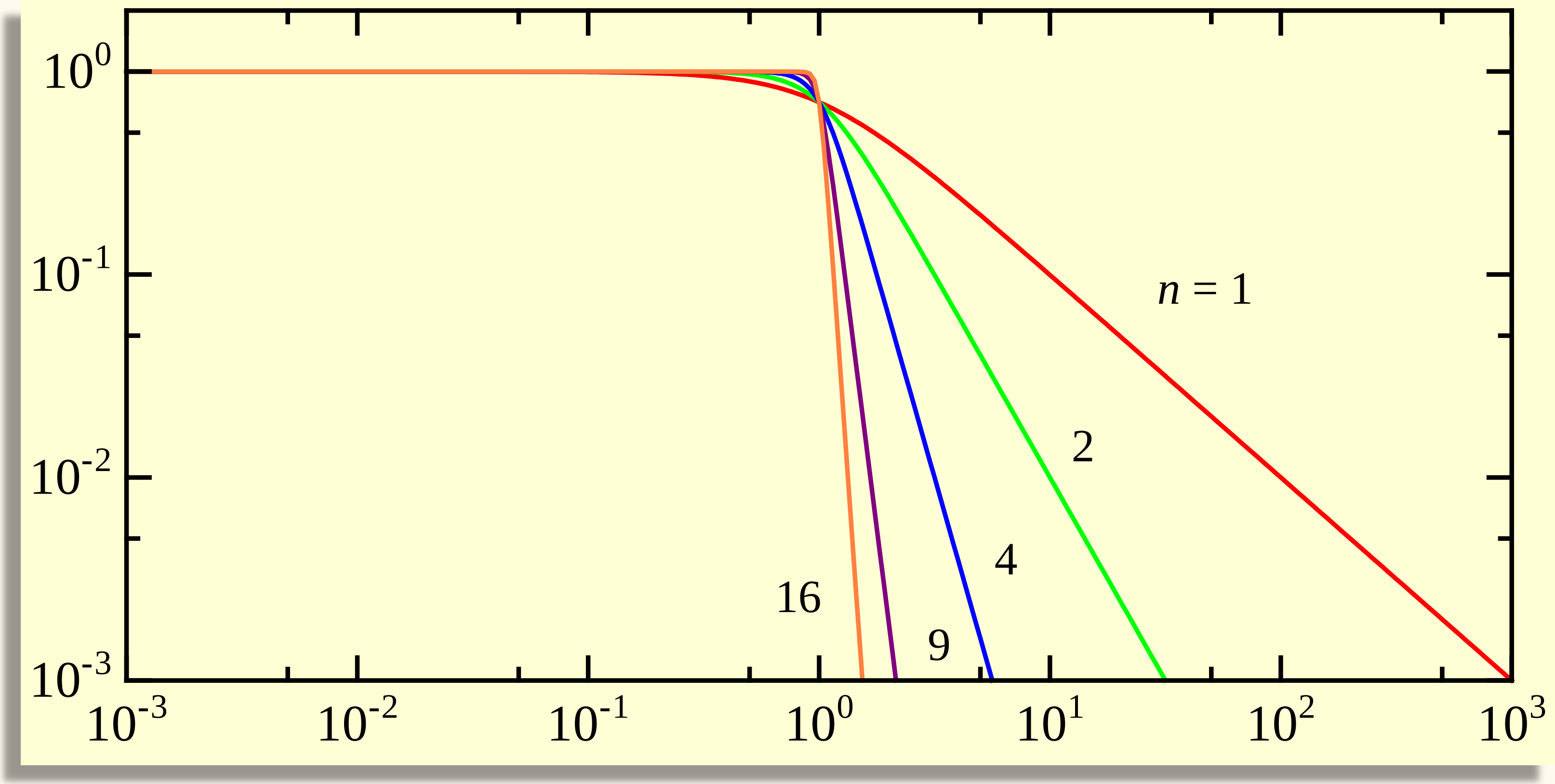




$$Z_1 Z_2 = R^2 (= K)$$

Butterworth Filter

$$G^2(i\omega/\omega_0) = |H(i\omega)|^2 = \frac{1}{1+(\omega/\omega_0)^{2n}}$$



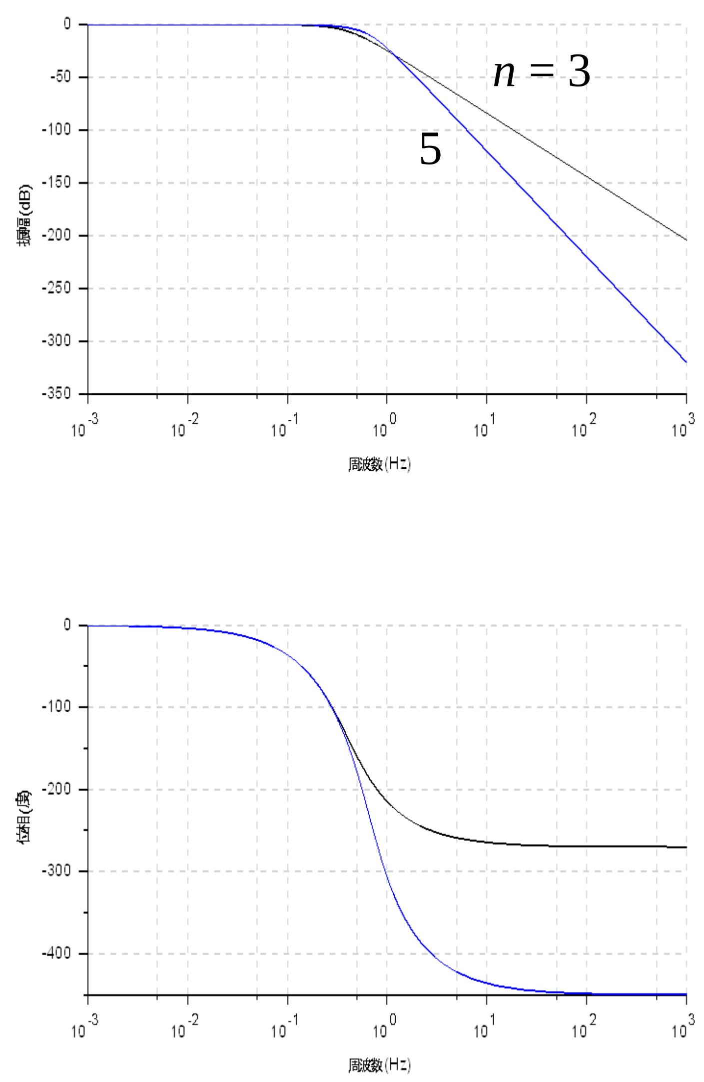
Bessel Filter

Inverse Bessel Polynomial

$$B_0 = 1, \quad B_1(s) = s + 1$$

$$B_n(s) = (2n-1)B_{n-1}(s) + B_{n-2}(s)s^2$$

$$\Xi(s) = \frac{B_n(0)}{B_n(s)}$$

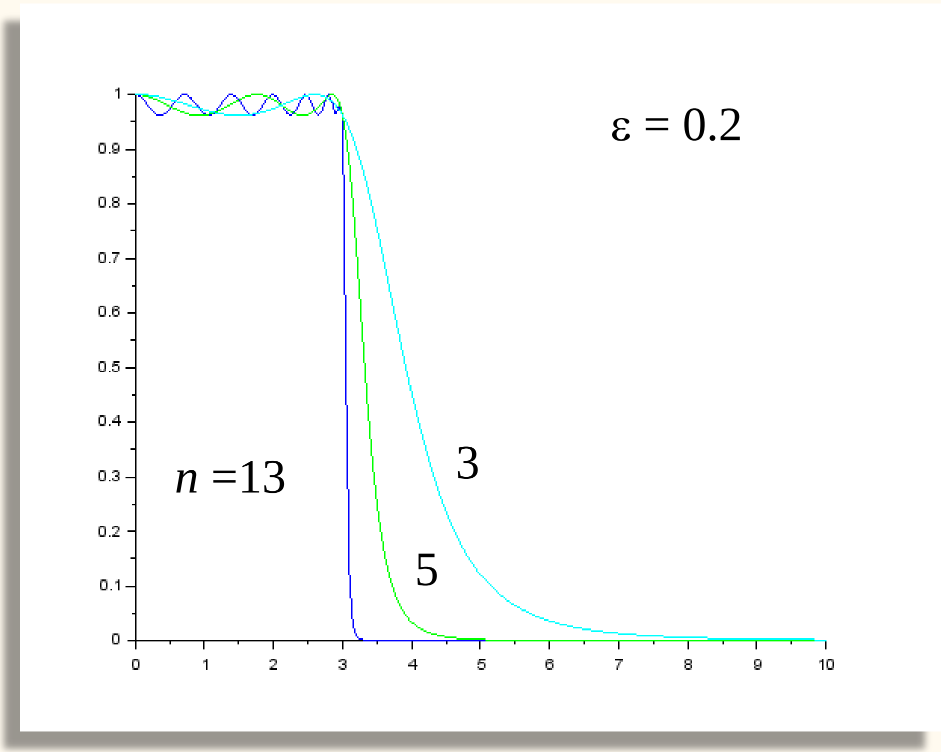


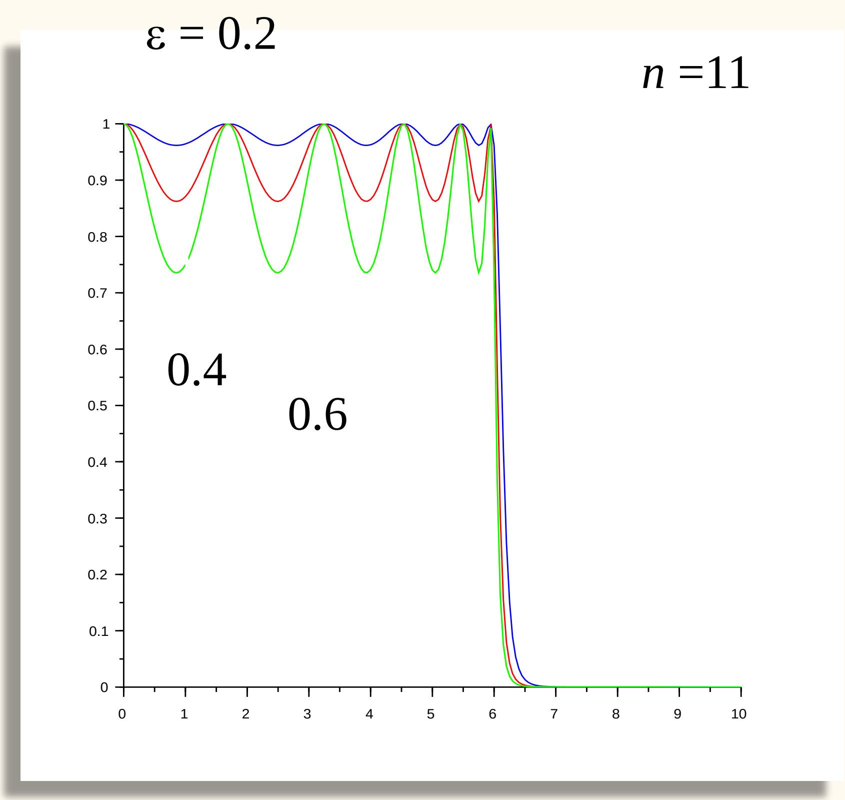
Chebyshev Filter

$$G_n(i\Omega) = |H_n(i\Omega)| = \frac{1}{\sqrt{1+\epsilon^2 T_n^2(\Omega)}}$$

ε: Ripple coefficeint

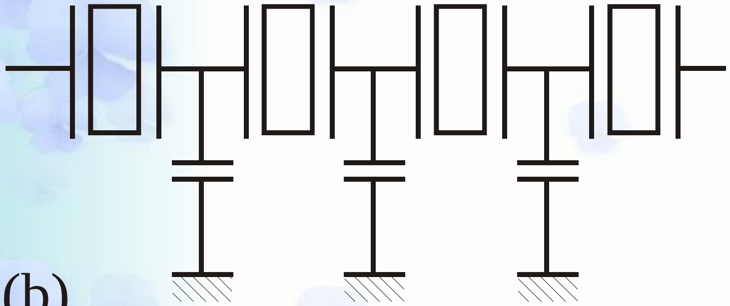
T_n: n-th order Chebyshev polynomial



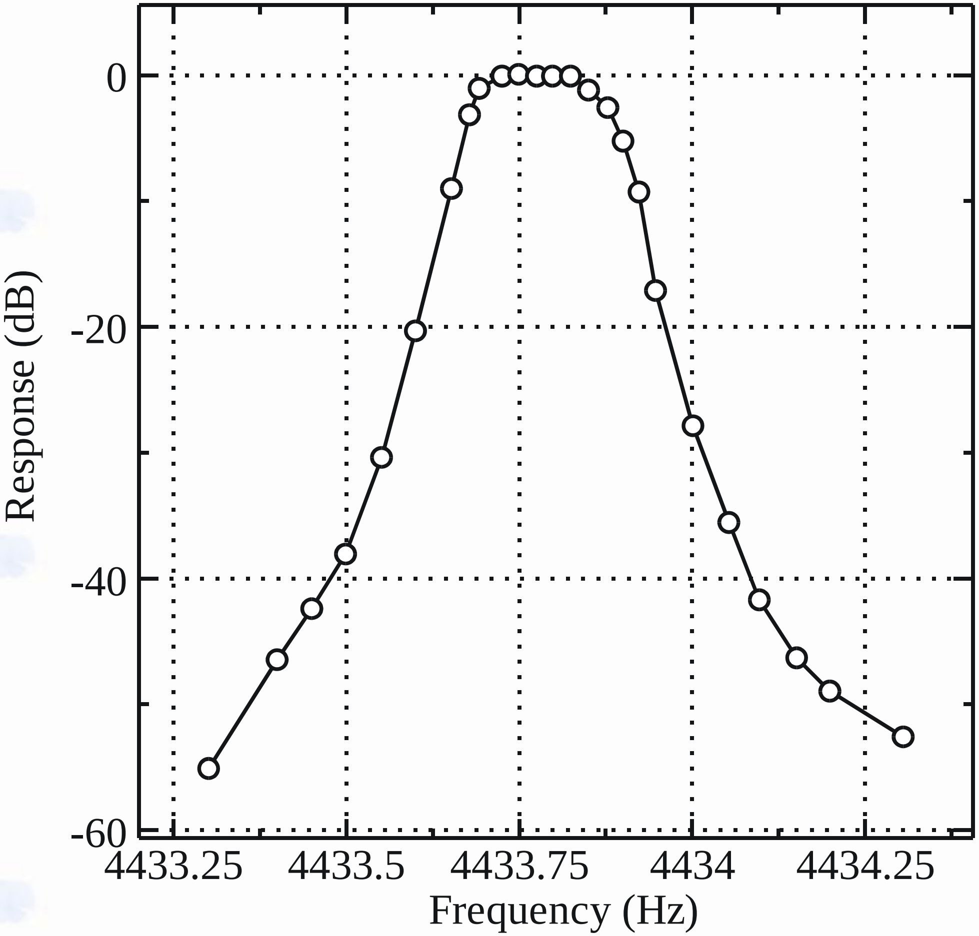


Quartz crystal filter

(a)



(b)



(c)

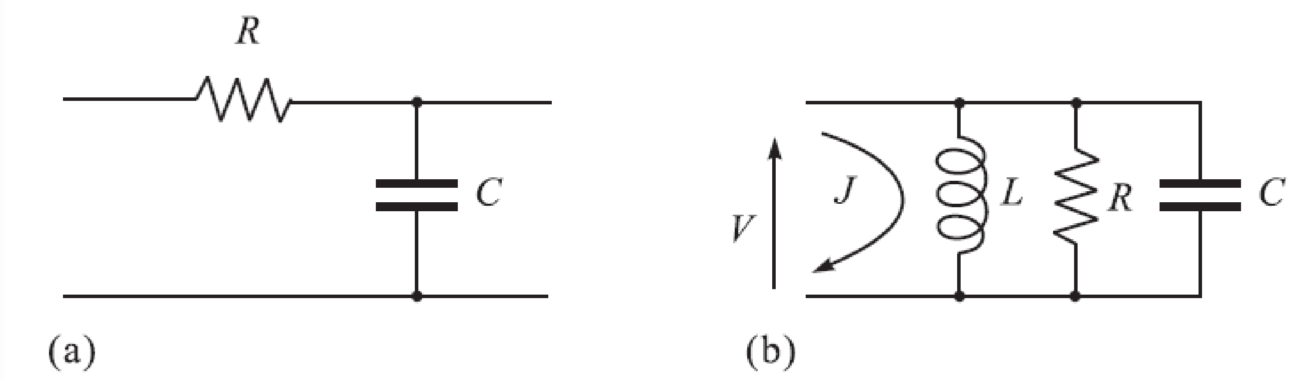
Packaged filters

Web selection [page](http://www.minicircuits.com/products/Filters.shtml)

http://www.minicircuits.com/products/Filters.shtml

Mini-Circuits
Band Pass
19.2 – 23.6MHz 50Ohm

Classification with the number of energy storages



(a) Single energy storage

$$\Xi(s) = \frac{1}{1 + s/s_0}$$

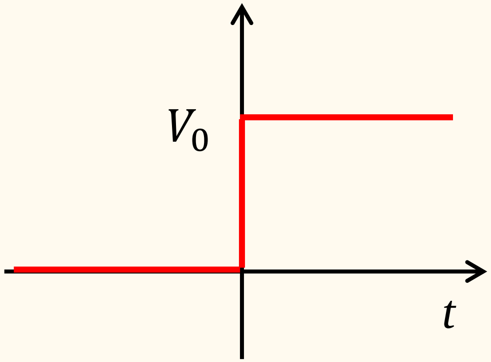
(b) Double energy storage

$$\Xi(s) = \frac{1}{b + s + as^{-1}}$$

過渡応答 (Transient Response)

$$w(t) = \int_{-\infty}^{\infty} \Xi(i\omega)U(i\omega)e^{i\omega t}\frac{d\omega}{2\pi}$$

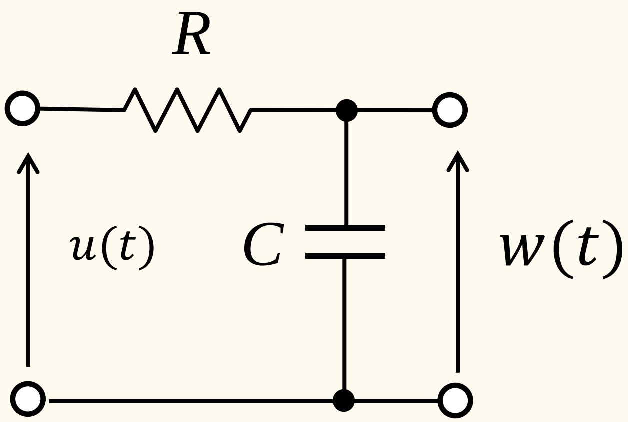
Heaviside



$$H(t) = \begin{cases} 0 & t < 0, \\ 1/2 & t = 0, \\ 1 & t > 0 \end{cases}$$

$$\mathcal{F}\{H(t)\} = \frac{1}{i\omega} + \pi\delta(\omega)$$

Simple application

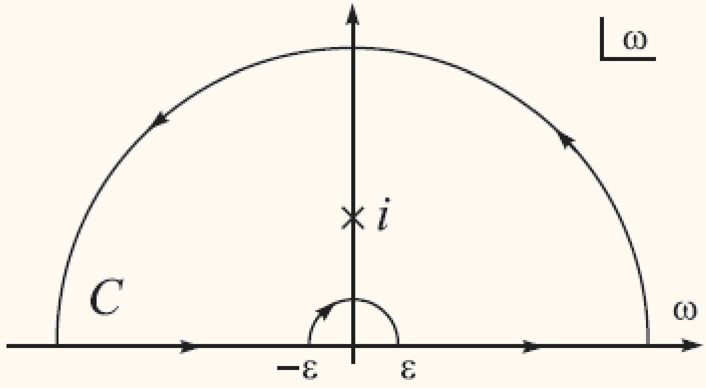


$$V = V_0 \left[1 - \exp\left(-\frac{t}{CR}\right)\right]$$

$$g(t) = \int_{-\infty}^{\infty} \frac{1}{1+i\omega}\left[\frac{1}{i\omega} + \pi\delta(\omega)\right] e^{i\omega t}\frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{(i-\omega)\omega}\frac{d\omega}{2\pi} + \frac{1}{2}$$

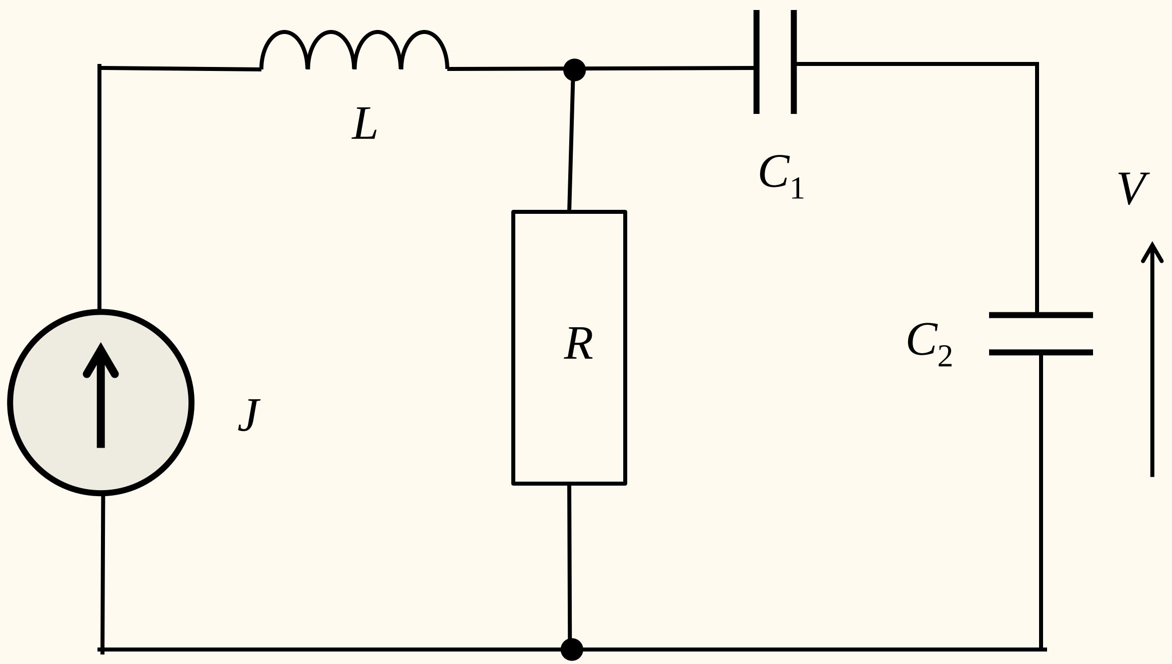
$$-2\pi i \frac{e^{-t}}{2\pi i} - \lim_{\epsilon\to 0}\left[\int_{\pi}^{0} \frac{e^{i\epsilon e^{i\theta} t}}{\epsilon e^{i\theta}(\epsilon e^{i\theta} - i)} \frac{i\epsilon e^{i\theta} d\theta}{2\pi}\right] = -e^{-t} - \frac{1}{2}$$



$$g(t) = -e^{-t}$$

Exercise B-1

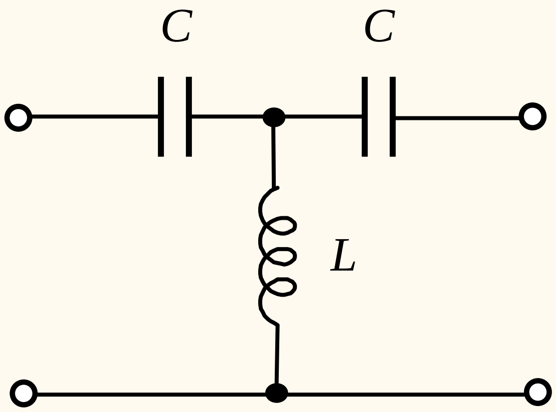
Calculate the voltage V over capacitor C_2 by using Norton theorem.



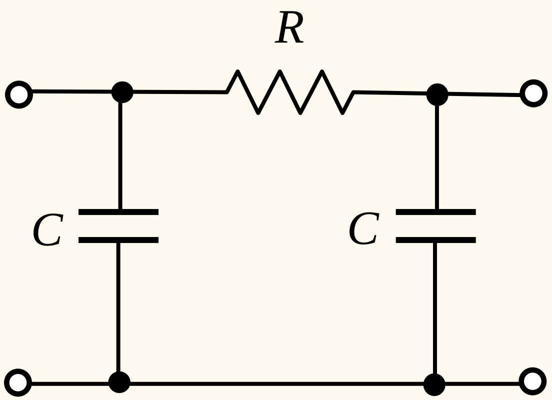
Exercise B-2

Obtain F-matrices for the circuits below.

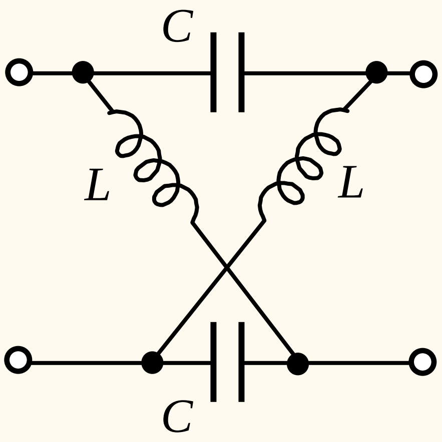
(a)



(b)

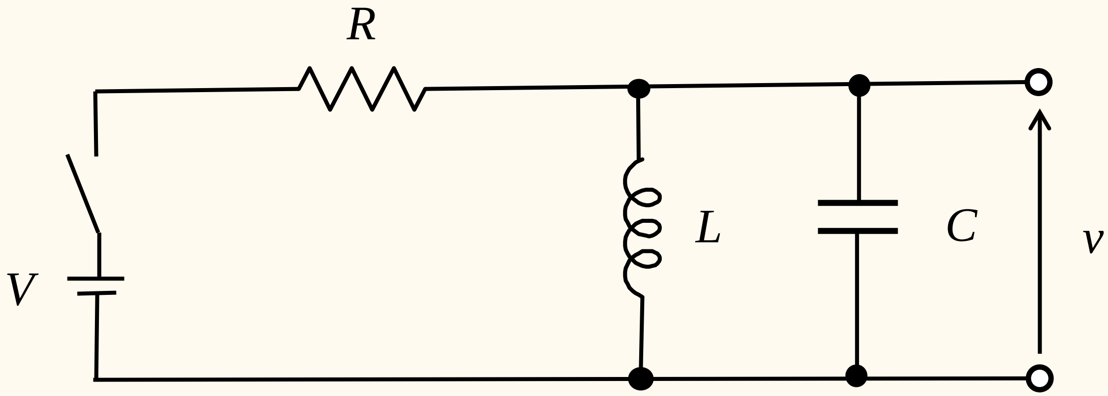


(c)



Exercise B-3

The switch below is turned on at $t = 0$.
Obtain the time evolution of voltage v henceforth.



電子回路論第5回

Electric Circuits for Physicists

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Outline

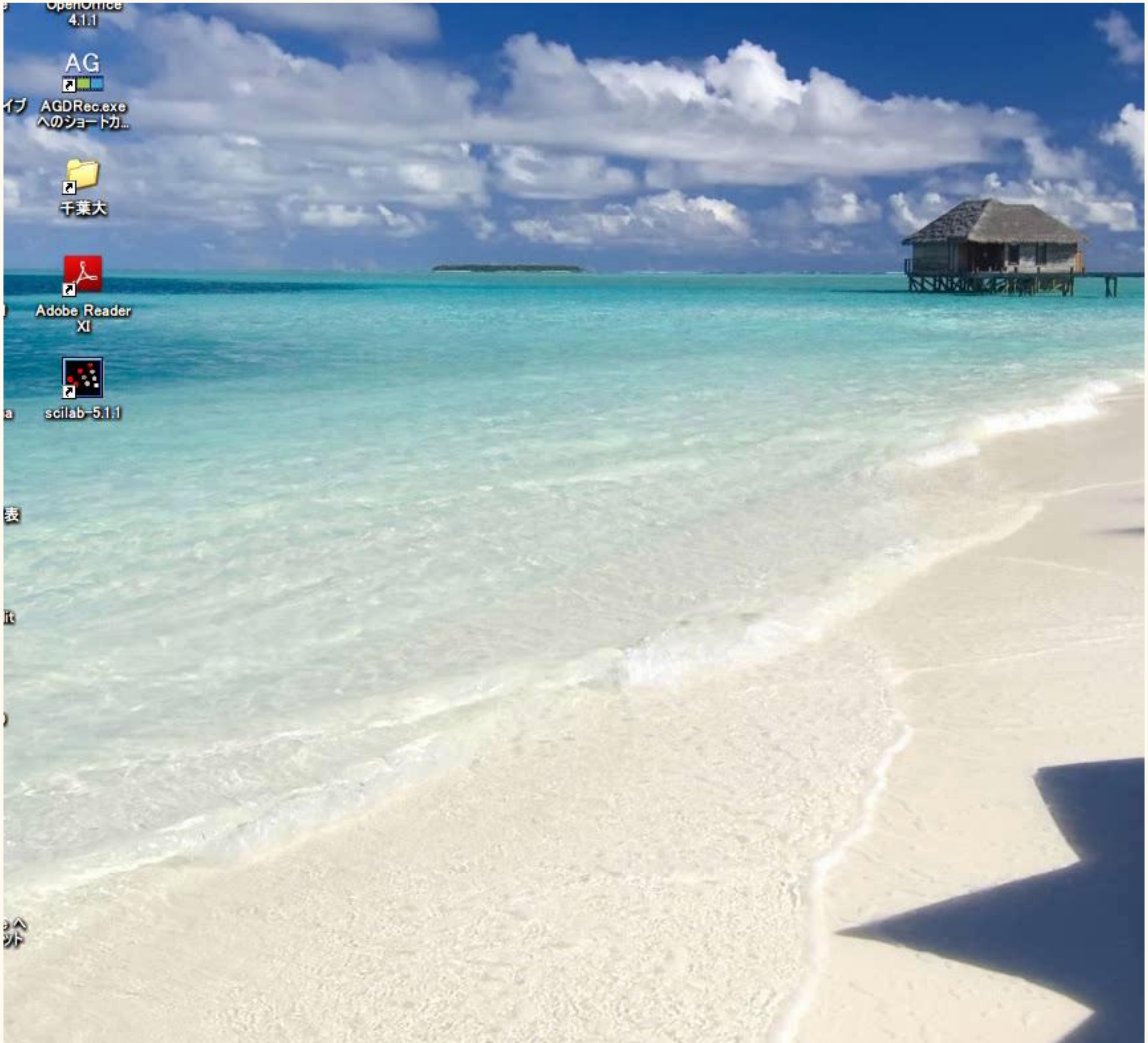
Introduction of a freeware “Scilab”

Ch.4 Amplification circuit

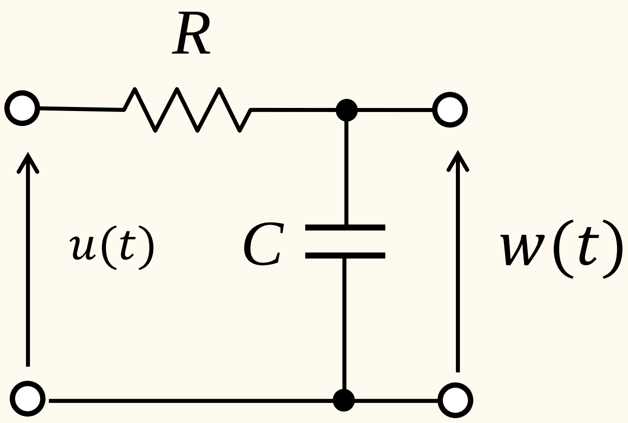
- 4.1 Amplification and system stability
  - 4.1.1 What is amplifier?
  - 4.1.2 Feedback
  - 4.1.3 Stability of feedback
- 4.2 Operational amplifier (OP-amp)
  - 4.2.1 Linear model of OP-amp
  - 4.2.2 Package
  - 4.2.3 Circuit examples
  - 4.2.4 Datasheet
  - 4.2.5 Stability

A convenient freeware: Scilab

Transfer function analysis with Scilab



Simple application

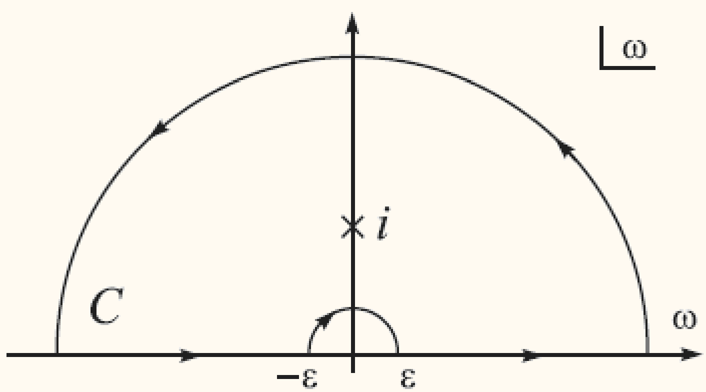


$$V = V_0\left[1 - \exp\left(-\frac{t}{CR}\right)\right]$$

$$g(t) = \int_{-\infty}^{\infty} \frac{1}{1+i\omega}\left[\frac{1}{i\omega} + \pi\delta(\omega)\right] e^{i\omega t}\frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{(i-\omega)\omega}\frac{d\omega}{2\pi} + \frac{1}{2}$$

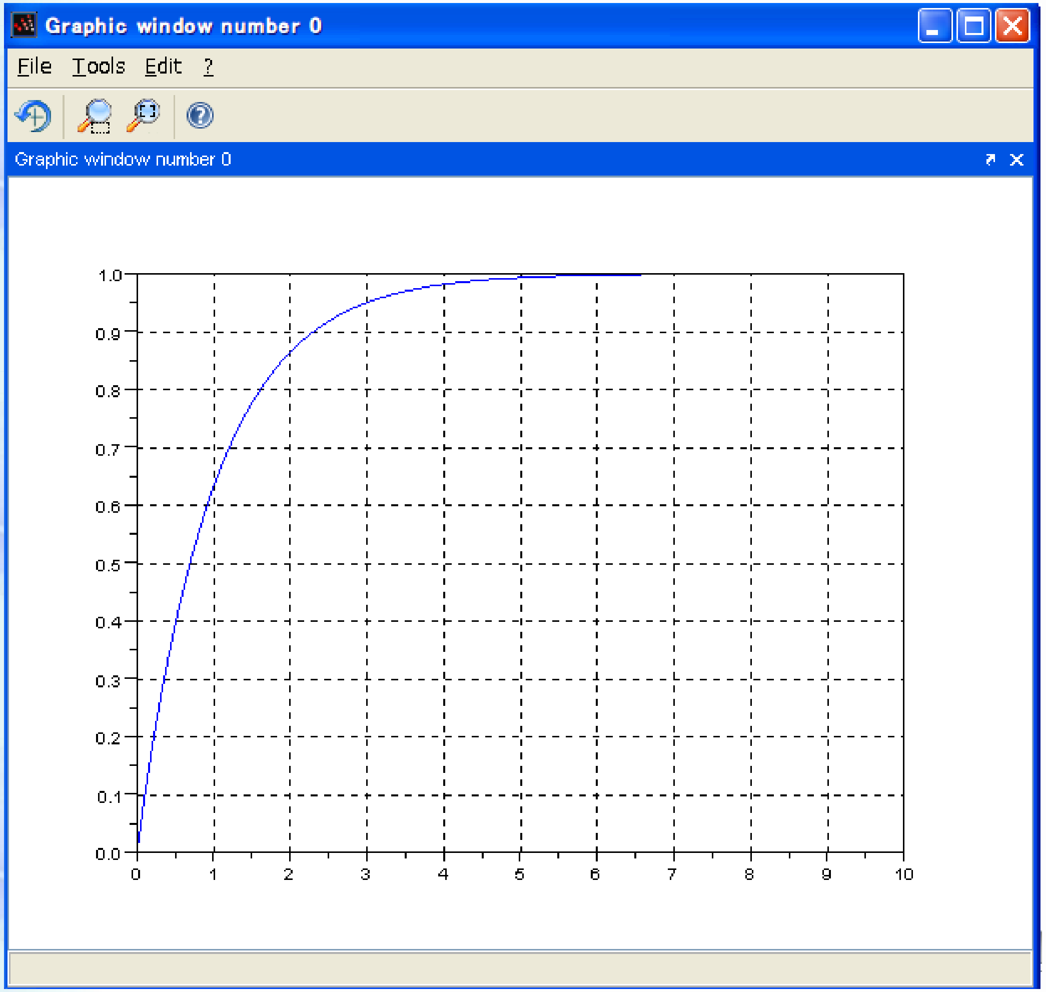
$$-2\pi i \frac{e^{-t}}{2\pi i} - \lim_{\epsilon\to 0}\left[\int_{\pi}^{0} \frac{e^{i\epsilon e^{i\theta}t}}{\epsilon e^{i\theta}(\epsilon e^{i\theta} - i)} \frac{i\epsilon e^{i\theta}d\theta}{2\pi}\right] = -e^{-t} - \frac{1}{2}$$



$$g(t) = -e^{-t}$$

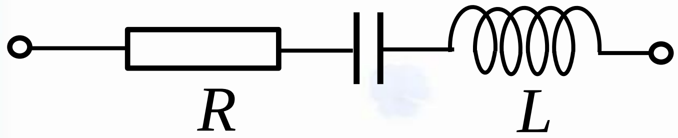
Transient response: Use of Scilab

$$\Xi(s) = \frac{1}{1+s}$$

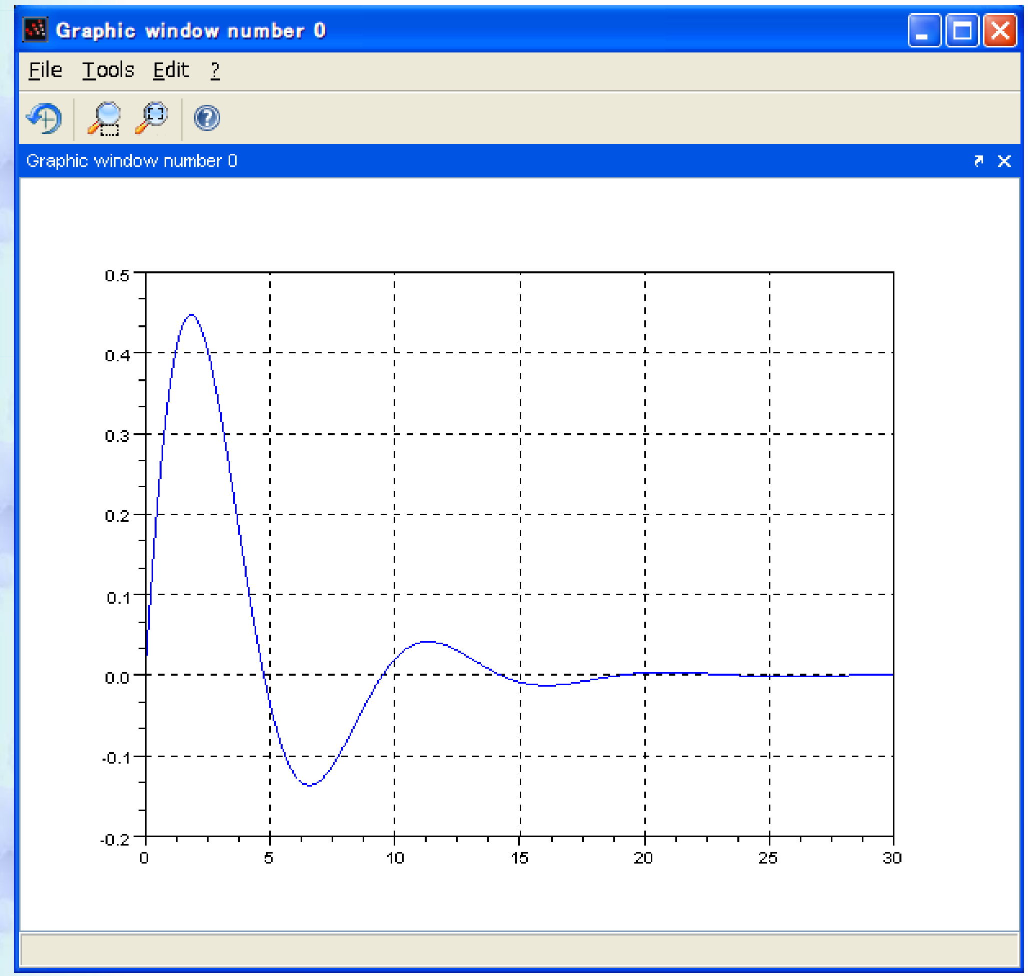



```
-->s=poly(0,'s');

-->G=1/(1+s);

-->sys=syslin('c',G);

-->t=linspace(0,10,100);

-->y=csim('step',t,sys);

-->plot(t,y)

-->xgrid()
```

Transient response: Use of Scilab

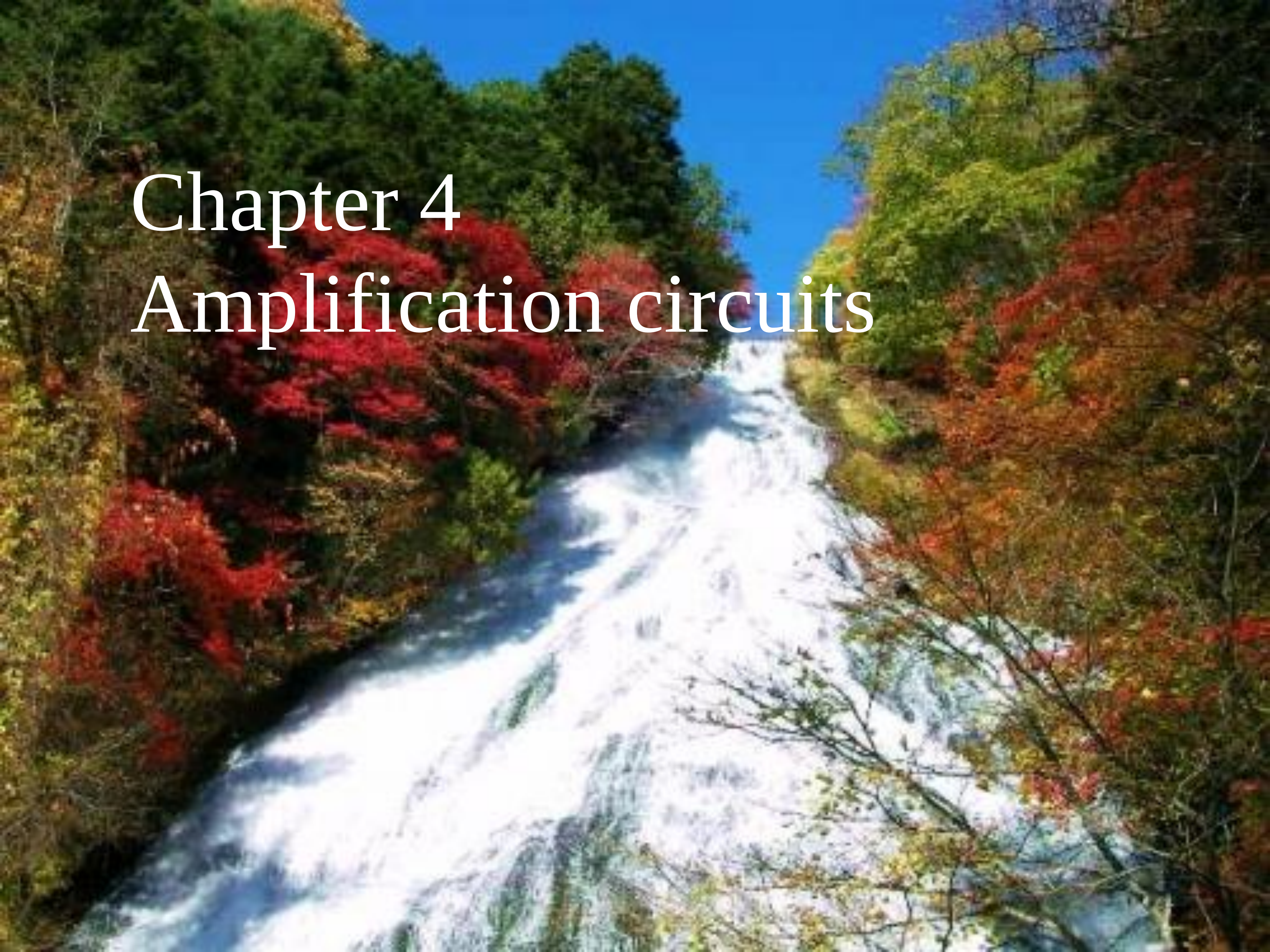


$$Y(s) = \frac{Cs}{LCs^2 + CRs + 1}$$



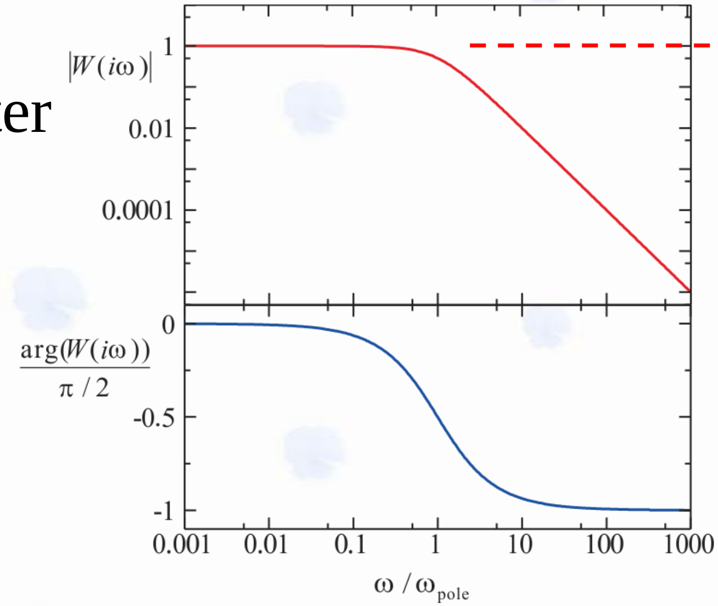

```
-->G=s/(1+s+2*s*s);

-->sys=syslin('c',G);

-->y=csim('step',t,sys);

-->plot(t,y)
```

Chapter 4
Amplification circuits



Linear amplifier

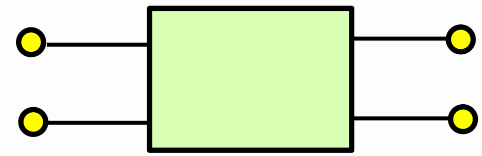
passive filter



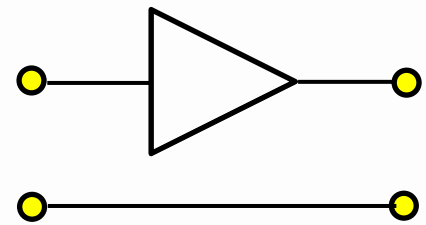
gain = 1

gain >1 ⟶ amplifier

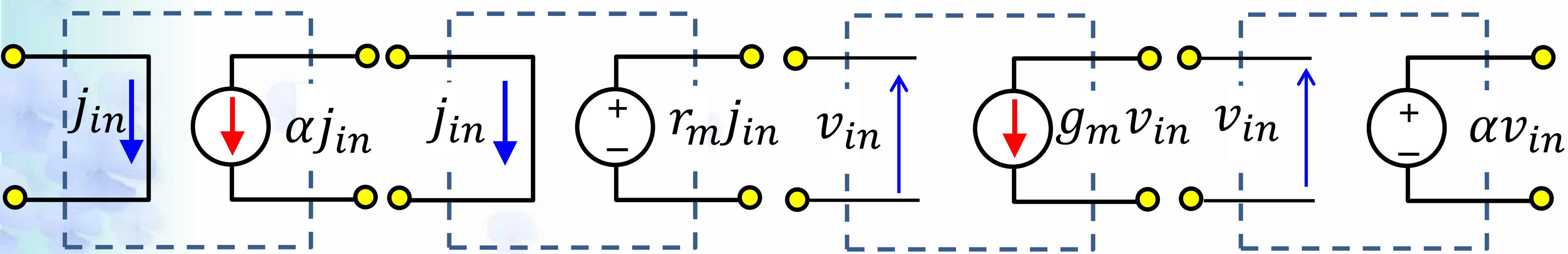
four terminal circuit model



Circuit symbol



Controlled power source models



J - *J* *J* - *V* *V* - *J* *V* - *V*

Gain, and "Unit" for gain

Voltage gain: $\left|\dfrac{v_{out}}{v_{in}}\right|$ Current gain: $\left|\dfrac{j_{out}}{j_{in}}\right|$ Power gain: $\left|\dfrac{v_{out}j_{out}}{v_{in}j_{in}}\right|$

When we say "the gain of the amplifier …", the gain means power gain.

quantity Q, unit Q_0 : Q in log scale: $L = \log_{10}\dfrac{Q}{Q_0}$ (B, bel)

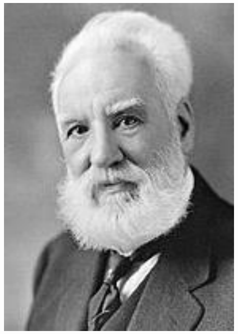
cf. deca- 10

1/10

dB : (decibel)

From: G. Bell

Alexander Graham Bell
1847 - 1922



$$G = 10 \times \log_{10}\left(\frac{v_{\rm out}}{v_{\rm in}}\right)^2 = 20\log_{10}\frac{v_{\rm out}}{v_{\rm in}}$$

---

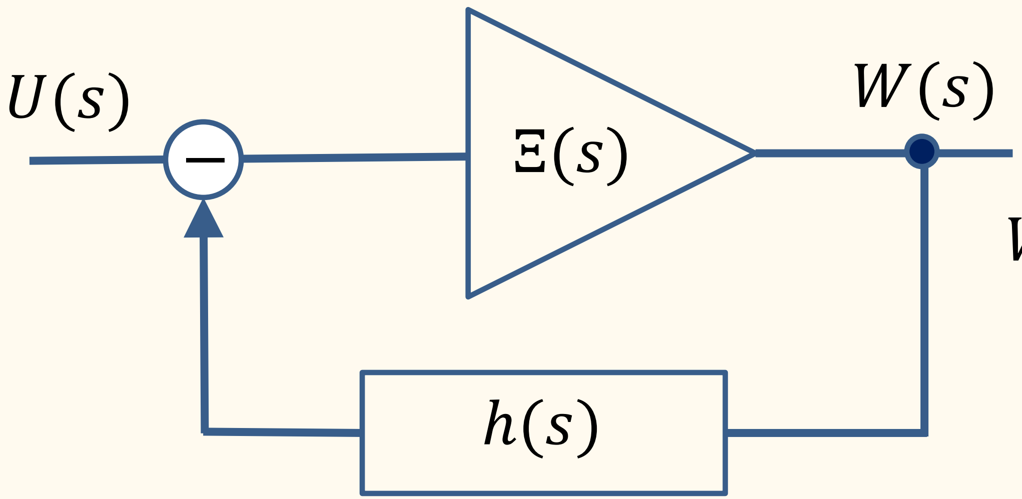
dB units: dBm (1mW: 0dBm), dBv (1V: 0dBv), etc.

Feedback circuit

Feedforward

⇕

Feedback



$$W(s) = \Xi(s)U(s)$$

$$W(s) = \Xi(s)[U(s) - h(s)W(s)]$$

$$W(s) = \frac{\Xi(s)}{1 + \Xi(s)h(s)}U(s)$$

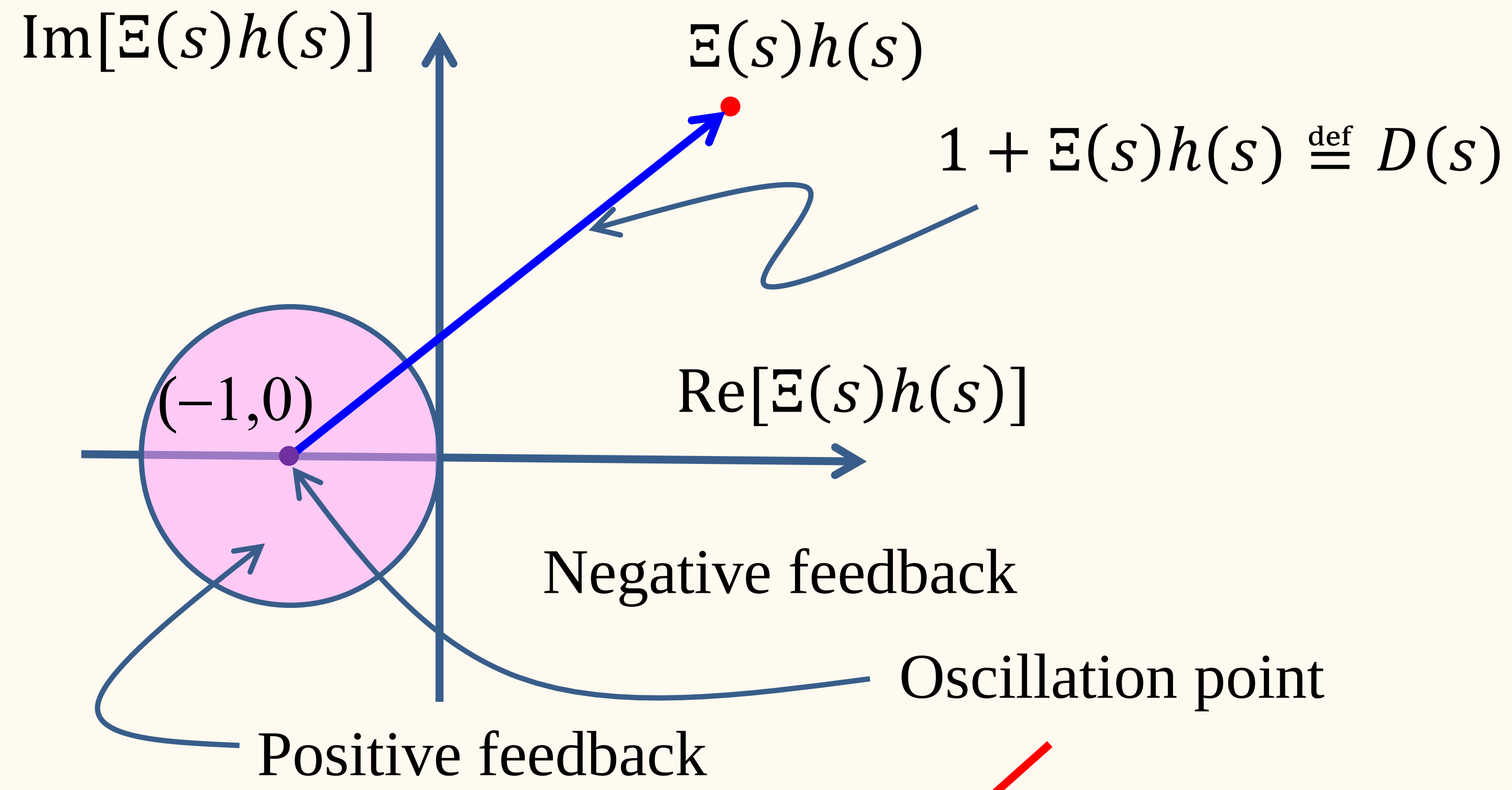
$$\stackrel{\text{def}}{=} G(s)U(s)$$

$|1 + \Xi(s)h(s)| > 1$: Negative feedback, < 1: Positive feedback

$$|\Xi(s)| \gg 1 \to G(s) \approx \frac{1}{h(s)}$$

Condition for negative feedback

$|1 + \Xi(s)h(s)| > 1$: Negative feedback, < 1: Positive feedback



If $\Xi(s)h(s) = -1$ has solutions, the circuit may be unstable.

How can we judge? → Criteria (Routh-Hurwitz, Nyqust, Liapunov, …)

Zeros and poles of $D(s)$

Assumption 1: $\Xi(s), \Xi(s)h(s)$ are stable

→ Poles are on the left half plane of s.

Assumption 2: $\Xi(i\omega), \Xi(i\omega)h(i\omega) \to 0$ for $|\omega| \to \infty$

$\Xi(s) = \frac{Q(s)}{P(s)},\ h(s) = \frac{q(s)}{p(s)}$: $P(s), Q(s), p(s), q(s)$ polynomials

$\deg(P) > \deg(Q)\,, \deg(p) \geq \deg(q)$

$$D(s) = 1 + \Xi(s)h(s) = \frac{P(s)p(s)}{P(s)p(s) + Q(s)q(s)}$$

$$D(s) = D_0 \frac{(s-\beta_1)\cdots(s-\beta_n)}{(s-\alpha_1)\cdots(s-\alpha_n)}$$

The same order

Zeros and poles of $D(s)$

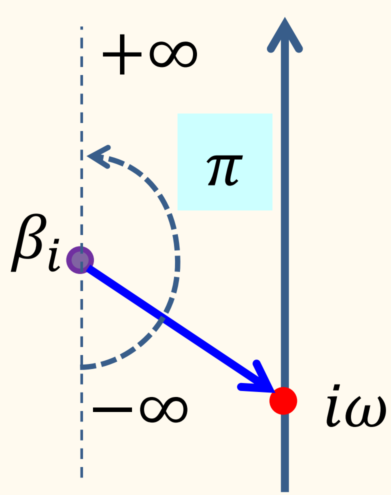
$$D(s) = D_0 \frac{(s-\beta_1)\cdots(s-\beta_n)}{(s-\alpha_1)\cdots(s-\alpha_n)}$$

$\{\beta_i\}$: Zeros of $D(s)$ → Poles of $G(s)$

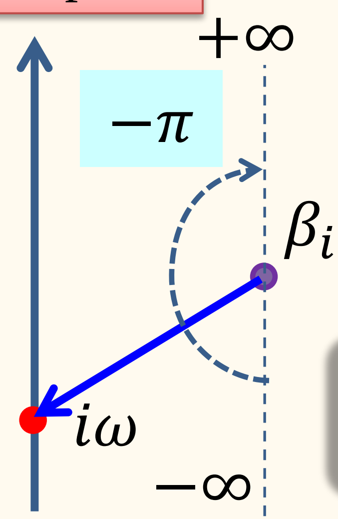
$\exists \beta_i \in$ right half plane of s → The circuit is unstable.

$$\arg(D) = \sum_{i=1}^{n} \arg(s-\beta_i) - \sum_{i=1}^{n} \arg(s-\alpha_i)$$

Left half plane



Right half plane



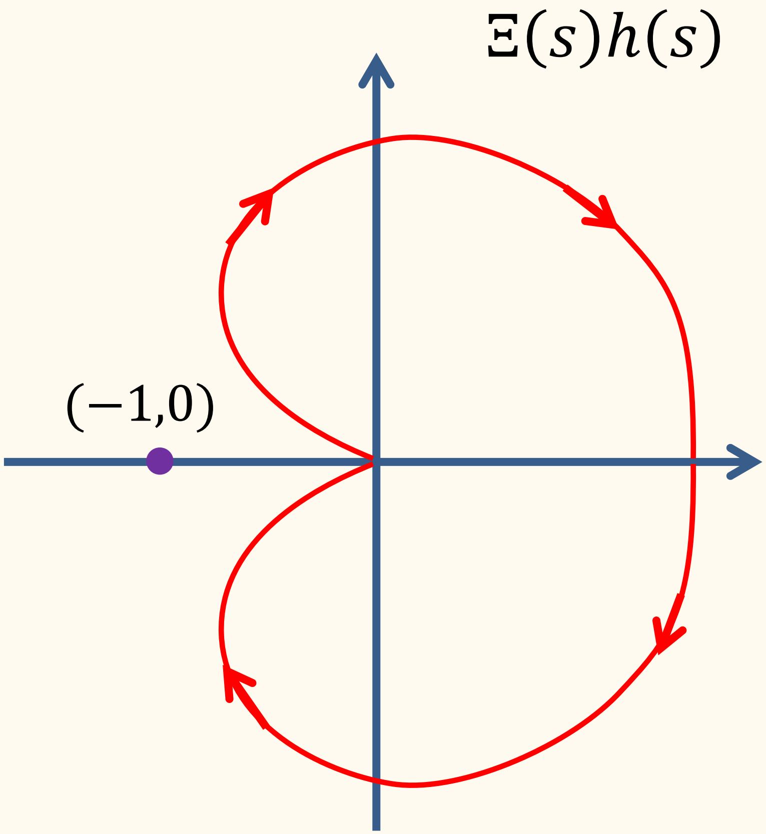
$s = i\omega$ (on imaginary axis)

$\omega: -\infty \rightarrow +\infty$

Number of zeros on the right half plane: m

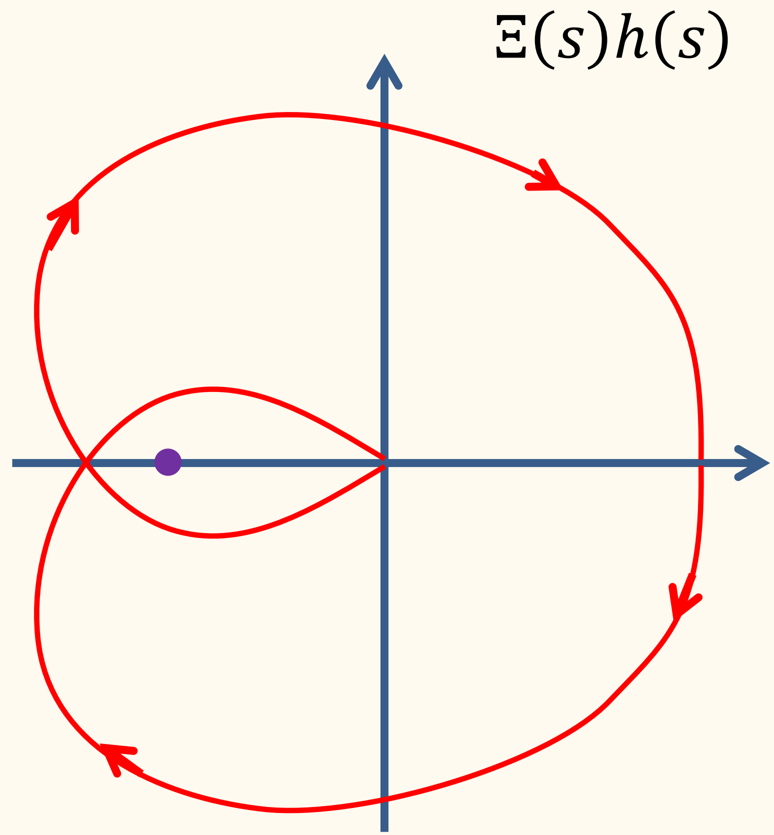
$$\Delta \arg(D) = (n-m)\pi - m\pi - n\pi = -2m\pi$$

Nyquist Plot and Criterion



$\Delta\,\arg(D) = 0$

Stable

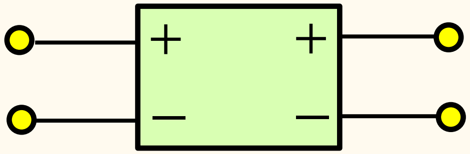


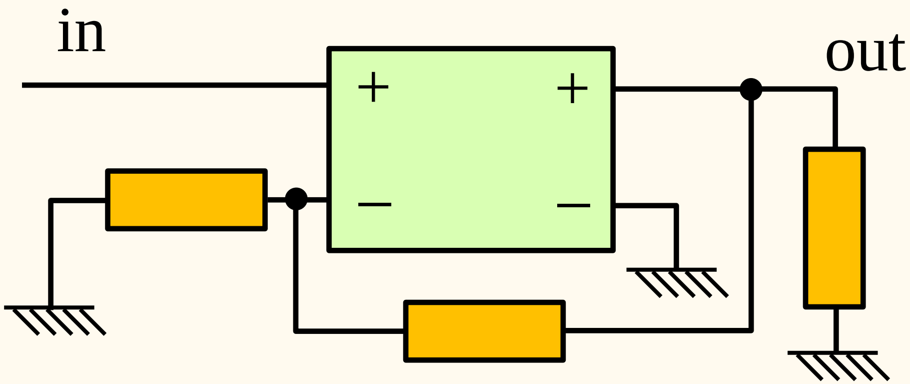
$\Delta\,\arg(D) = -4\pi$

Unstable

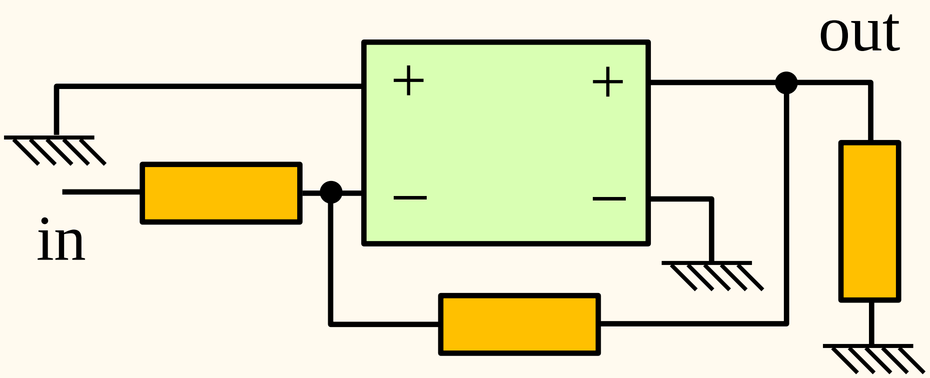
Harry Nyquist
(1889–1976)

Feedback in terminal-pair circuits with resistors

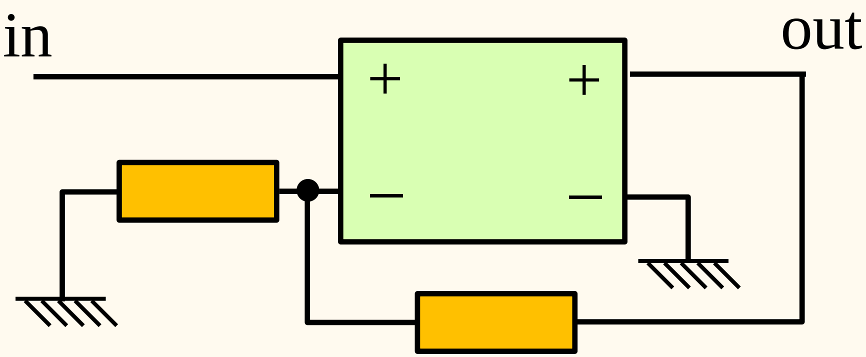




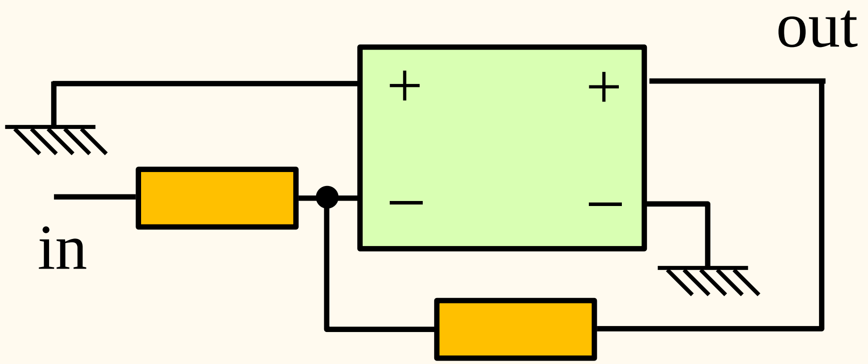
(i) input: parallel, output: parallel



(ii) input: series, output: parallel

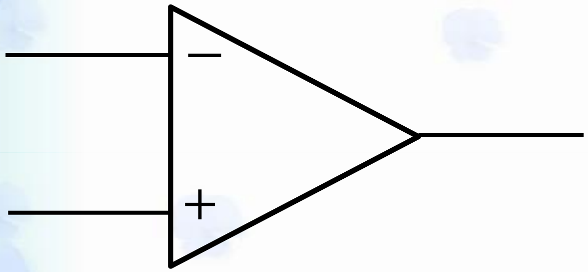


(iii) input: parallel, output: series



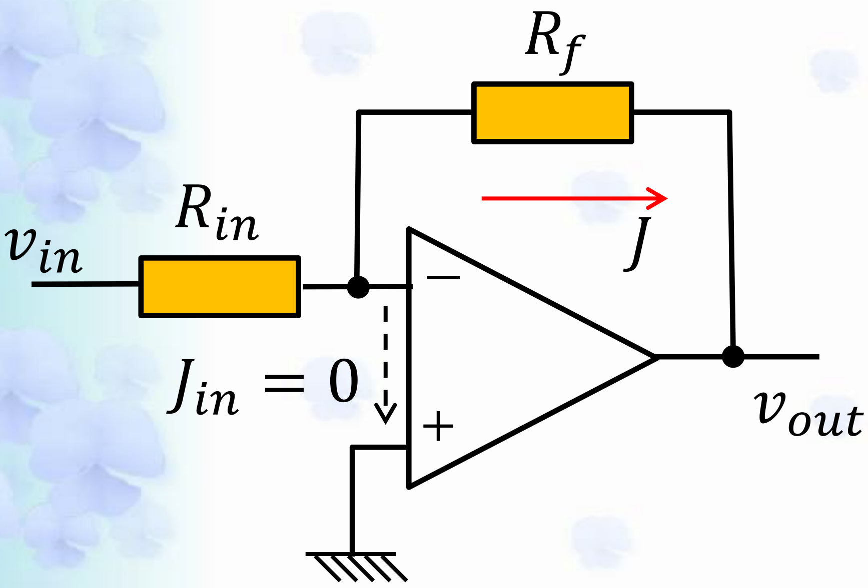
(iv) input: series, output: series

Operational amplifier (OP amp.)



- Differential amplifier
- Input impedance ~∞
- Open loop gain $A_o \gg 1$
- Output resistance ≈ 0

Case (iv)



$$A_\mathrm{o} \gg 1 \quad \therefore V_- \approx V_+ = 0$$

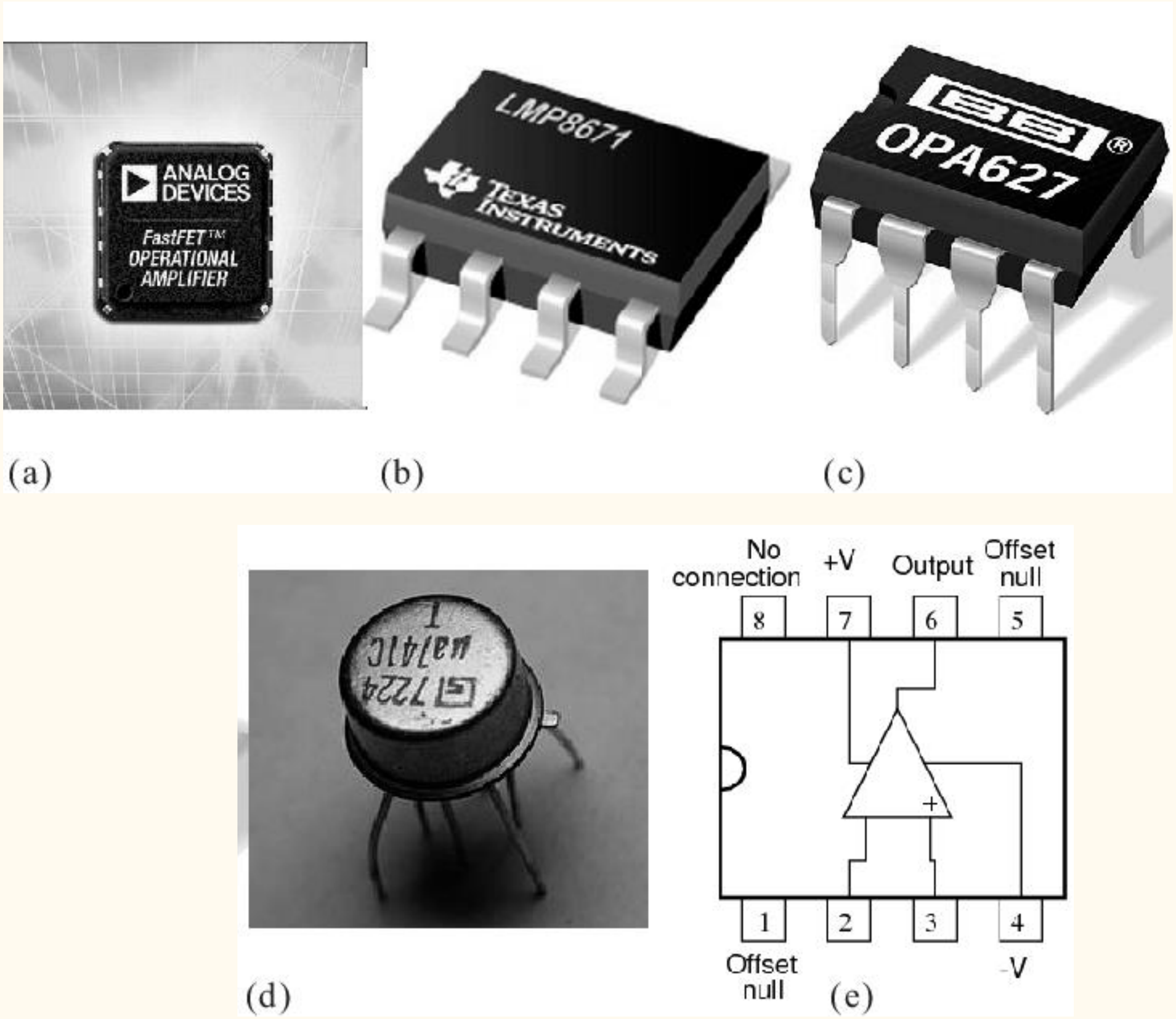
Virtual short circuit

$$J = -\frac{v_\mathrm{out}}{R_f} = \frac{v_\mathrm{in}}{R_\mathrm{in}}$$

$$\therefore v_\mathrm{out} = -\frac{R_f}{R_\mathrm{in}} v_\mathrm{in}$$

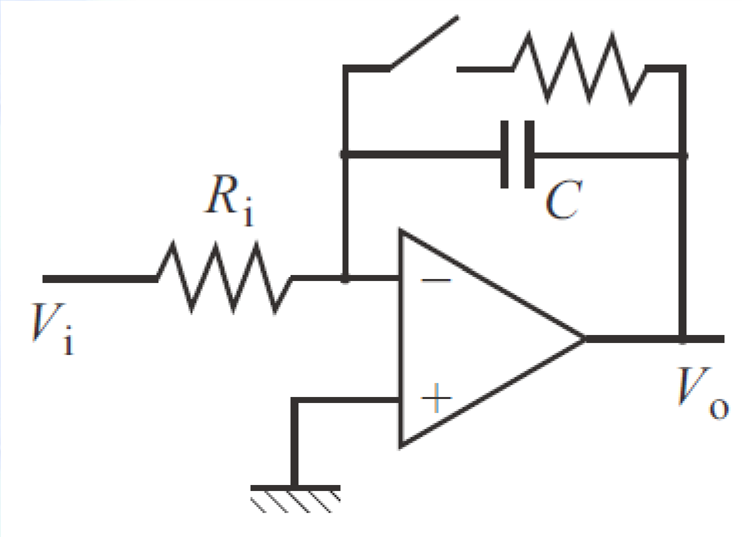
Inverting amplifier

Opamp packages



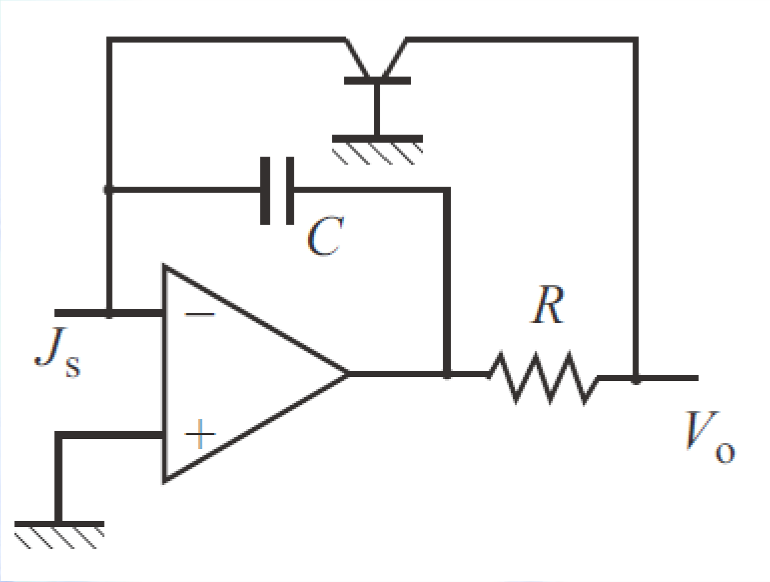
(a) (b) (c) (d) (e)

Various applications of OP amps



$$V_{\text{out}}(t) = -\frac{Q}{C} = -\frac{1}{C}\int_0^t \frac{V_i(\tau)}{R_\text{i}} d\tau$$
$$= -\frac{1}{CR_\text{i}}\int_0^t V_\text{i}(\tau) d\tau$$

Integration circuit



$$V\text{out} = -V\text{BE} = -\frac{k_\text{B}T}{e}\ln\left(\frac{J_s}{J_0}+1\right)$$

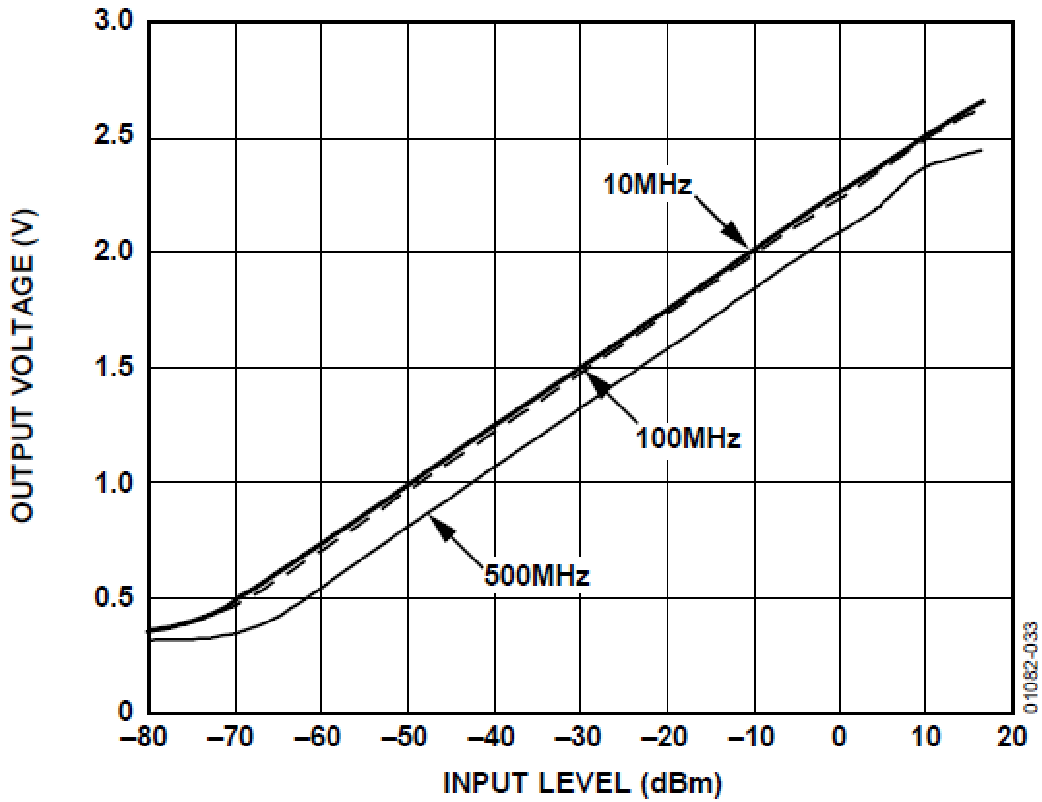
Logarithmic amplifier

Logarithmic Amplifier

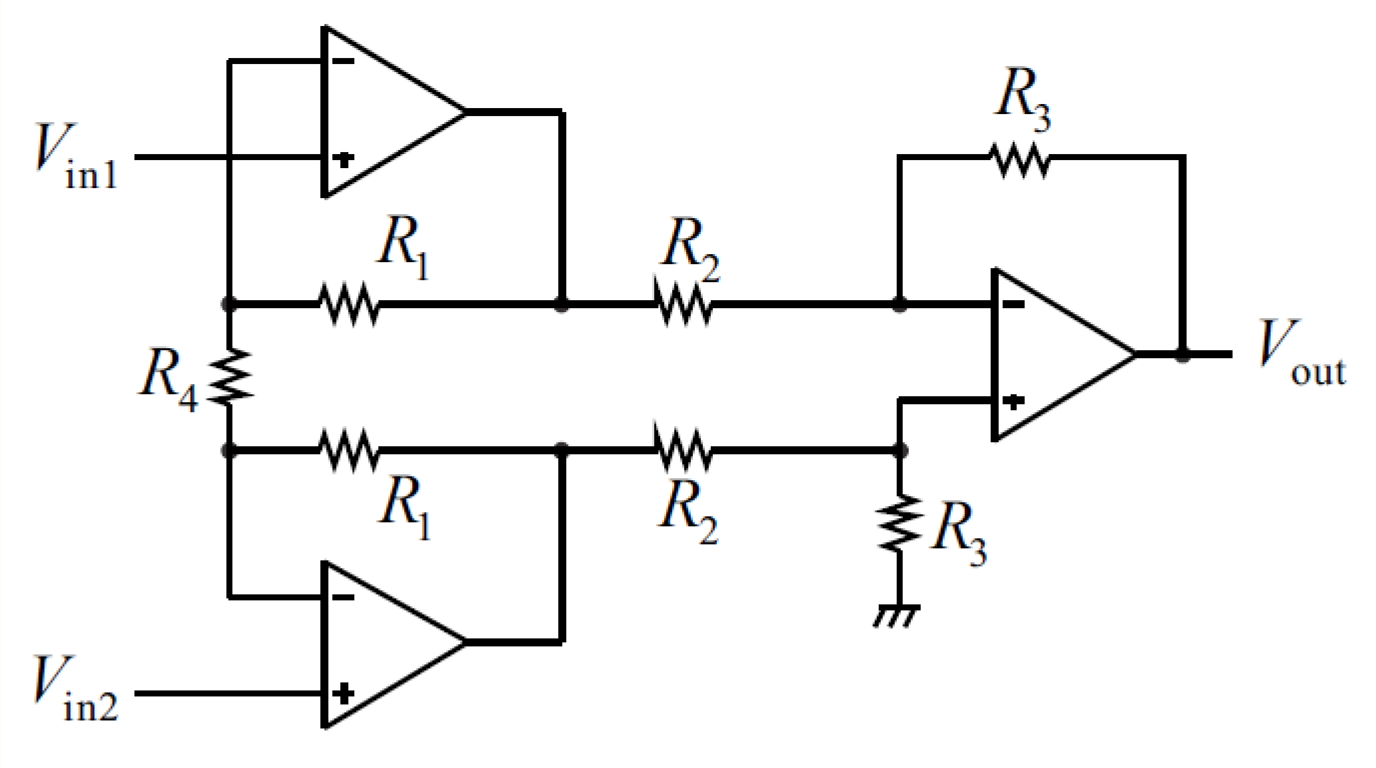
ANALOG DEVICES

Low Cost, DC to 500 MHz, 92 dB Logarithmic Amplifier

Data Sheet **AD8307**



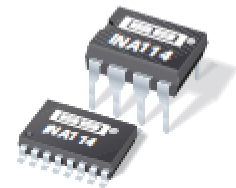
Instrumentation amplifier



$$V_{\text{out}} = -\frac{R_3}{R_2}\left(\frac{2R_1 + R_4}{R_4}\right)(V_{\text{in1}} - V_{\text{int2}})$$

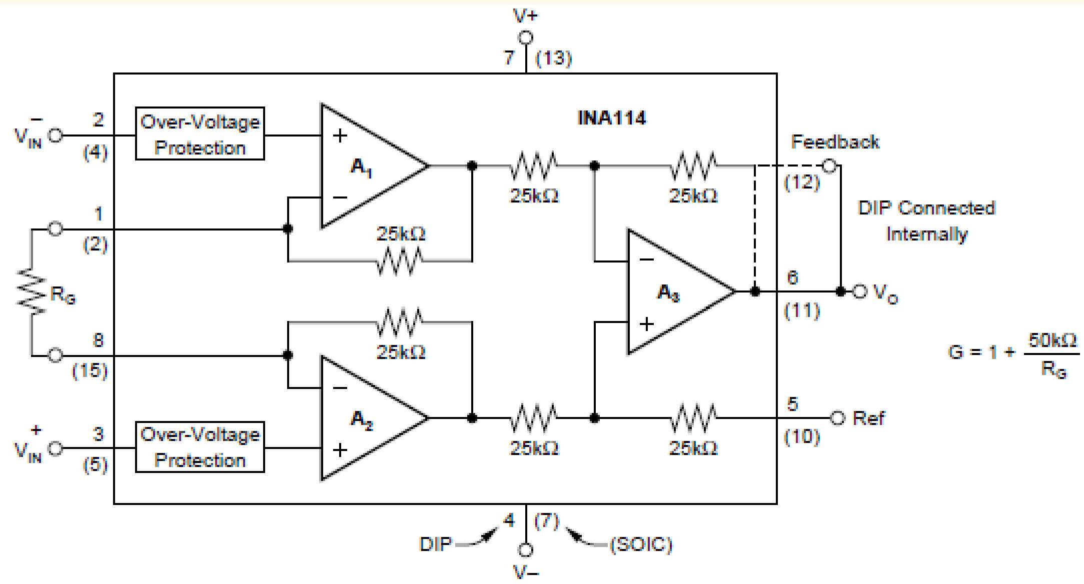
Instrumentation amplifier

BURR - BROWN®



INA114

Precision
INSTRUMENTATION AMPLIFIER



OP-amp data sheet

ANALOG DEVICES

Ultralow Offset Voltage Operational Amplifier

Data Sheet **OP07**

FEATURES

- Low V_{OS}: 75 µV maximum
- Low V_{OS} drift: 1.3 µV/°C maximum
- Ultrastable vs. time: 1.5 µV per month maximum
- Low noise: 0.6 µV p-p maximum
- Wide input voltage range: ±14 V typical
- Wide supply voltage range: ±3 V to ±18 V
- 125°C temperature-tested dice

PIN CONFIGURATION

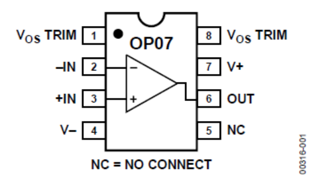
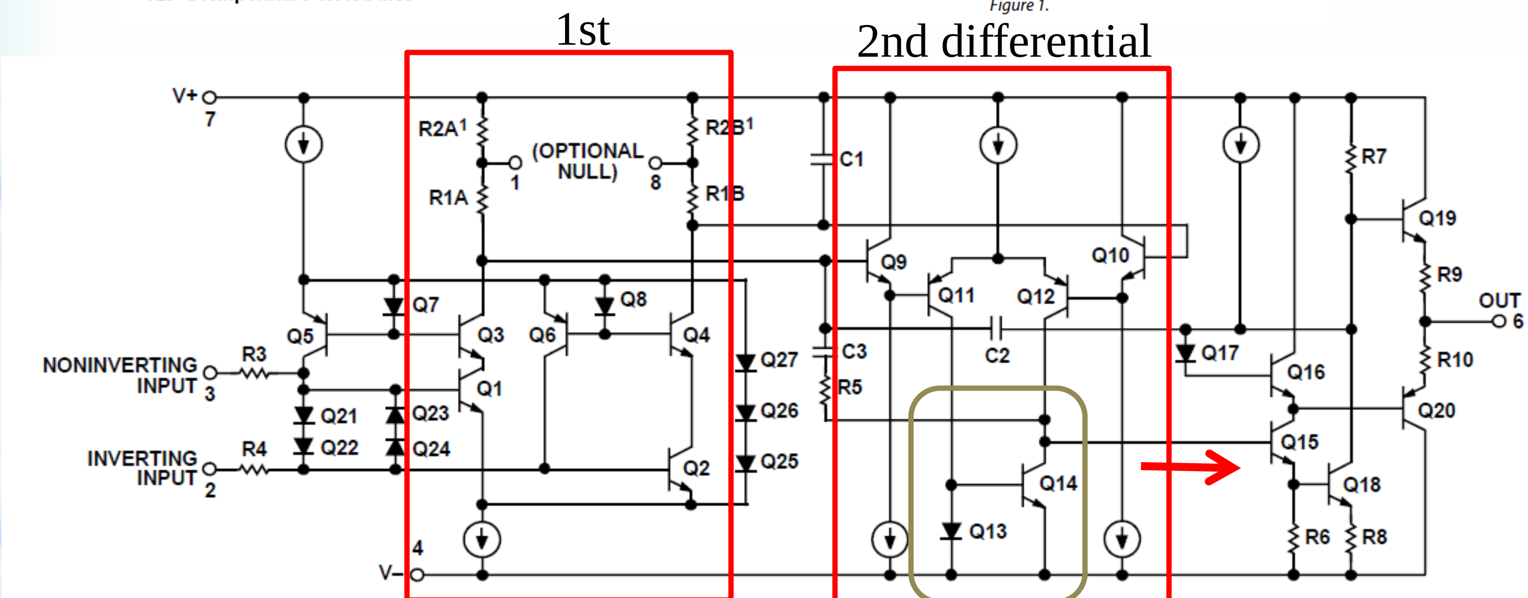


Figure 1.



¹ R2A AND R2B ARE ELECTRONICALLY ADJUSTED ON CHIP AT FACTORY FOR MINIMUM INPUT OFFSET VOLTAGE.

Figure 2. Simplified Schematic

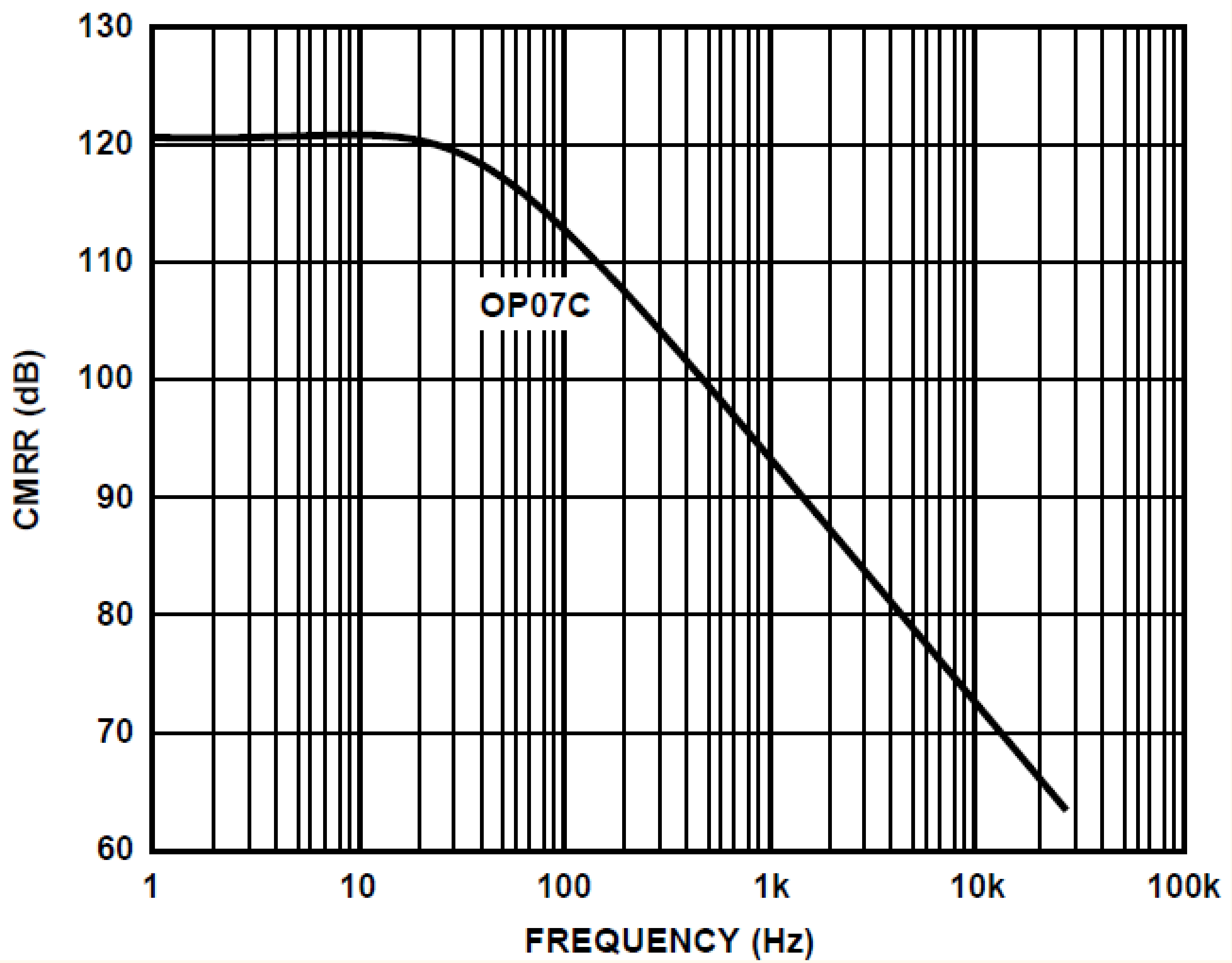
OP-amp data sheet

Parameters

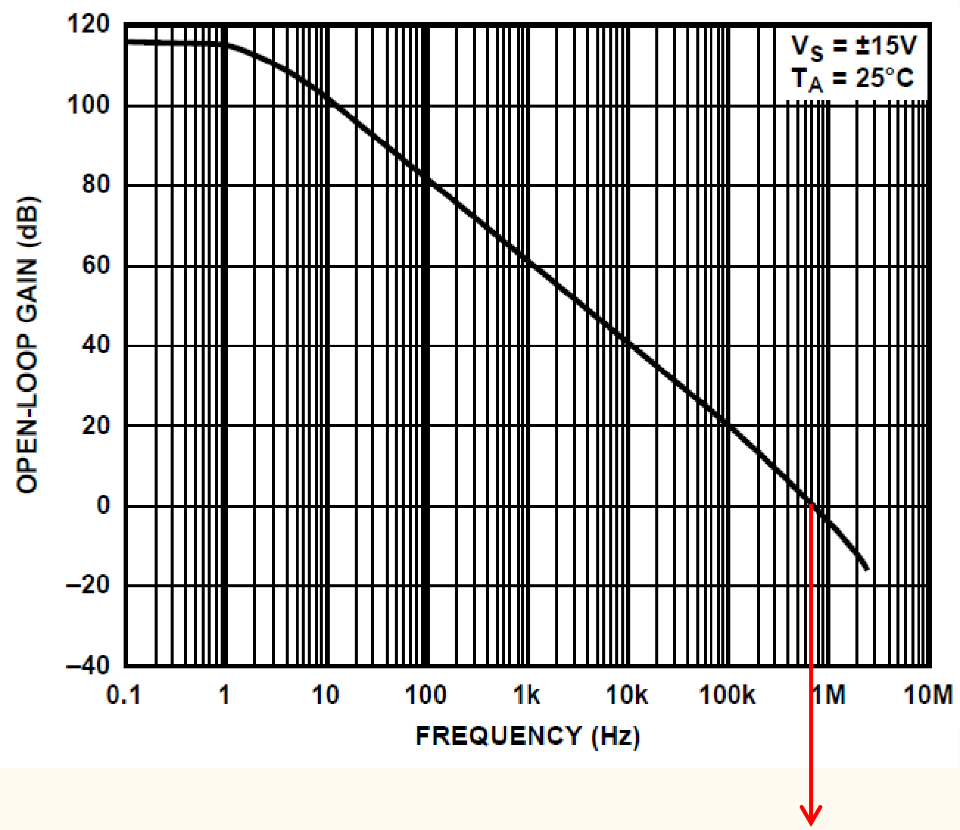
Parameter	Symbol	Conditions	Min	Typ	Max	Unit
INPUT CHARACTERISTICS						
$T_A = 25°C$						
Input Offset Voltage[1]	V_{OS}			60	150	μV
Long-Term V_{OS} Stability[2]	V_{OS}/Time			0.4	2.0	μV/Month
Input Offset Current	I_{OS}			0.8	6.0	nA
Input Bias Current	I_B			±1.8	±7.0	nA
Input Noise Voltage	e_n p-p	0.1 Hz to 10 Hz[3]		0.38	0.65	μV p-p
Input Noise Voltage Density	e_n	f_O = 10 Hz		10.5	20.0	nV/√Hz
		f_O = 100 Hz[3]		10.2	13.5	nV/√Hz
		f_O = 1 kHz		9.8	11.5	nV/√Hz
Input Noise Current	I_n p-p			15	35	pA p-p
Input Noise Current Density	I_n	f_O = 10 Hz		0.35	0.90	pA/√Hz
		f_O = 100 Hz[3]		0.15	0.27	pA/√Hz
		f_O = 1 kHz		0.13	0.18	pA/√Hz
Input Resistance, Differential Mode[4]	R_{IN}		8	33		MΩ
Input Resistance, Common Mode	R_{INCM}			120		GΩ

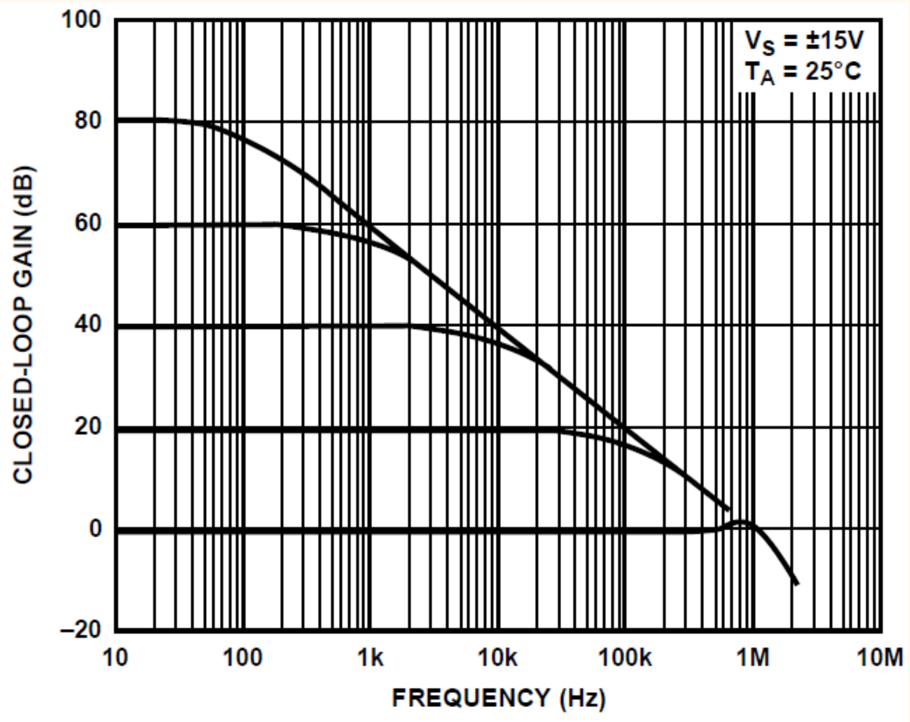
OP-amp data sheet

Common mode rejection ratio (CMRR)

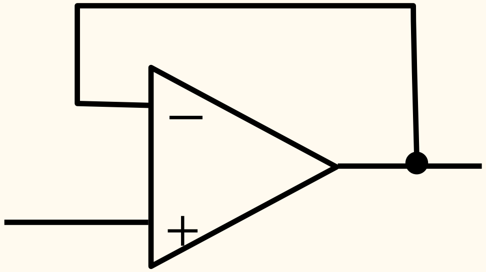


OP-amp data sheet



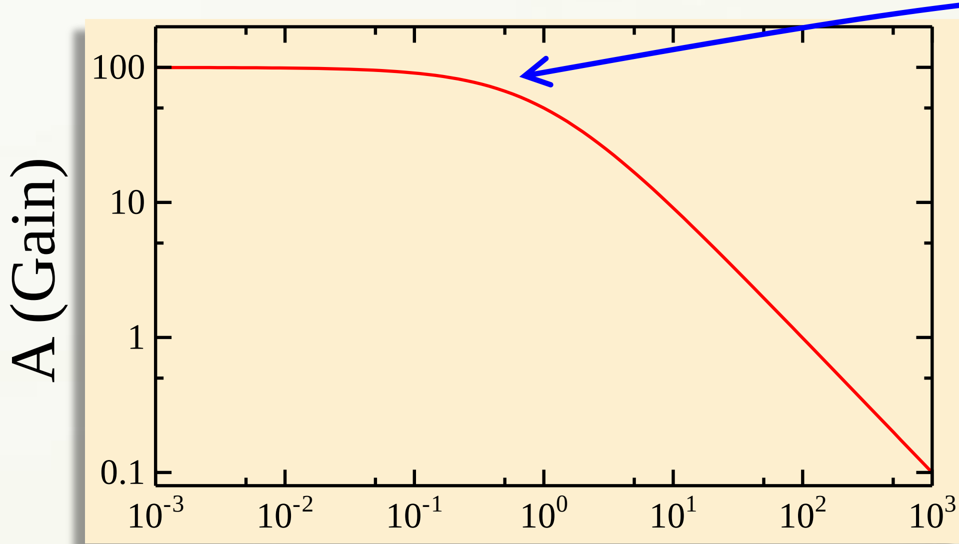


Unity gain frequency



Voltage follower

Frequency Dependent Characteristics of OP-Amps

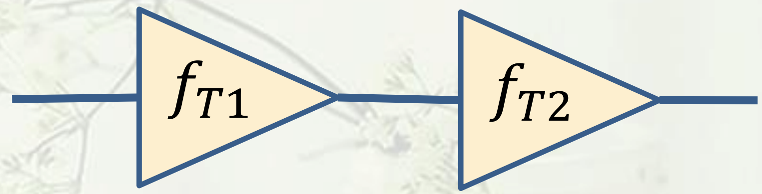


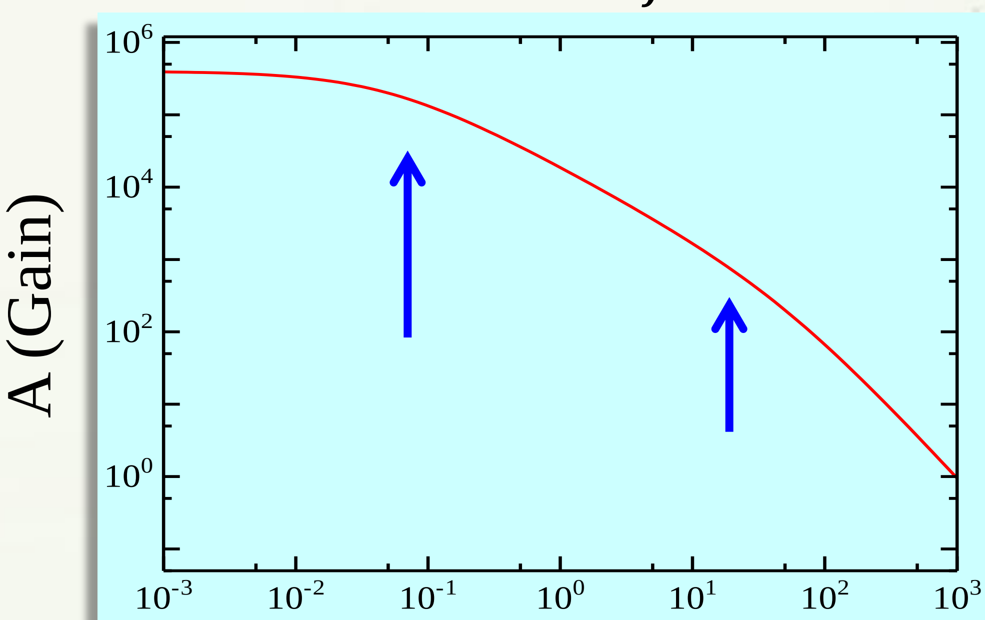
$\omega = 2\pi f$

Cut-off frequency

$\omega_T = 2\pi f_T$

Phase rotates by $\pi/2$





Multiple cut-off frequency:
Phase rotates more than π

If gain is larger than 1 at phase shift π :

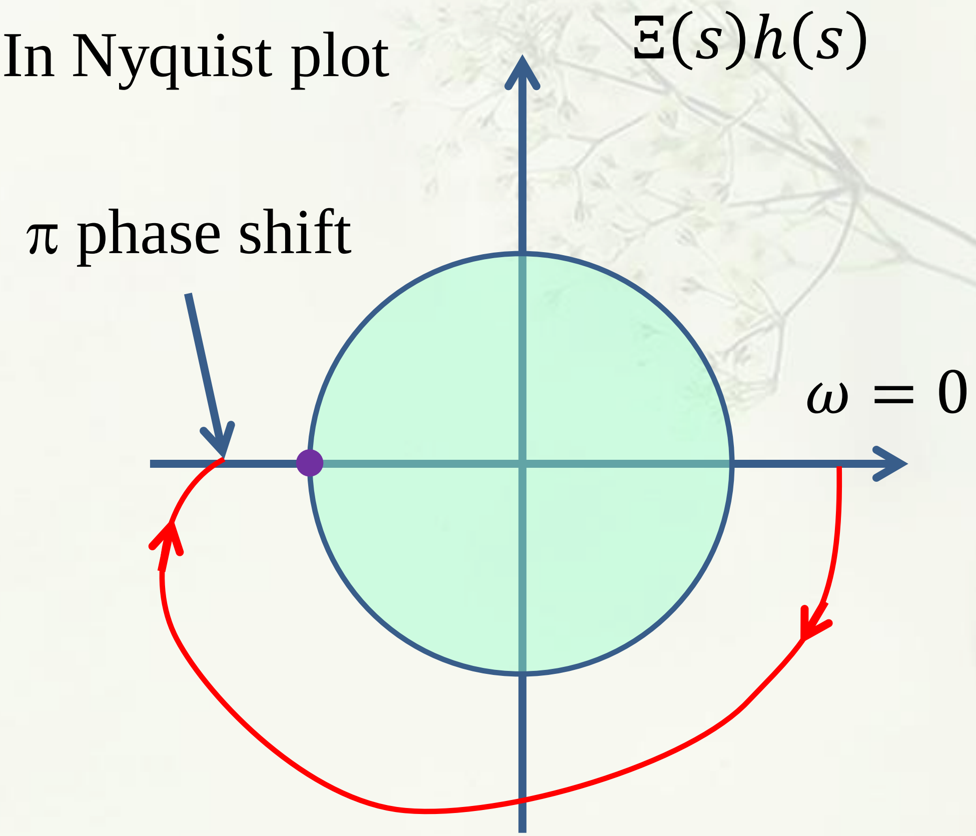
Dangerous!

Phase compensation

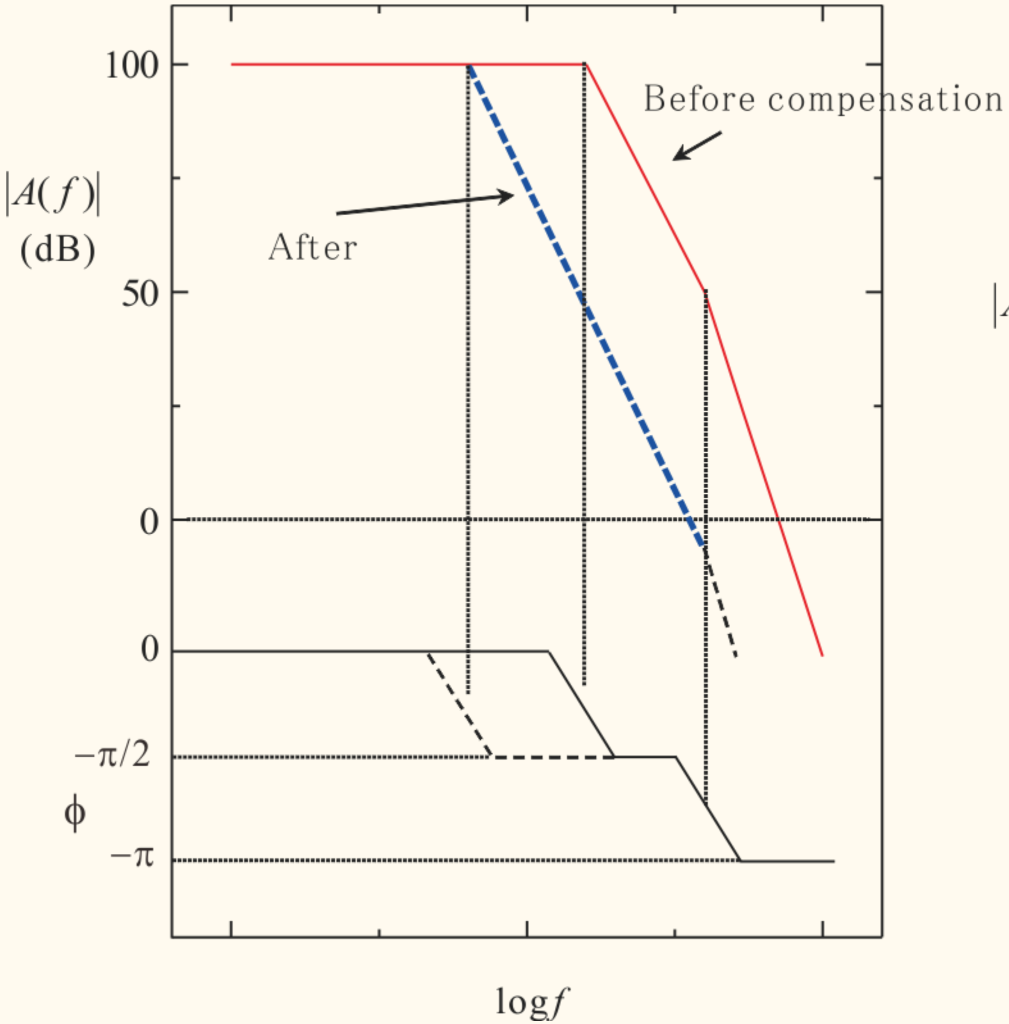
Why dangerous?

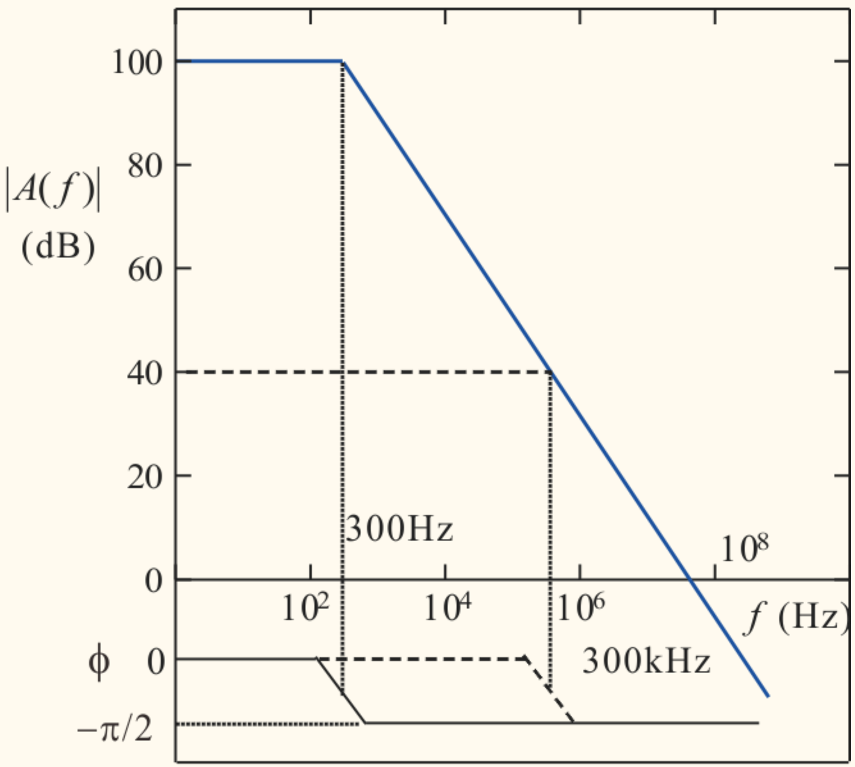
π phase shift: negative feedback → positive feedback

In Nyquist plot

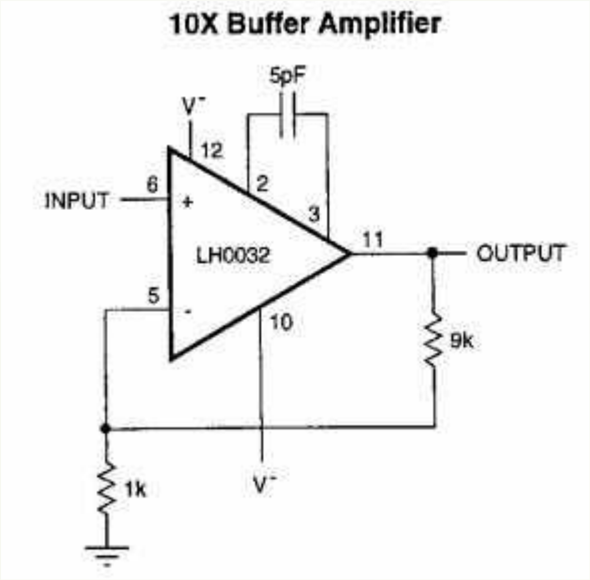


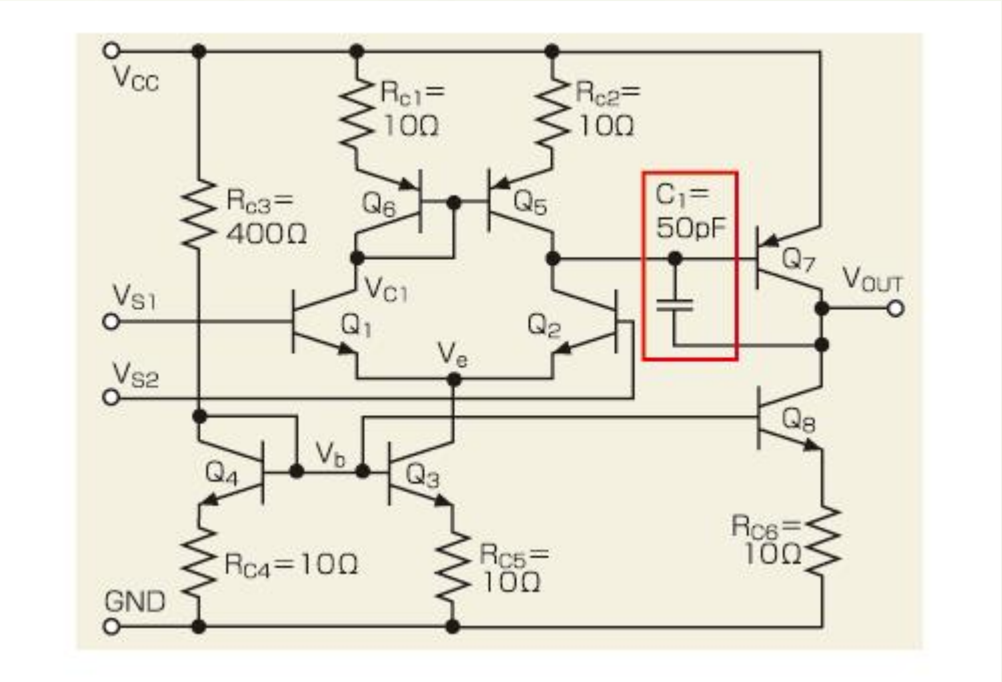
Phase compensation



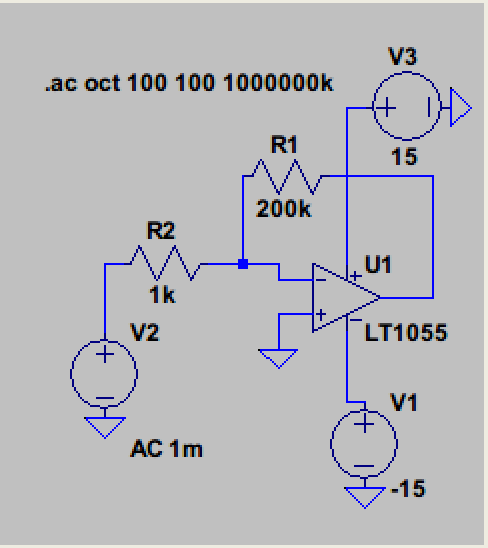


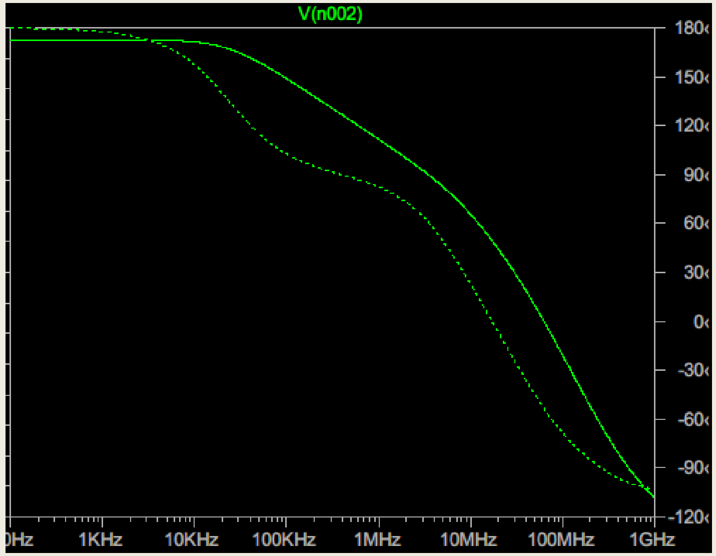
Phase compensation



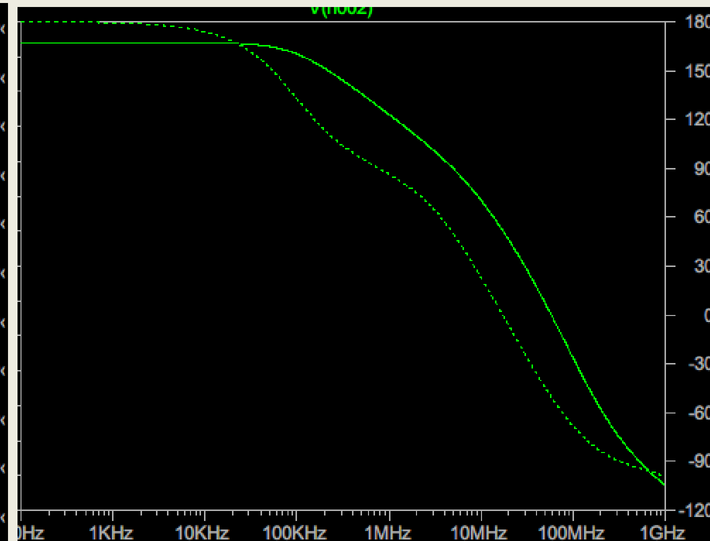


Inverting amplifier and cut-off frequency

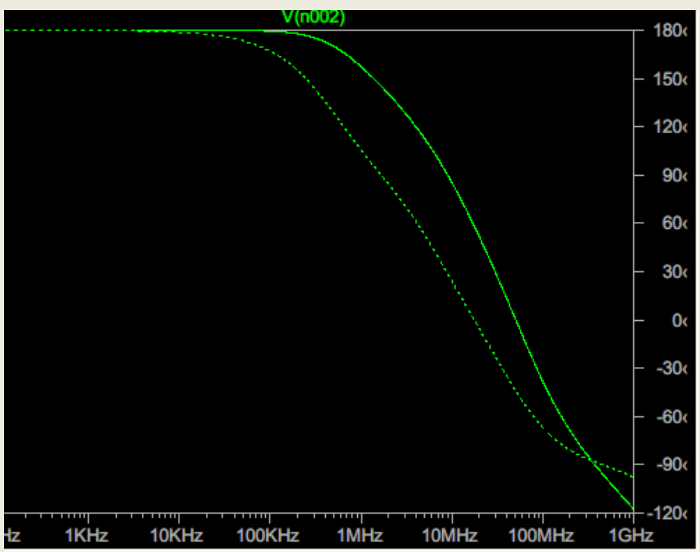




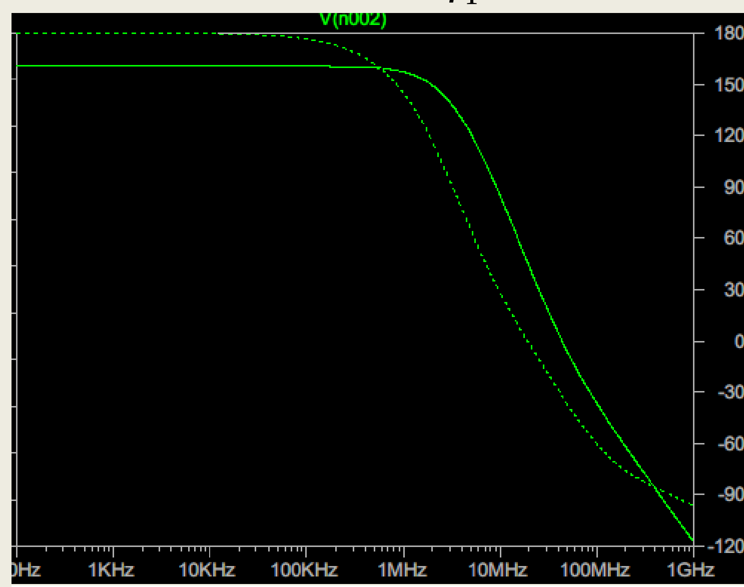
$A = 200$ $f_T = 30\text{kHz}$



$A = 50$ $f_T = 90\text{kHz}$

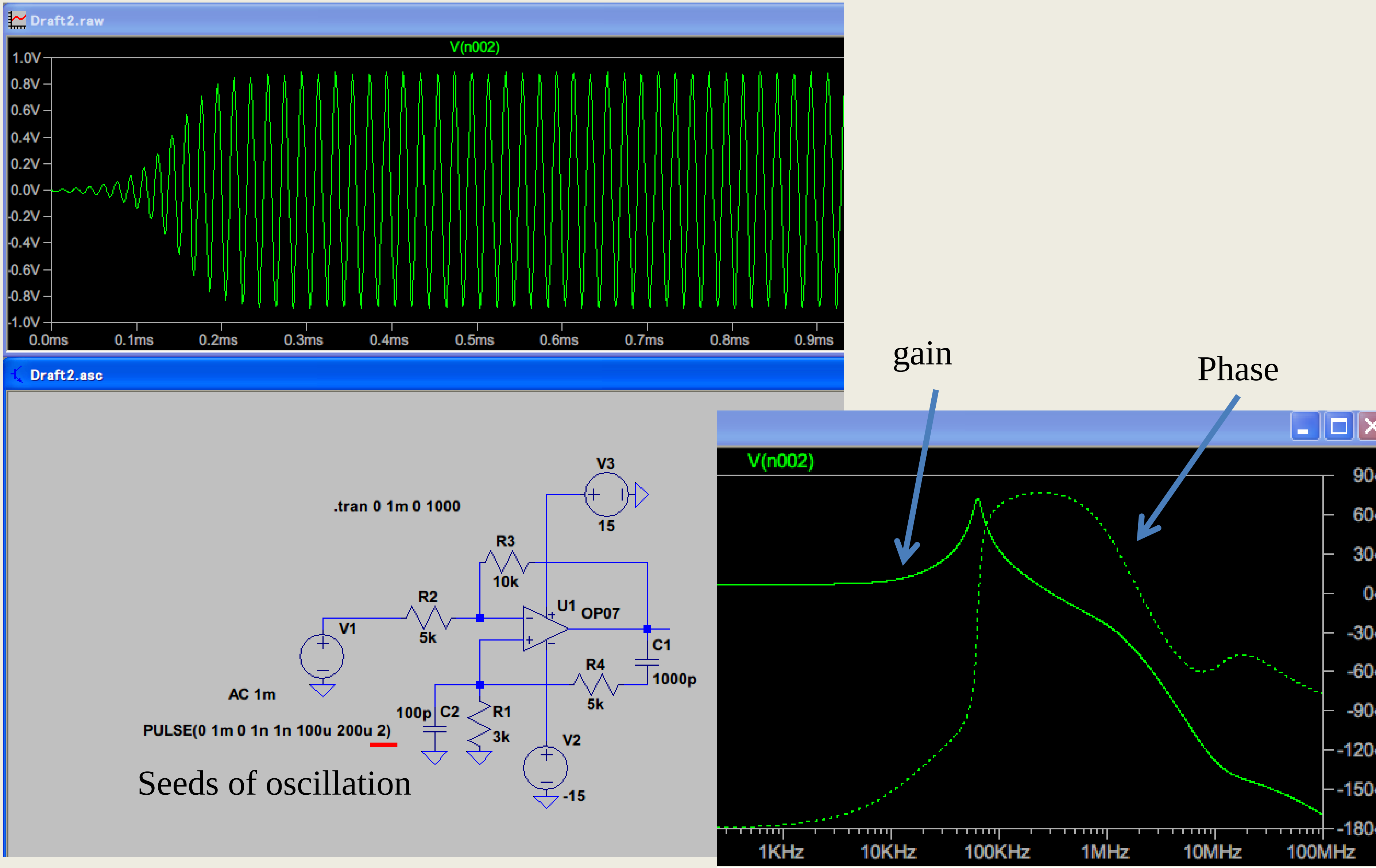


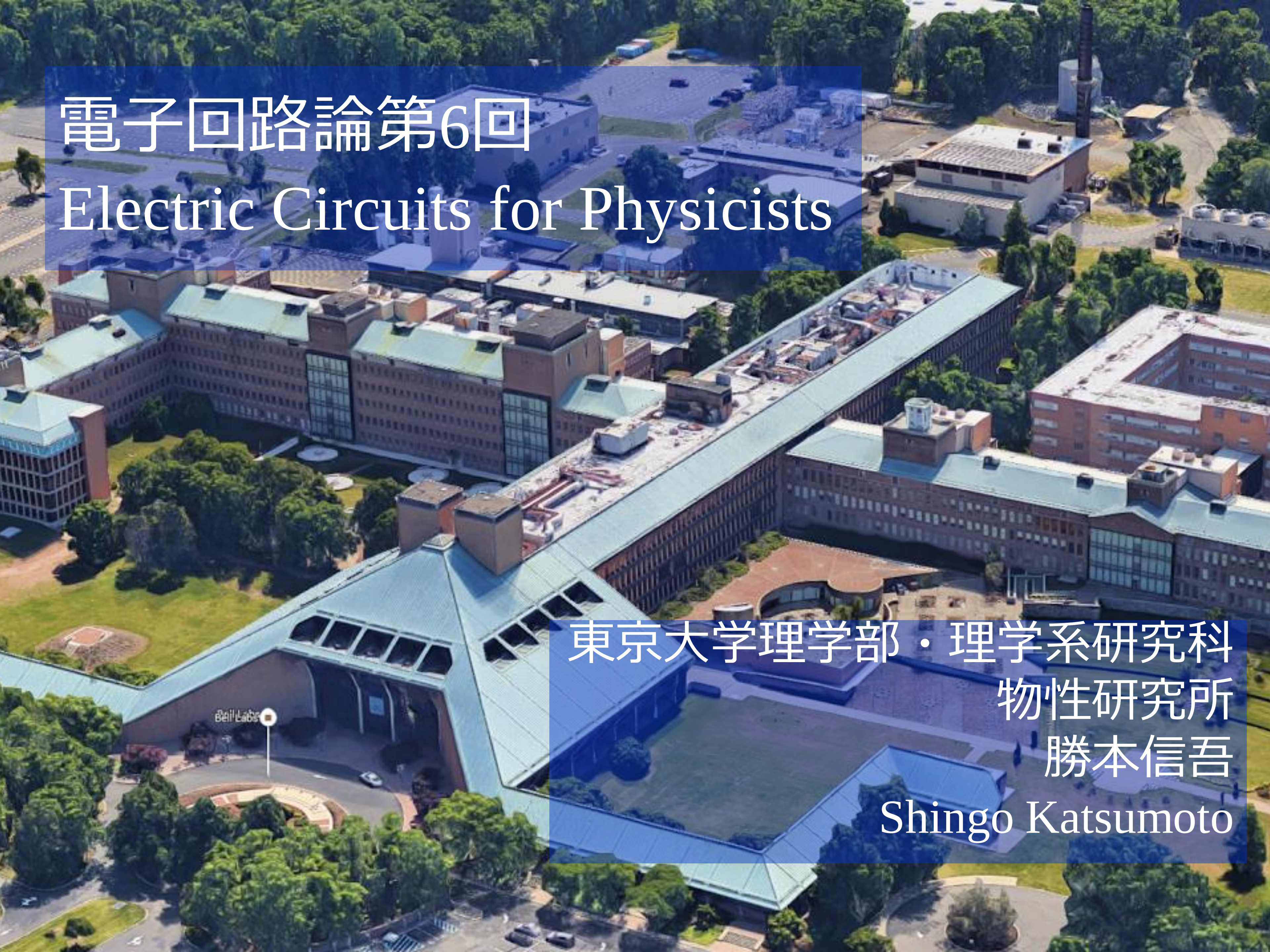
$A = 10$ $f_T = 300\text{kHz}$



$A = 2$ $f_T = 2\text{MHz}$

Oscillation of OPamp





電子回路論第6回
Electric Circuits for Physicists

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Outline

4.3 Feedback control

4.3.1 Disturbance and noise

4.3.2 PID control

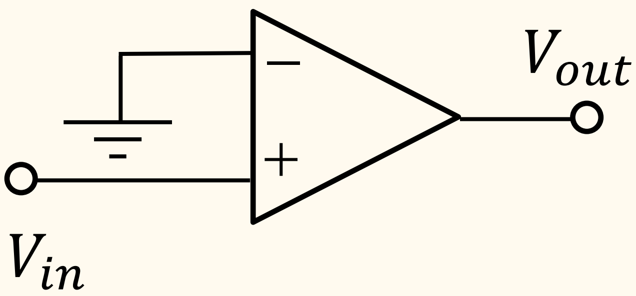
4.4 PN junction transistors

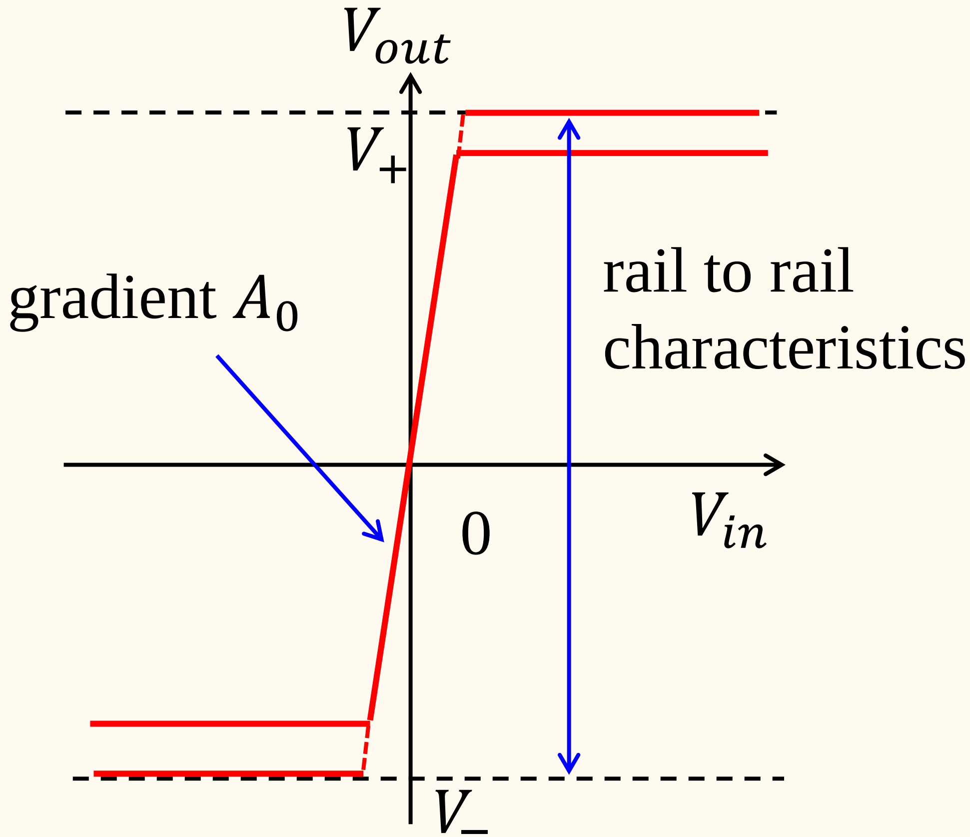
4.4.1 Diodes

4.4.2 Bipolar junction transistors

4.5 Field effect transistors

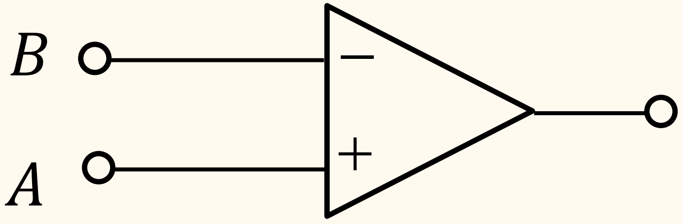
Comment: Use of OP-amp at saturation voltages



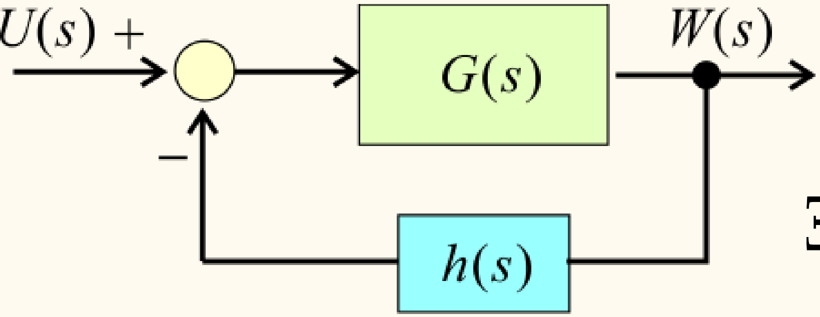


Compare V_{in} with 0

Comparator



Hurwitz criterion



Adolf Hurwitz
1859 - 1919

$$\Xi(s) = \frac{G(s)}{1 + h(s)G(s)}$$

Pole equation: $$\text{(denominator)} = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0$$
$$= a_n(s - p_1)\cdots(s - p_n) = 0$$

$\forall j = 0, 1, \cdots, n : \quad a_j > 0 \text{ (or } < 0)$ (Otherwise the system is unstable.)

Hurwitz matrix
$n \times n$

$$H = \left(\begin{array}{ccccc} a_{n-1} & a_{n-3} & a_{n-5} & \cdots & 0 \\ a_n & a_{n-2} & a_{n-4} & \cdots & 0 \\ \hline 0 & a_{n-1} & a_{n-3} & \cdots & 0 \\ 0 & a_n & a_{n-2} & \cdots & 0 \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_0 \end{array}\right)$$

Hurwitz criterion

Hurwitz determinants $H_j \equiv |H[1, \cdots, j; 1, \cdots, j]|$

$$H_1 = a_{n-1},\ H_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix},\ H_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix}, \cdots.$$

Hurwitz criterion

$$H_j > 0 \quad (j = 2, \cdots, n = 1)$$

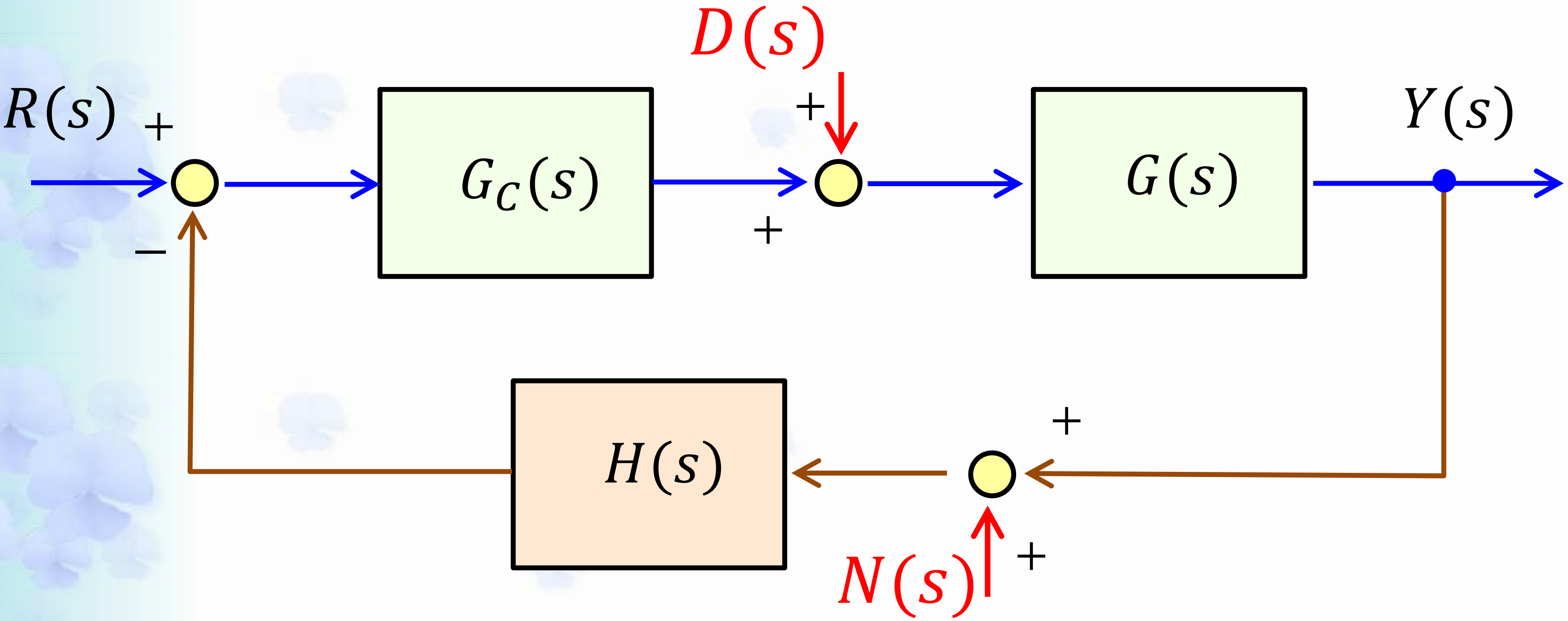
$H_1, H_n > 0$ is trivial from the assumption.

Another expression:

Divide the denominator to odd and even parts $O(s)$ and $E(s)$.
If the zeros of $O(s)$ and $E(s)$ are aligned on the imaginary axis alternatively, the system is stable.

Disturbance and noise on feedback control

Circuit treatment of fluctuations:
- Prepare external power sources
- Express them as transfer functions

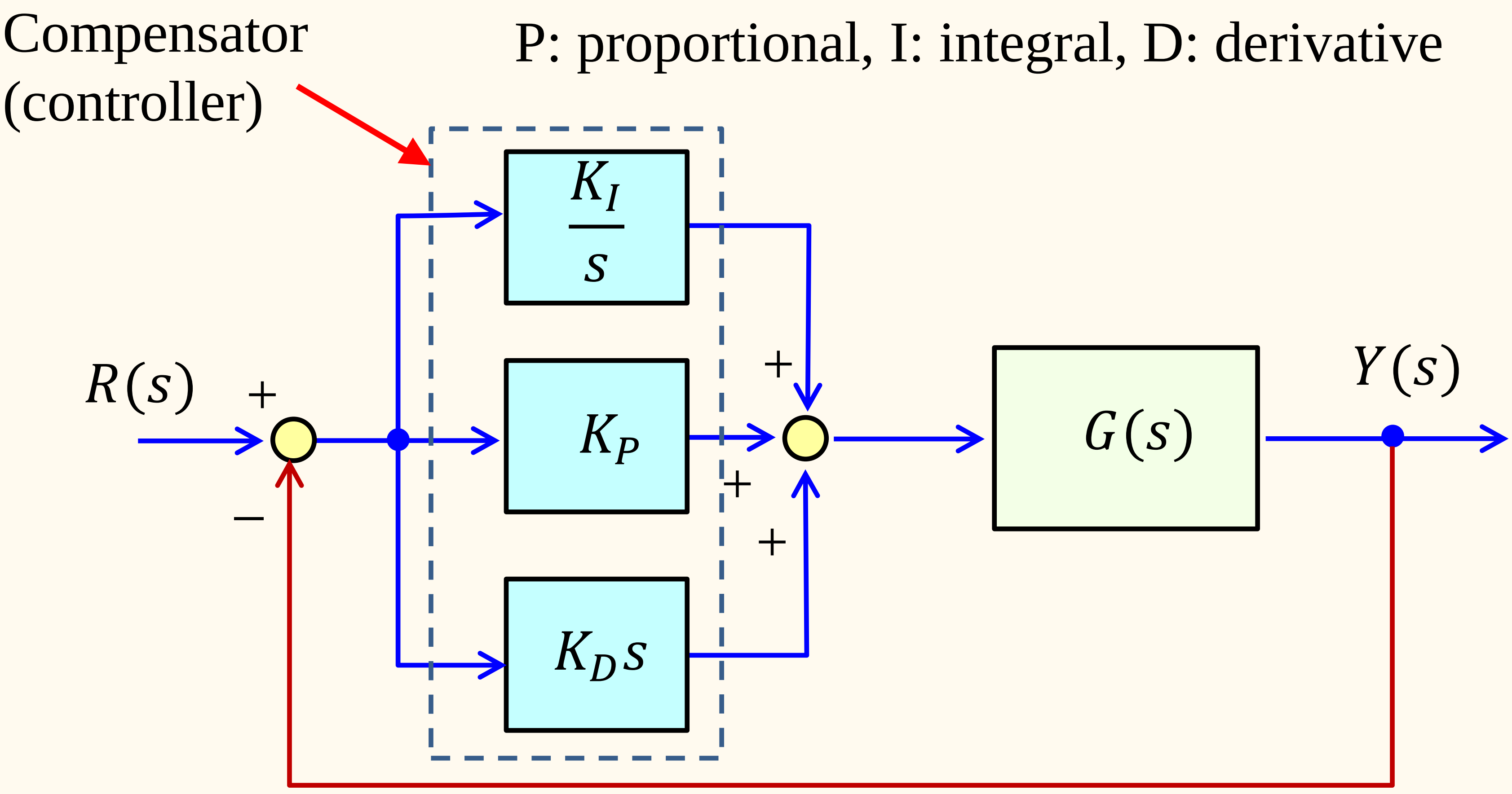


$$Y(s) = \frac{G(s)}{F(s)}[G_C(s)R(s) + D(s) + G_C(s)H(s)N(s)]$$

$$F(s) \equiv 1 + G_C(s)G(s)H(s)$$

PID control

P: proportional, I: integral, D: derivative

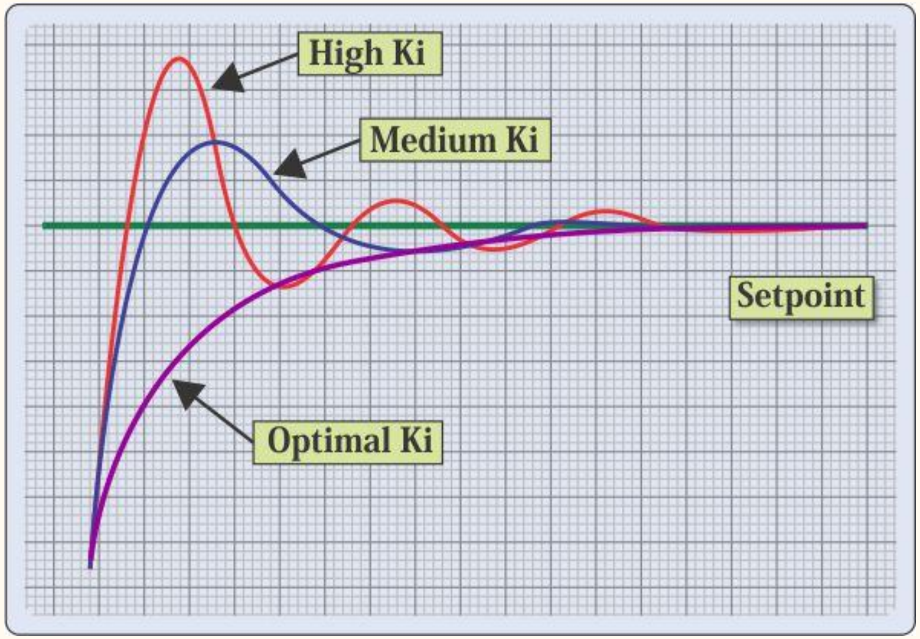


$$G_c(s) = K_P + \frac{K_I}{s} + K_D s$$

PID controllers

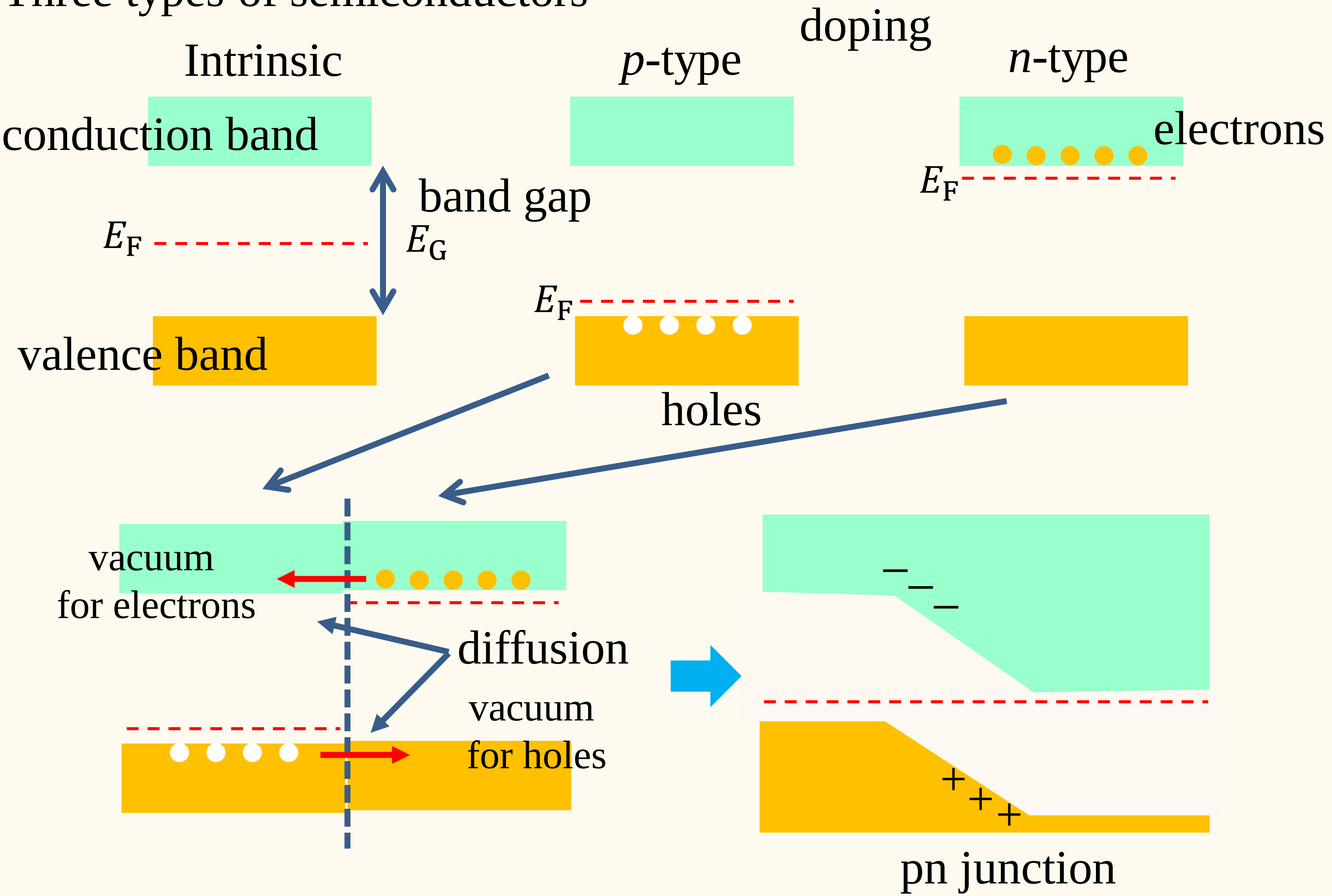
OMRON

invensys
EUROTHERM



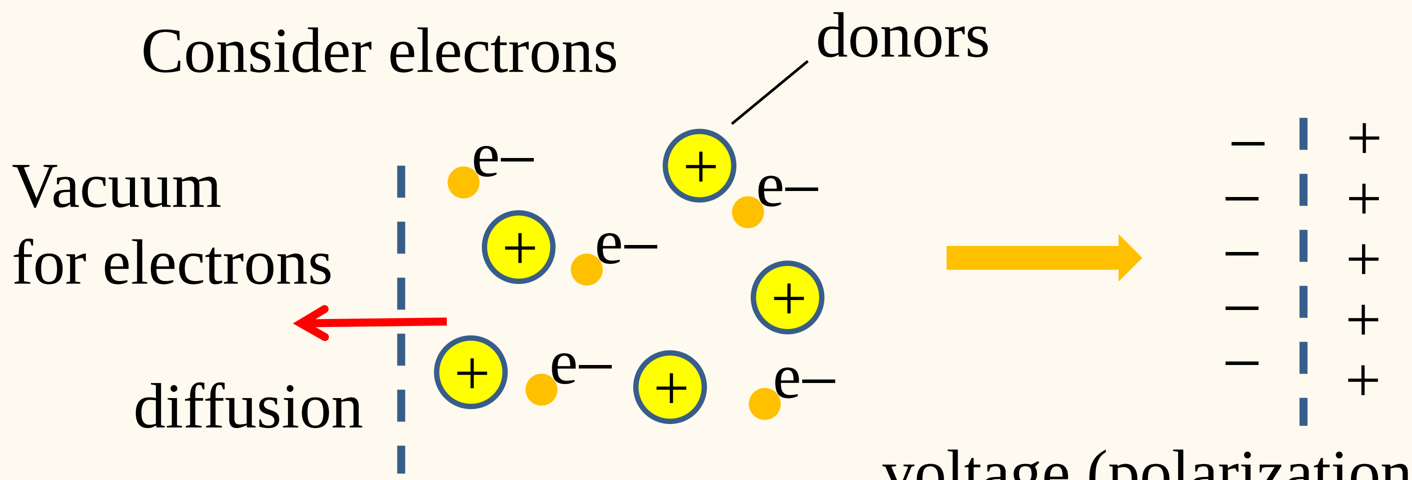
4.4 Example of active element: Transistors

Three types of semiconductors



pn junction thermodynamics

Consider electrons



voltage (polarization) → energy cost

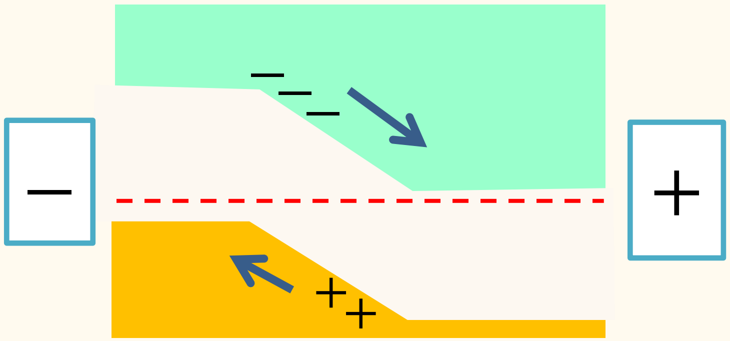
$$F = \underline{U} - T\underline{S}$$

Voltage (internal energy cost)

Diffusion (entropy)

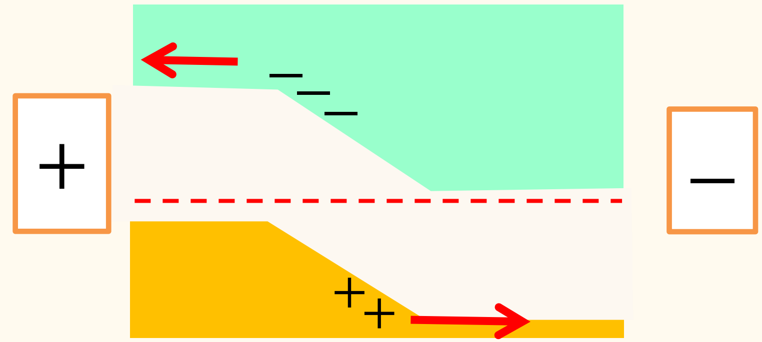
Minimization of F → Built-in (diffusion) voltage V_{bi}

4.4.1 I-V characteristics of pn junctions



Reverse bias

enhances V_{bi} : no go

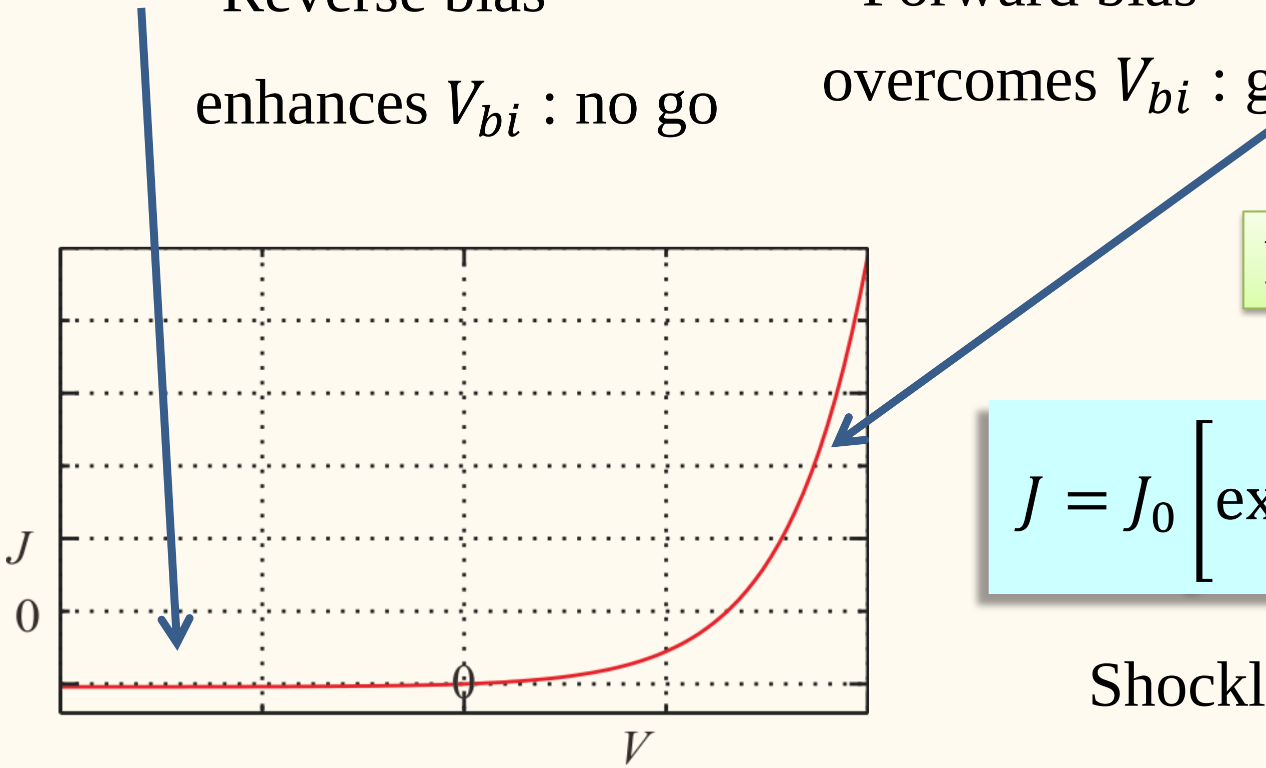


Forward bias

overcomes V_{bi} : go

Minority carrier injection

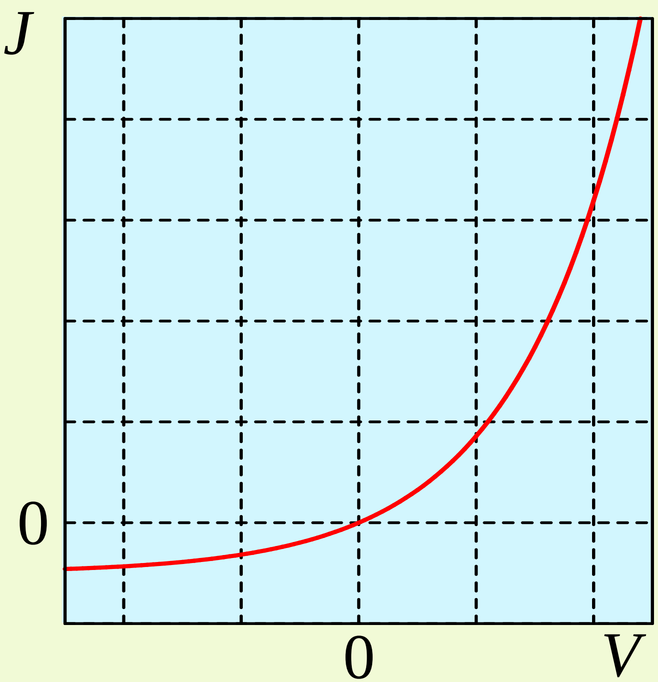
Rectification



$$J = J_0 \left[\exp\left(\frac{eV}{k_B T} \right) - 1 \right]$$

Shockley theory

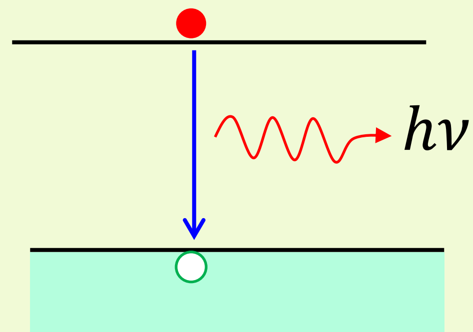
Injection of minority carriers



$$J = e(v_n n_p + v_p p_n)\left[\exp\frac{eV}{k_{\mathrm{B}}T} - 1\right]$$

minority carrier current

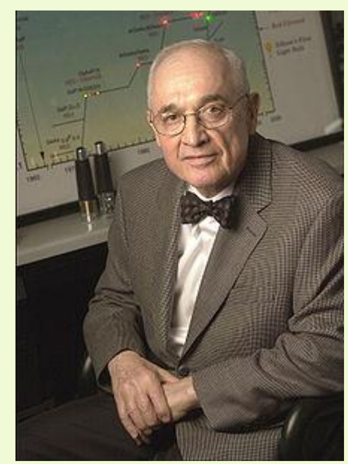
Barrier overflow



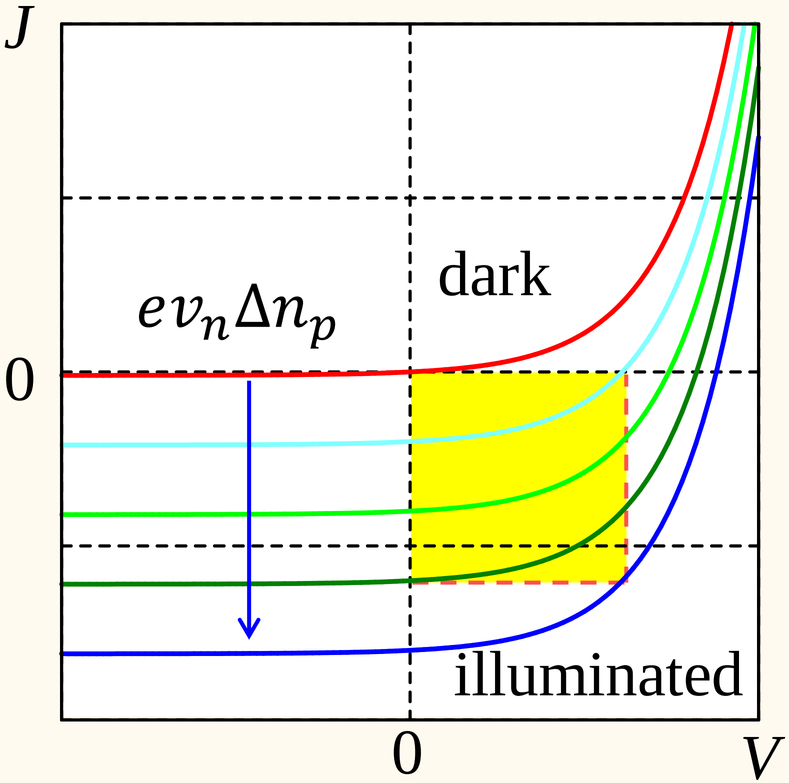
light emitting diode

Fate of injected minority carriers:
Radiative recombination

Nick Holonyak Jr.



Solar cell (injection of minority carriers with illumination)



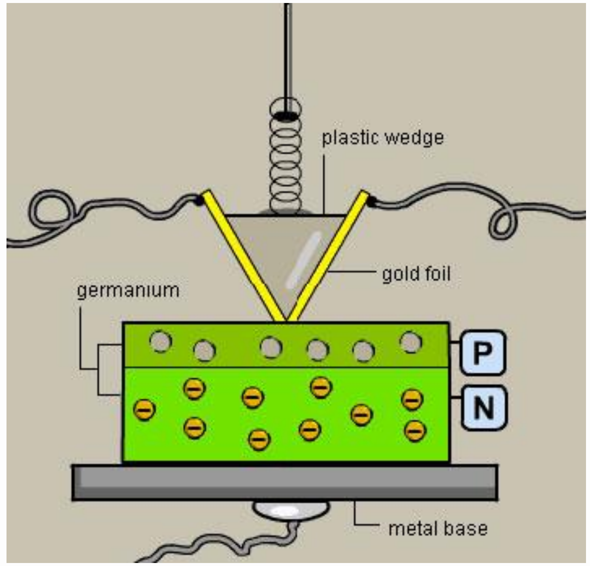
$$J_{e0} = ev_n n_p \left[\exp \frac{eV}{k_B T} - 1 \right]$$

$$J_e = ev_n n_p \exp \frac{eV}{k_B T} - ev_n(n_p + \Delta n_p) = J_{n0} - \underline{ev_n \Delta n_p}$$

External injection

Gerald Pearson,
Daryl Chapin
and Calvin Fuller
at Bell labs. 1954

4.3.2 Discovery and invention of bi-polar transistors

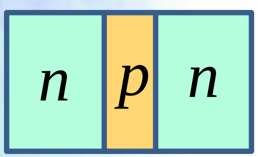


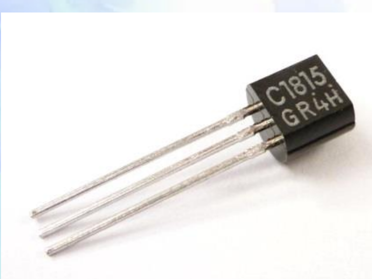
The first point contact transistor

(Dec. 1947
The paper published in June 1948.)

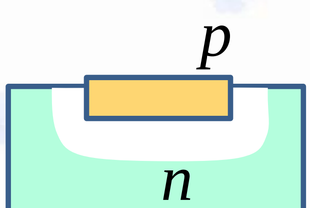
John Bardeen, William Shockley, Walter Brattain 1948 Bell Labs.

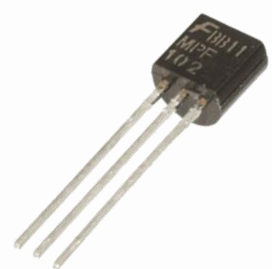
Bipolar junction transistor



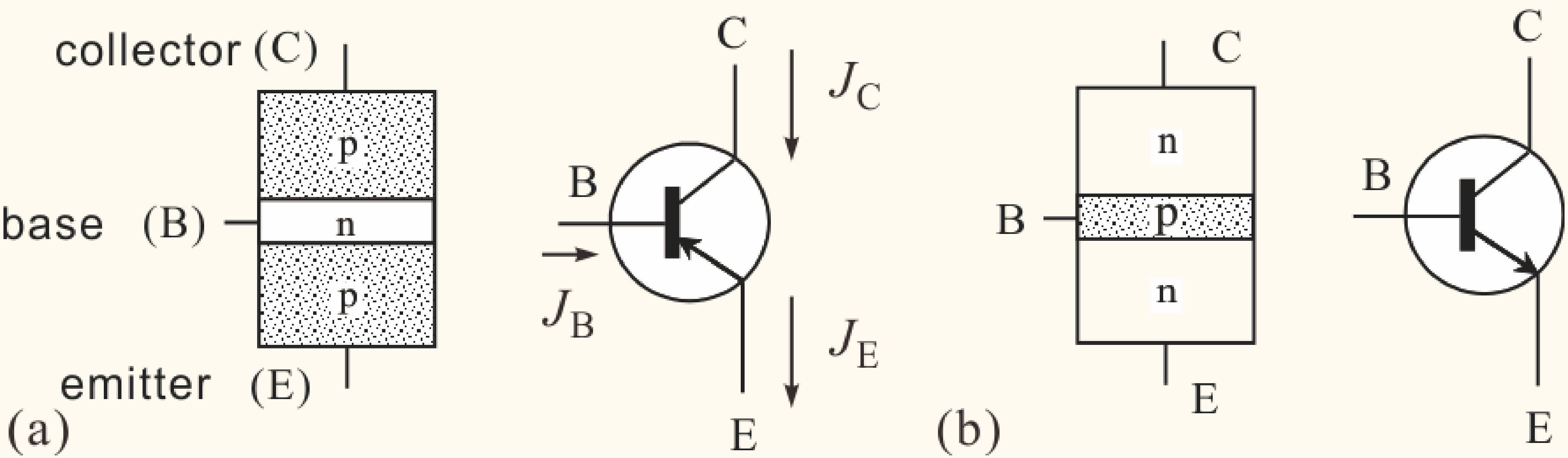


Field effect transistor





Bipolar transistor structures and symbols



PNP type

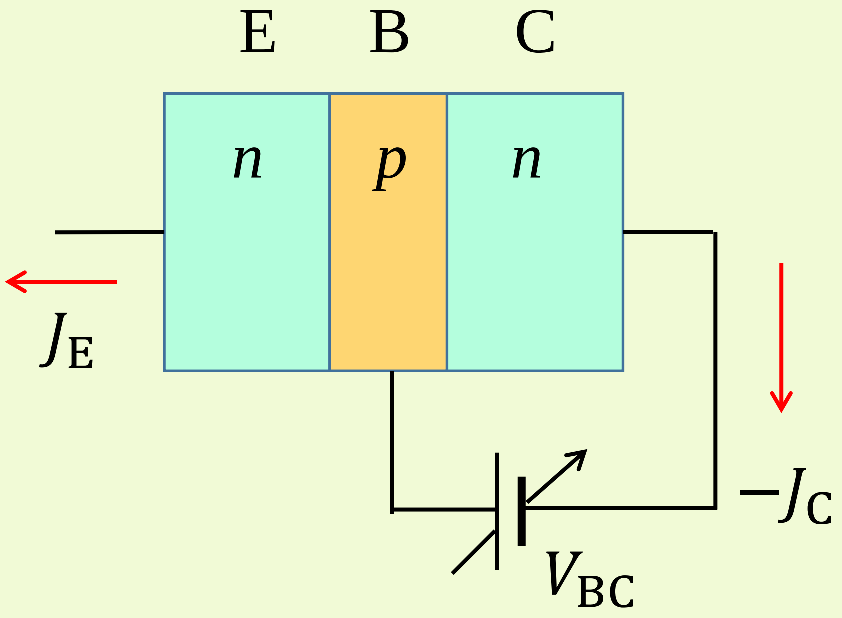
$$L_{\mathrm{B}} < L_h$$

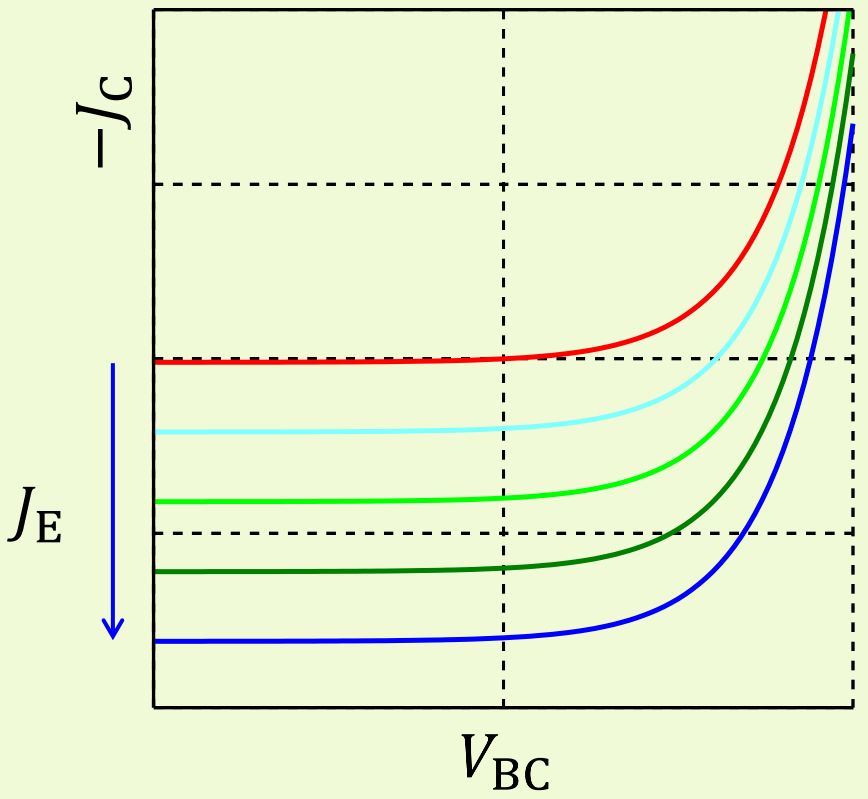
NPN type

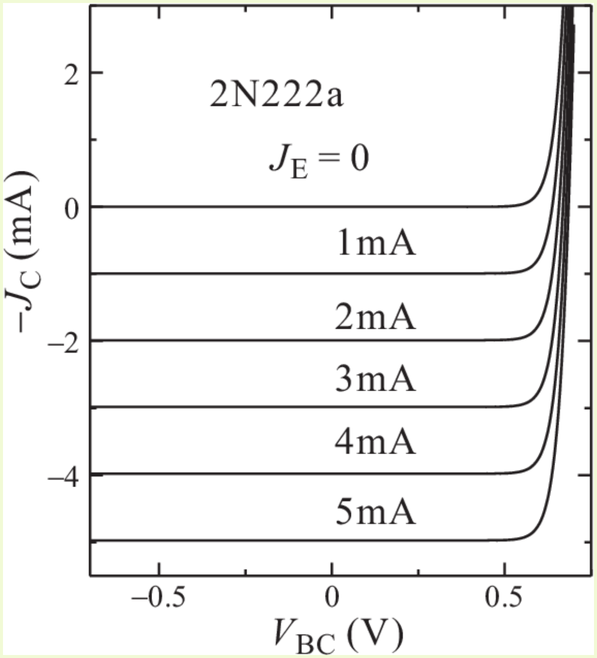
$$L_{\mathrm{B}} < L_e$$

Similar characteristics PNP and NPN: complementary

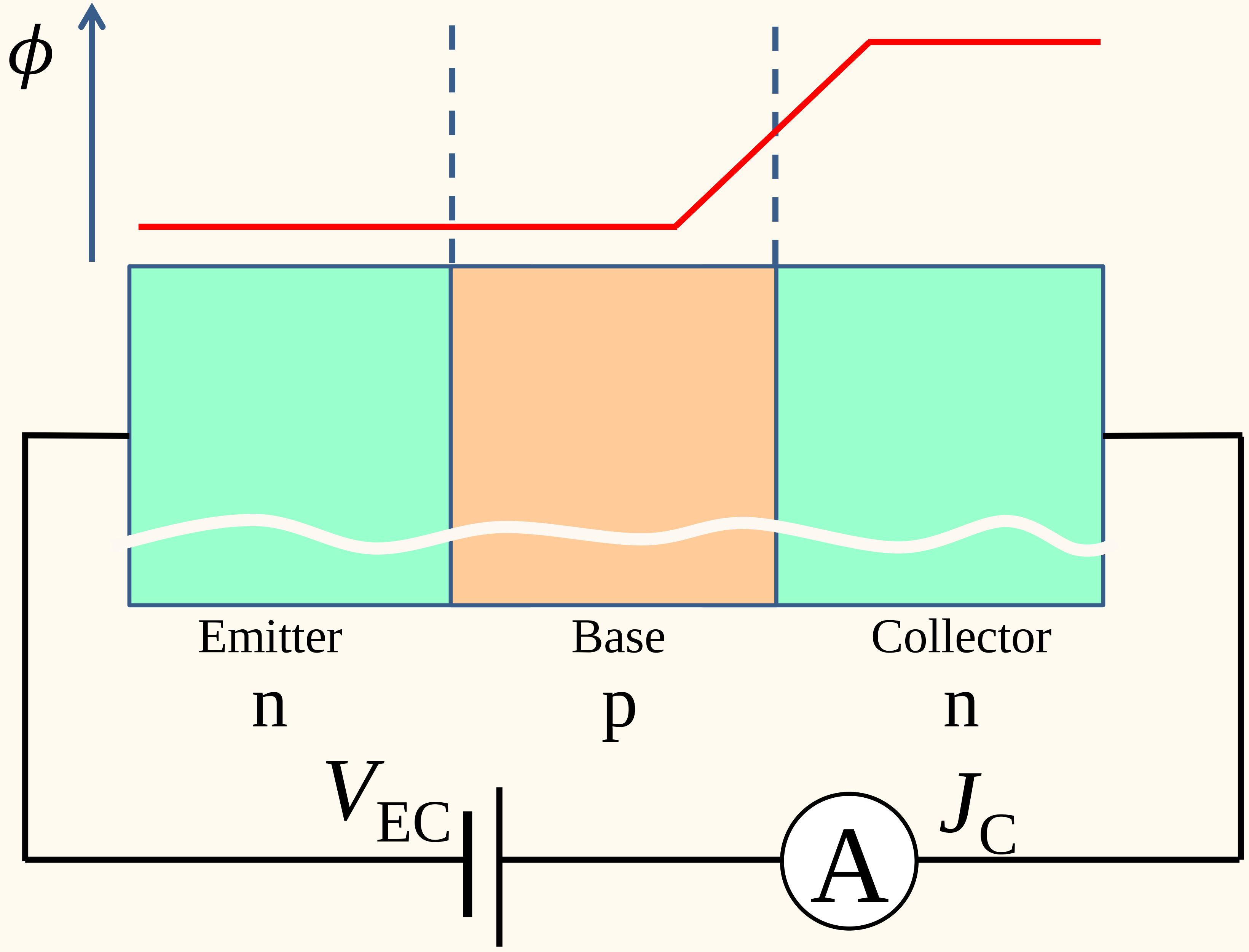
Base-Collector characteristics



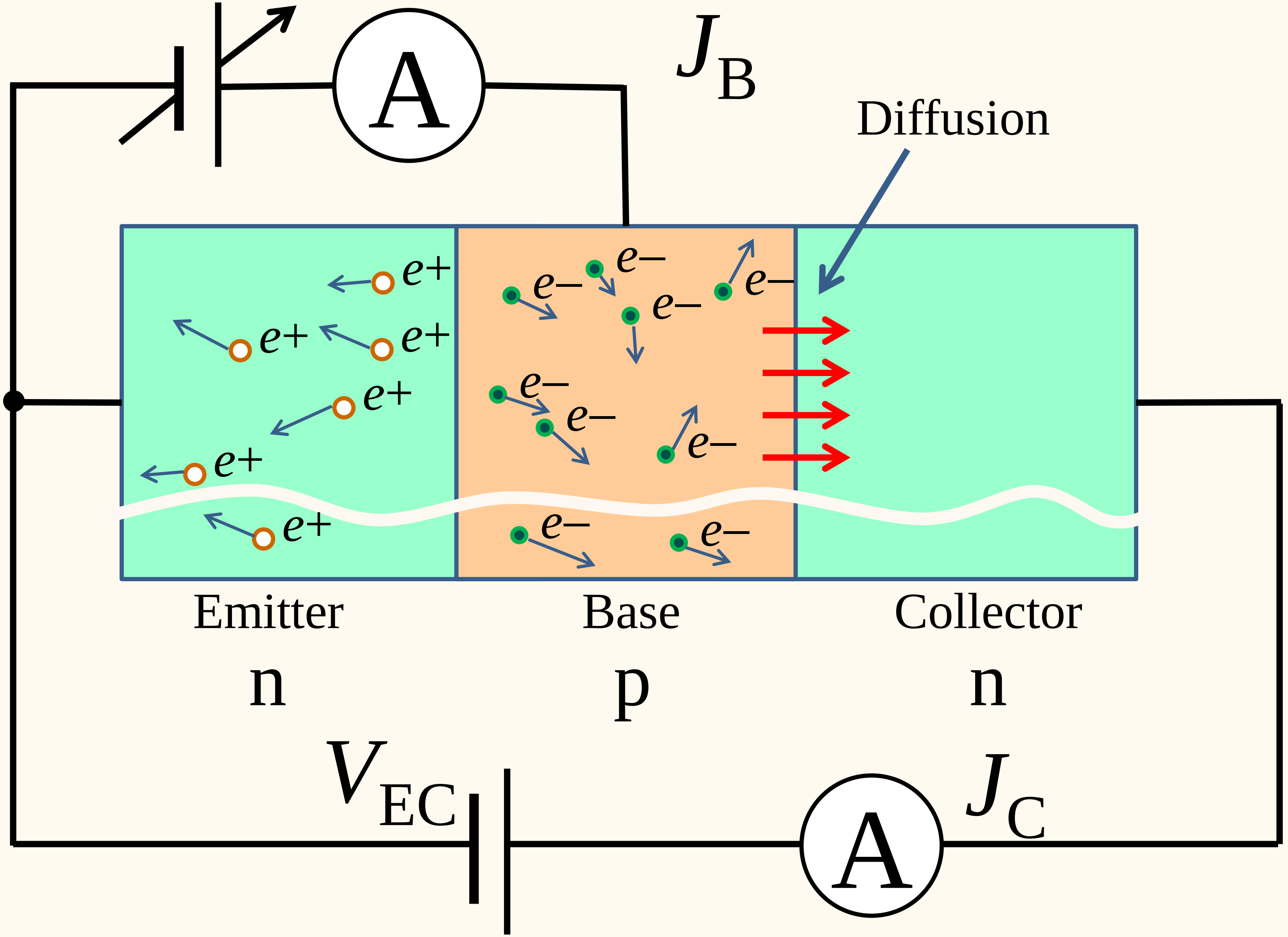




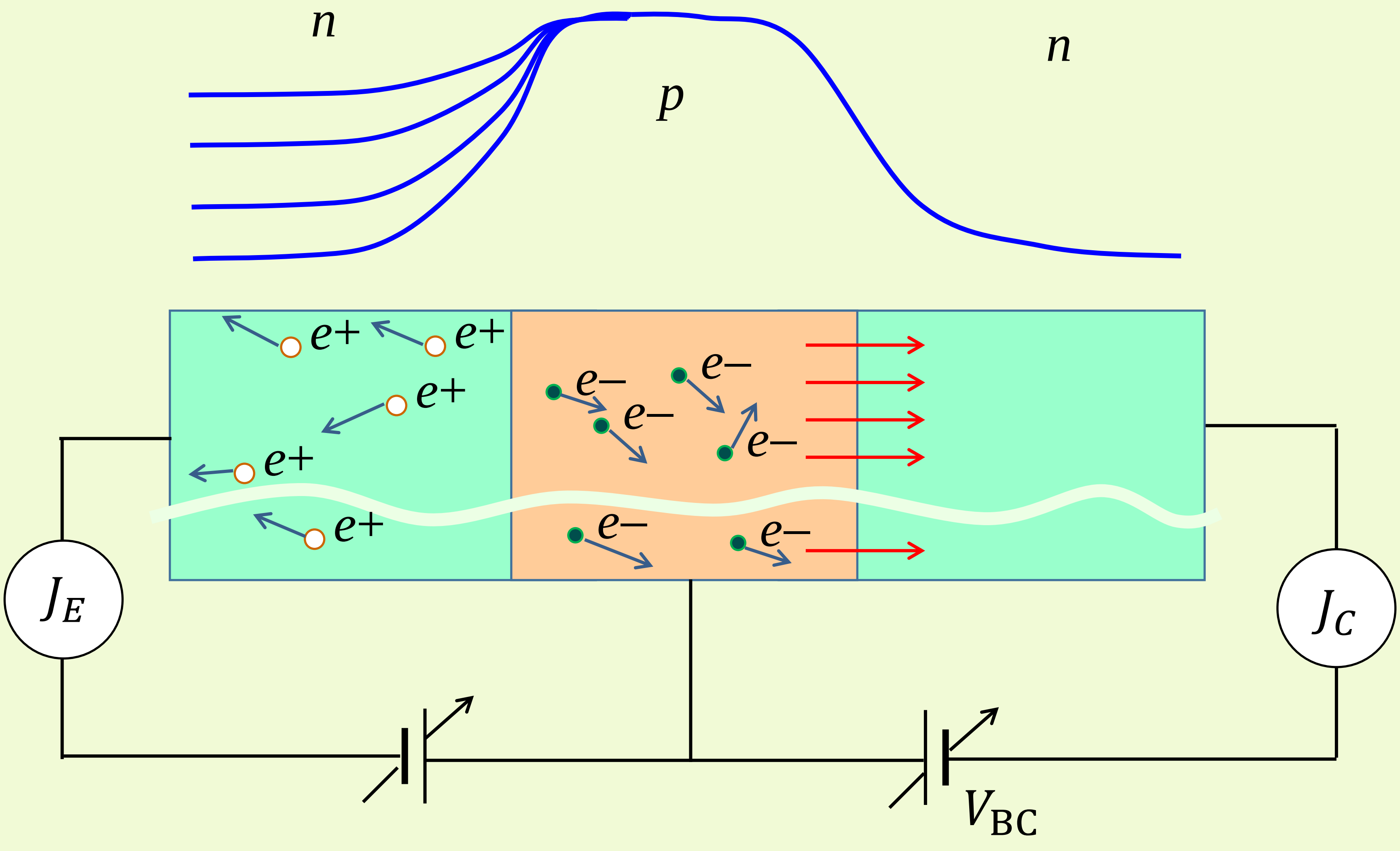
How a bipolar transistor amplifies?



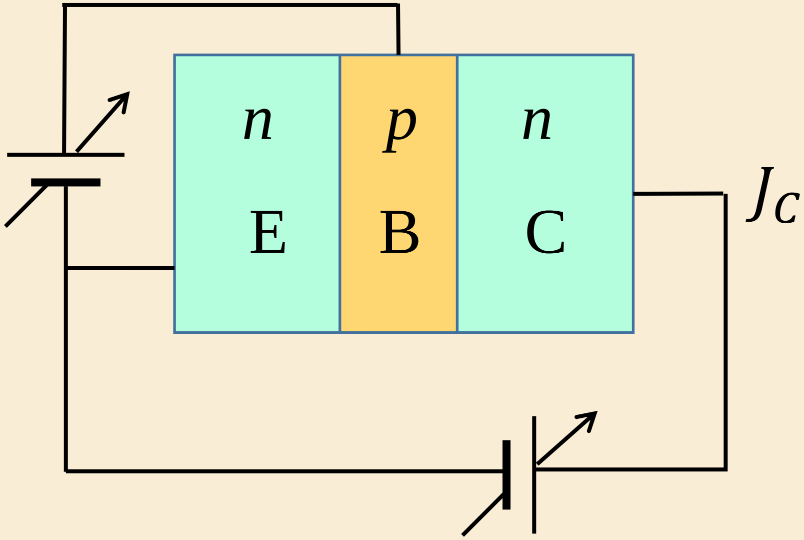
How a bipolar transistor amplifies?

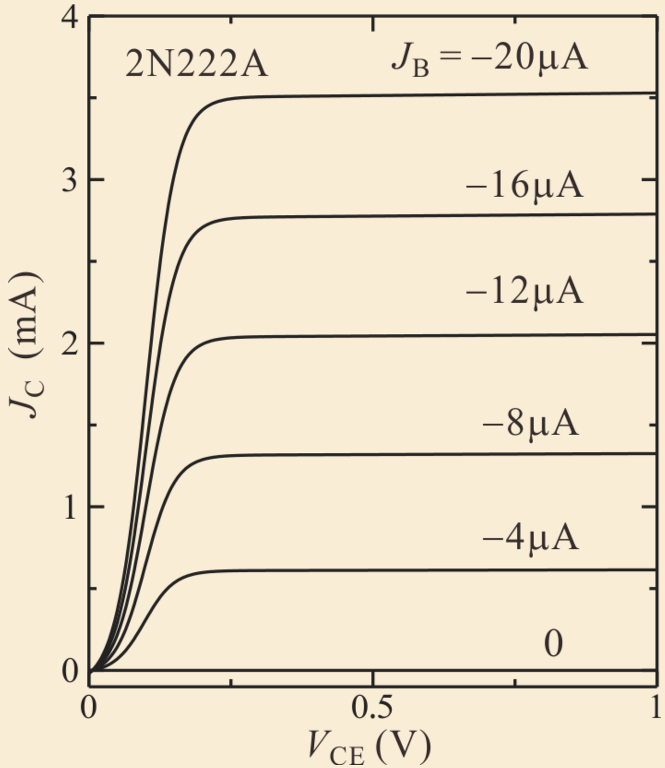


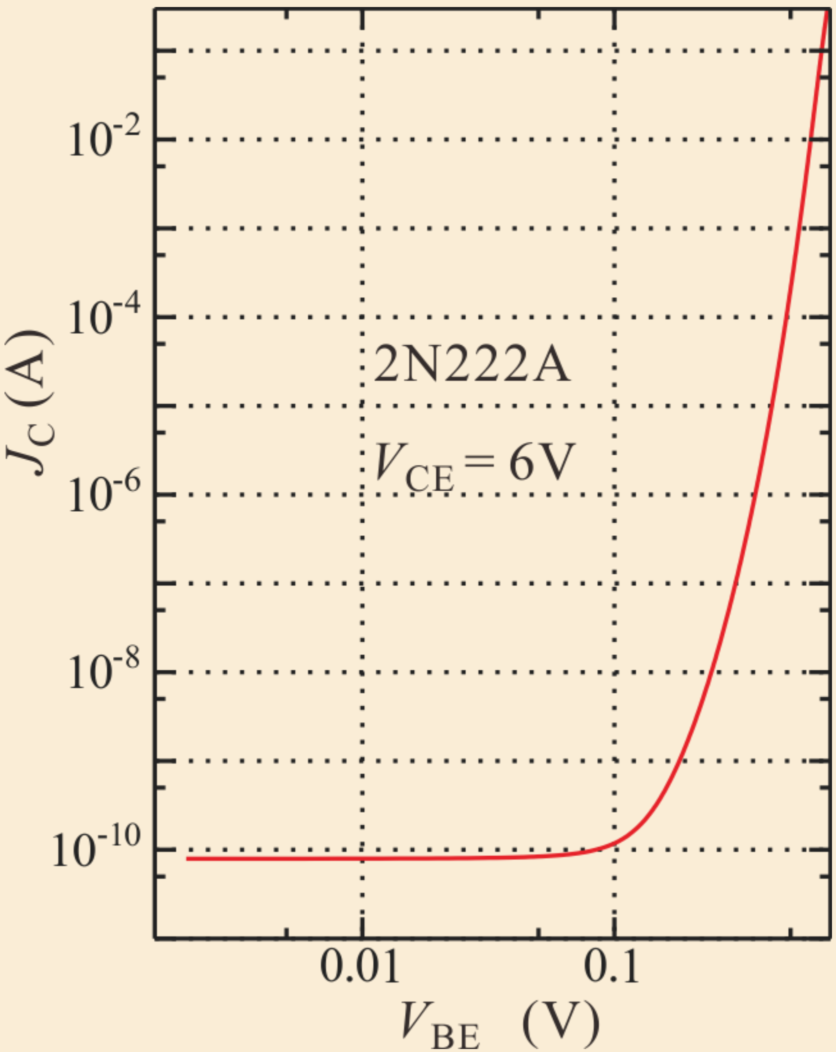
Base-Collector characteristics



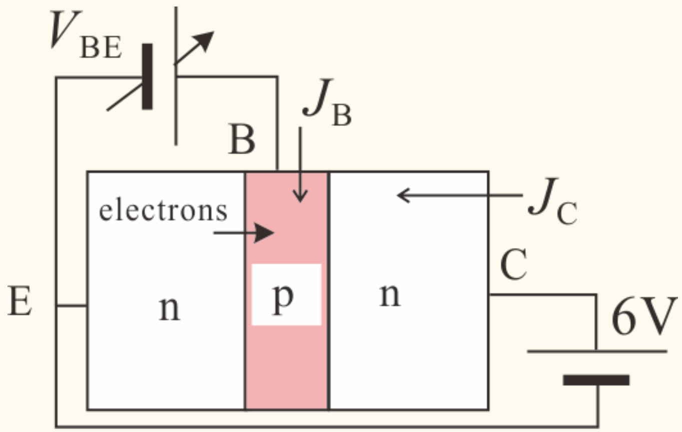
Collector-Emitter characteristics





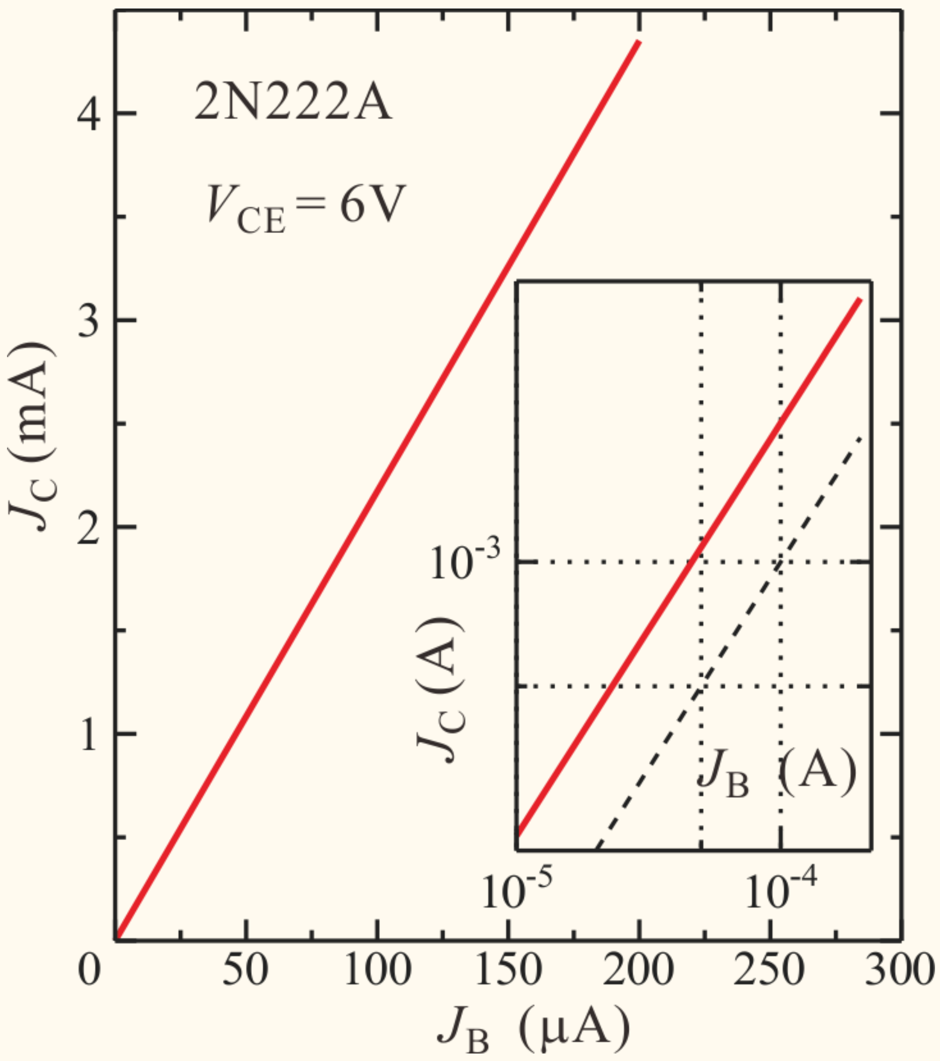


Current amplification : Linearize with quantity selection

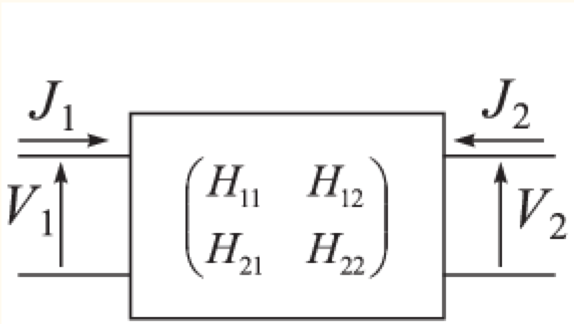


$$J_C = h_{FE}\, J_B$$

Emitter-common current gain

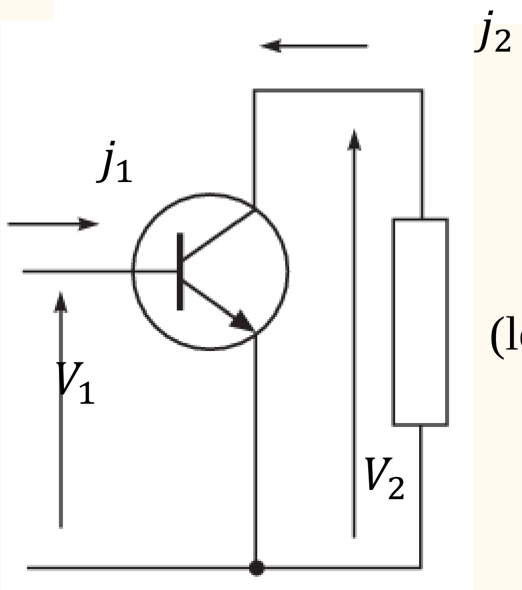


Linear approximation of bipolar transistor



Hybrid matrix

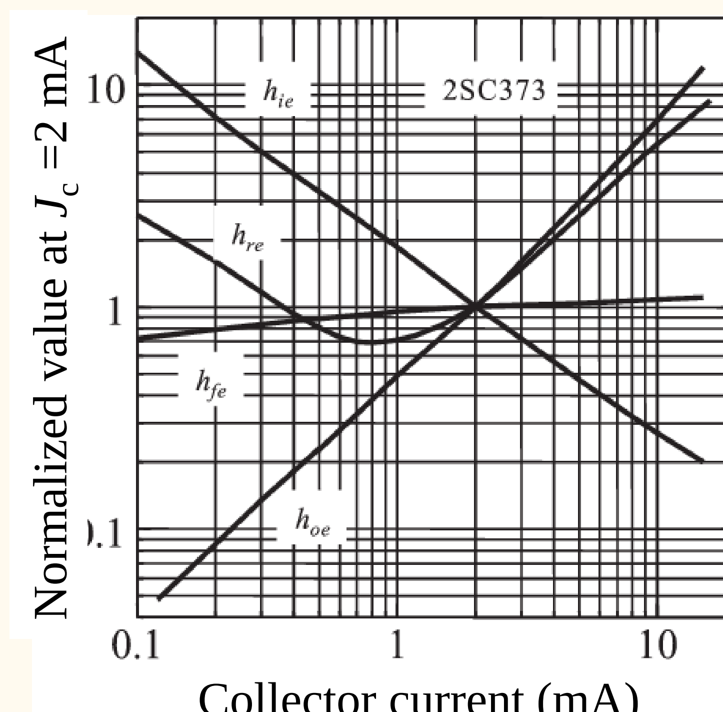
$$\begin{pmatrix} V_1 \\ J_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} J_1 \\ V_2 \end{pmatrix}.$$



$$\begin{pmatrix} v_1 \\ j_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} j_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} h_i & h_r \\ h_f & h_o \end{pmatrix} \begin{pmatrix} j_1 \\ v_2 \end{pmatrix}$$

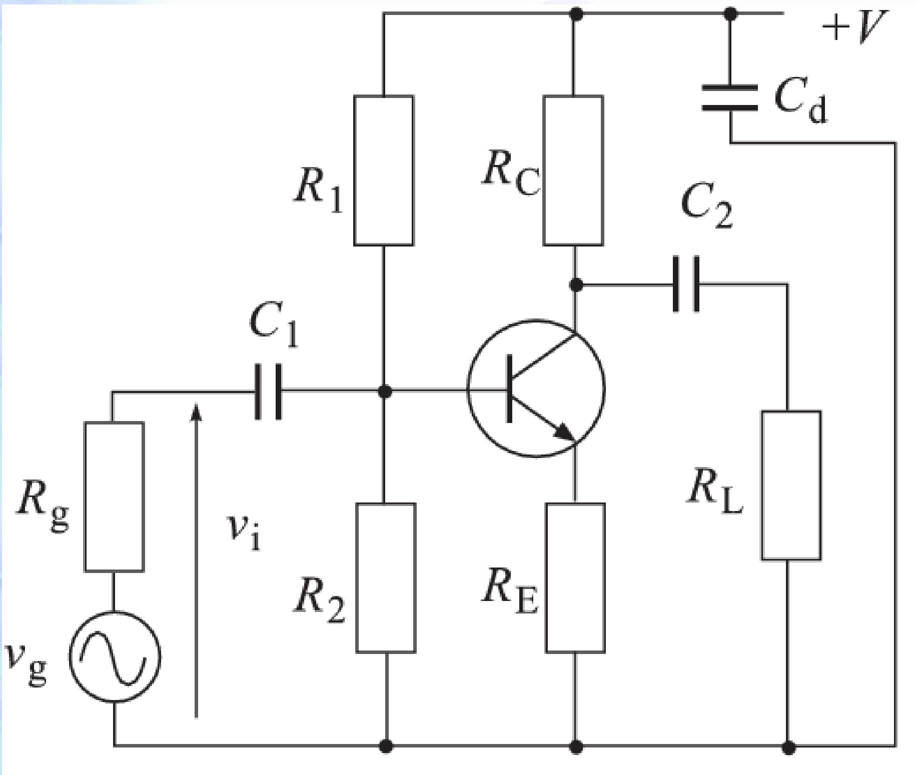
h-parameters

(lower case:
local linear approximation)



Concept of bias circuits for non-linear devices

Common emitter amplifier

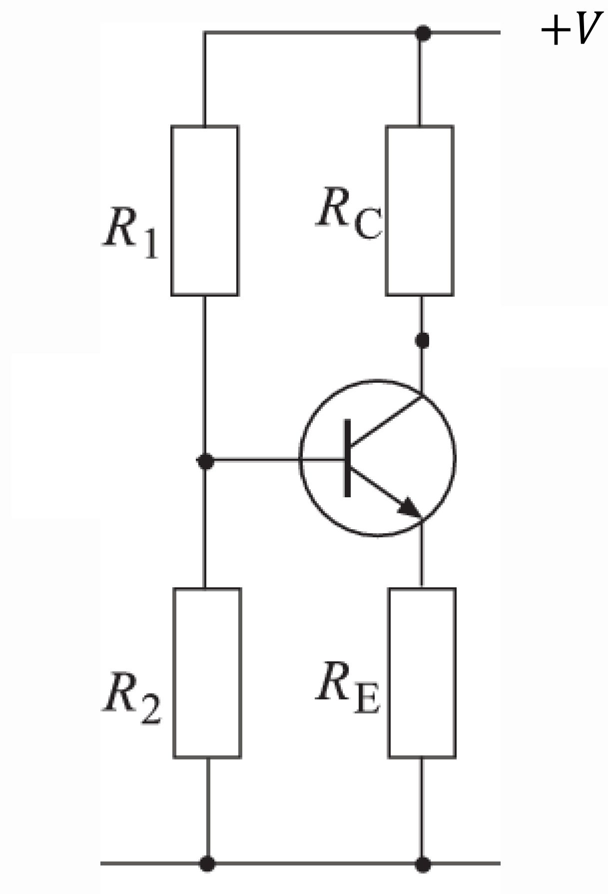


For small amplitude (high-frequency) circuits

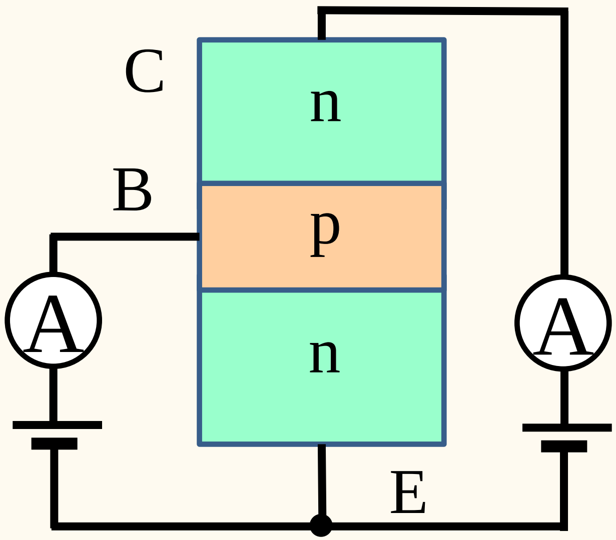
All the capacitors can be viewed as short circuits.

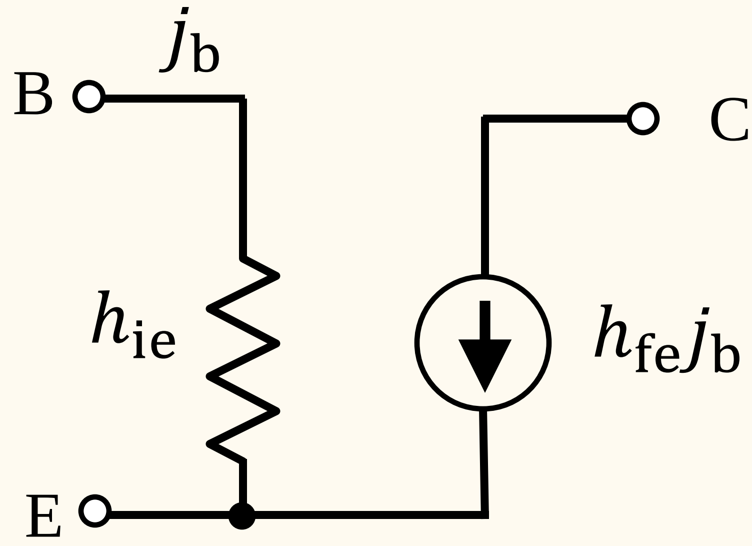
For bias (dc) circuits

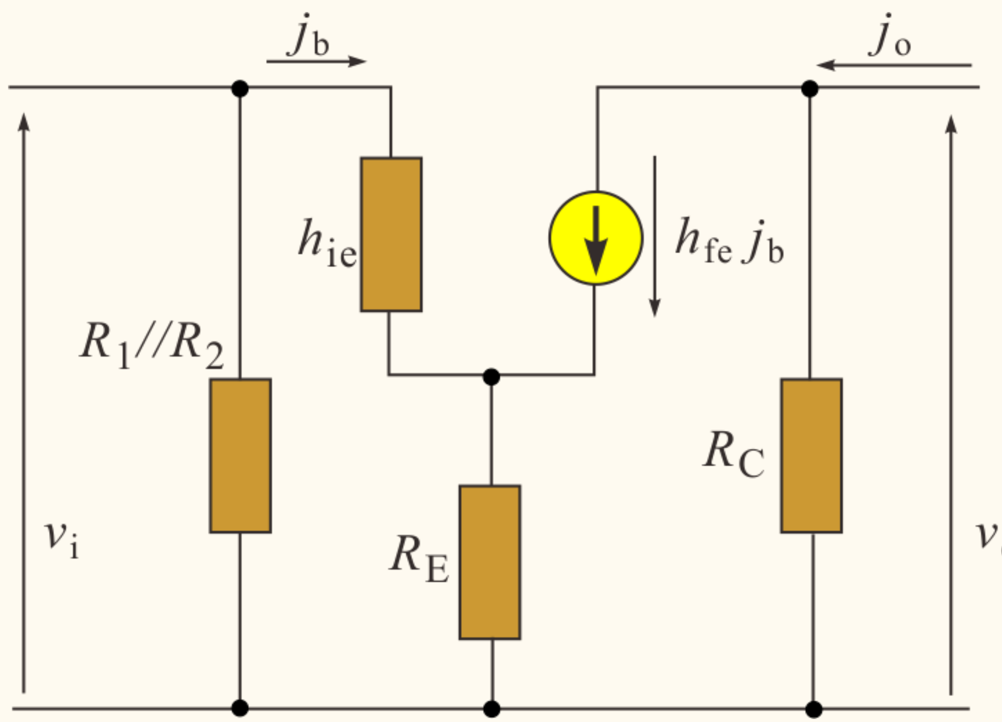
All the capacitors can be viewed as break line.



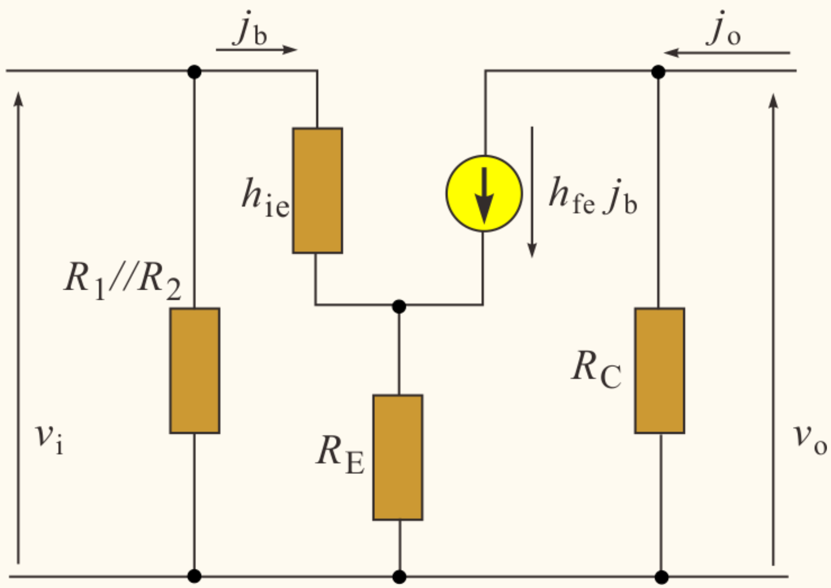
Concept of equivalent circuit

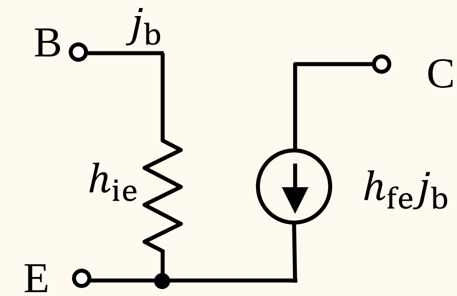


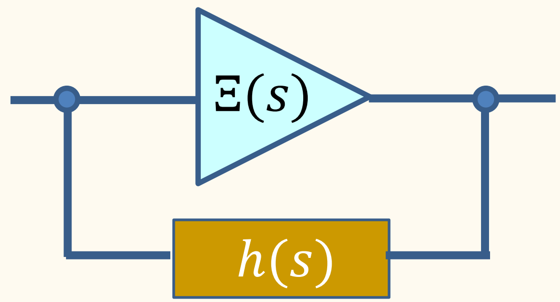


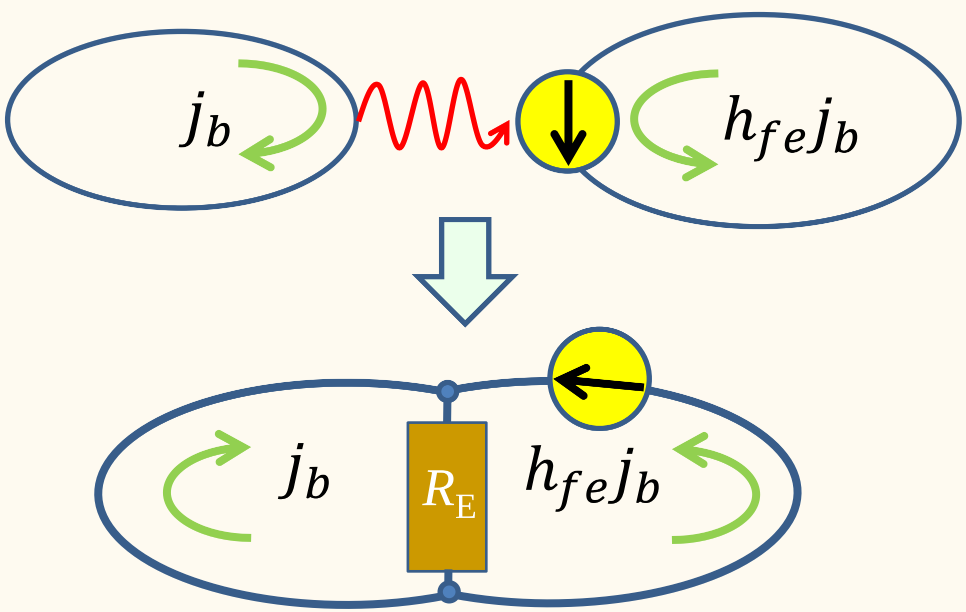


Concept of equivalent circuit: Where is feedback?



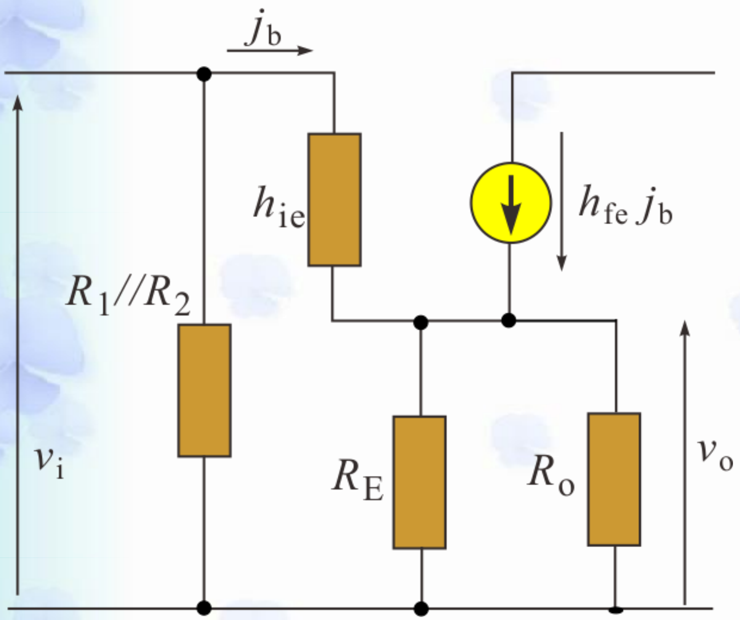






$$
\begin{aligned}
A &= \frac{v_o}{v_i} \\
&= \frac{h_{fe} R_C}{h_{ie} + R_E(1 + h_{fe})} \\
&\approx \frac{R_C}{R_E} \qquad h_{fe} \gg 1
\end{aligned}
$$

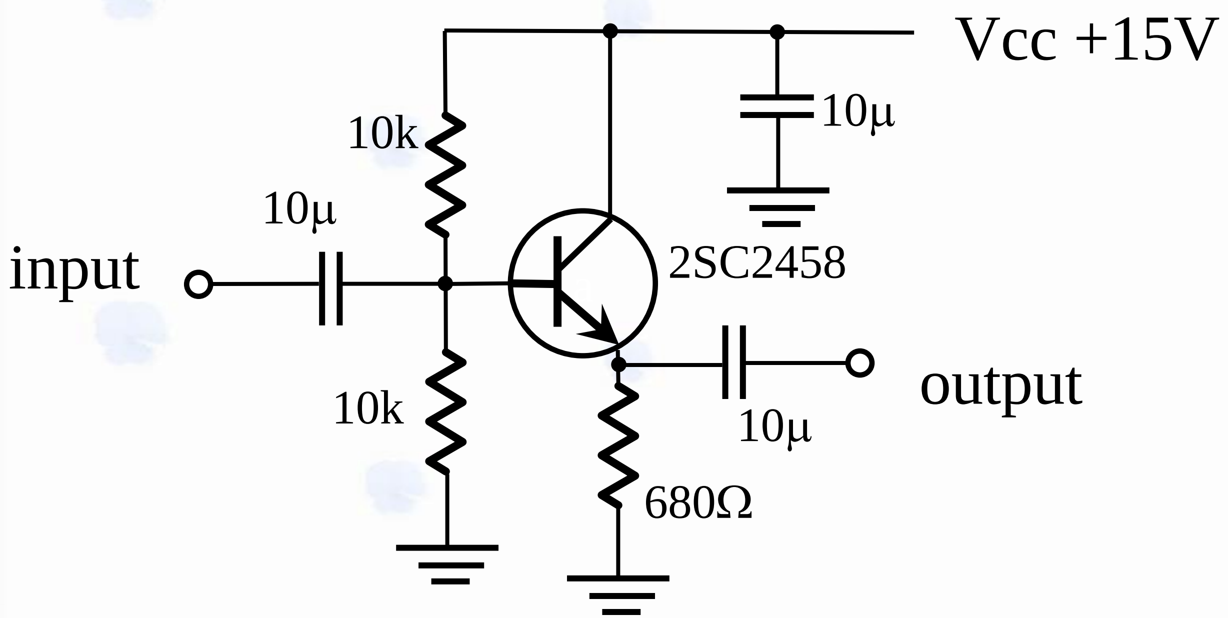
Current amplification: Emitter follower



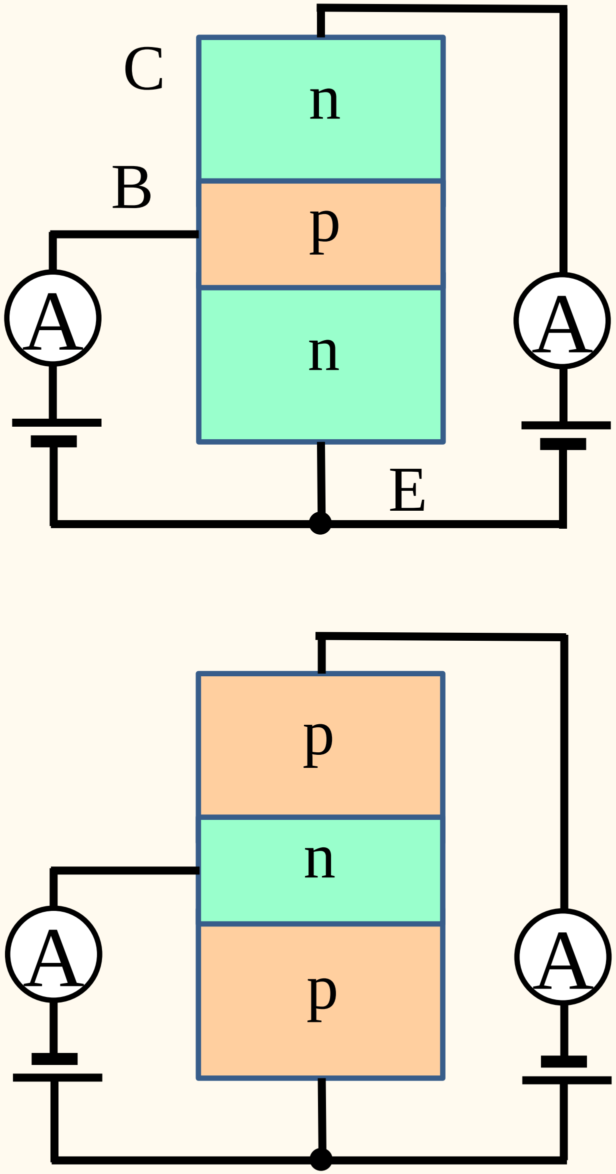
$$\frac{v_o}{v_i} = \frac{j_b(1 + h_{fe})(R_E \parallel R_o)}{j_b[h_{ie} + (1 + h_{fe})(R_E \parallel R_o)]}$$
$$\approx 1 \qquad (h_{fe} \gg 1)$$

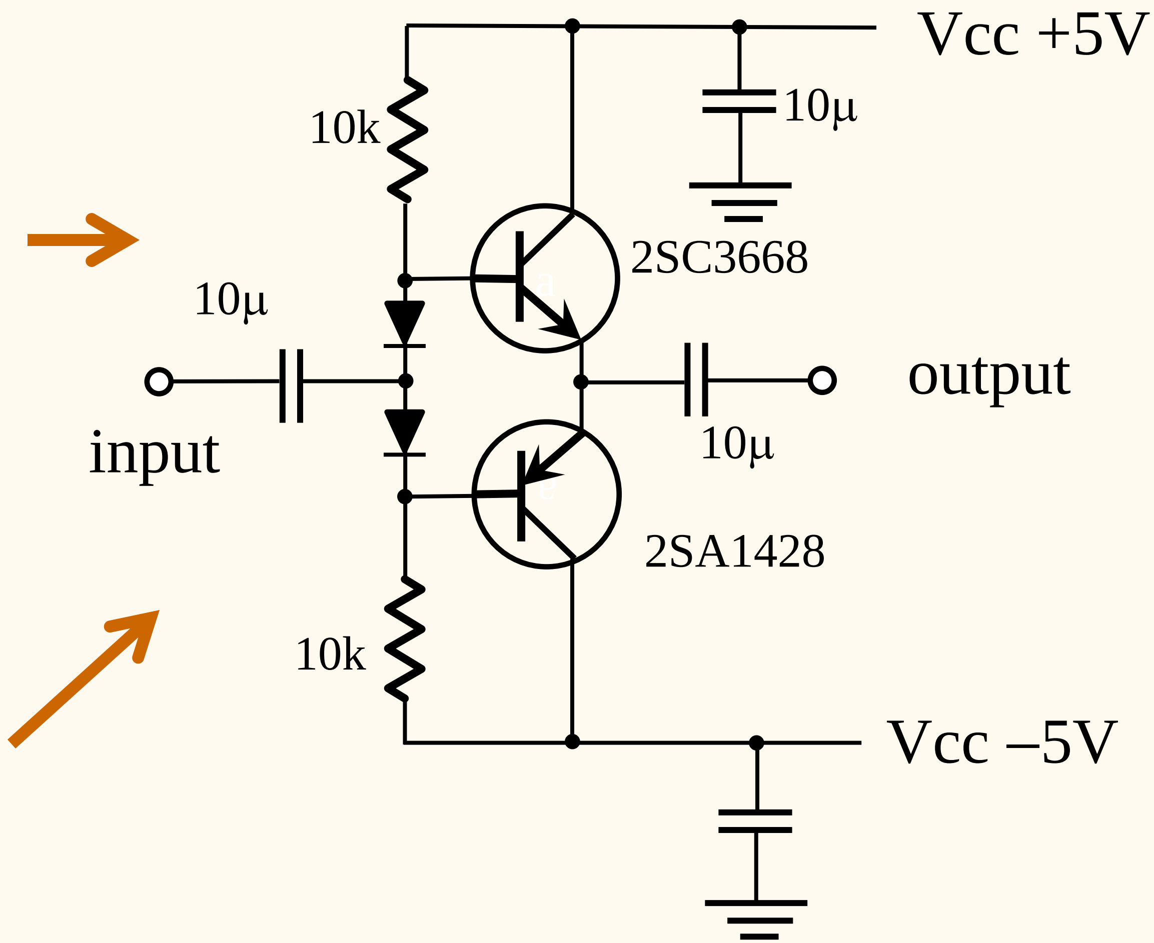
v_o does not depend on load resistance

⇒ Very low output resistance



Complementary transistors



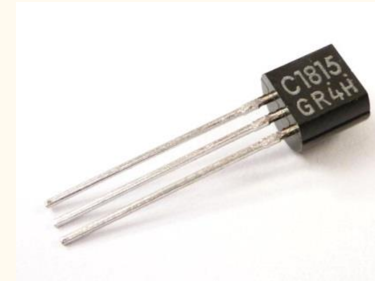


Symmetric characteristics: Complementary

Symmetric: Small collector current
(idling current) for zero input.

Example of transistor datasheet

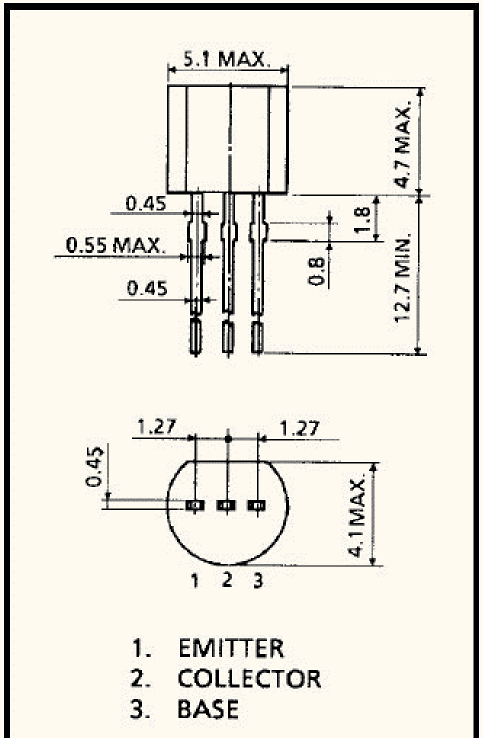
TOSHIBA 2SC1815(L)



TOSHIBA Transistor Silicon NPN Epitaxial Type (PCT process)

2SC1815(L)

Unit: mm



JEDEC	TO-92
JEITA	SC-43
TOSHIBA	2-5F1B

Audio Frequency Voltage Amplifier Applications

Low Noise Amplifier Applications

- High breakdown voltage, high current capability
  : V_{CEO} = 50 V (min), I_C = 150 mA (max)
- Excellent linearity of h_{FE}
  : h_{FE} (2) = 100 (typ.) at V_{CE} = 6 V, I_C = 150 mA
  : h_{FE} (I_C = 0.1 mA)/h_{FE} (I_C = 2 mA) = 0.95 (typ.)
- Low noise: NF = 0.2dB (typ.) (f = 1 kHz).
- Complementary to 2SA1015 (L). (O, Y, GR class).

Example of transistor datasheet

TOSHIBA 2SC1815(L)

TOSHIBA Transistor Silicon NPN Epitaxial Type (PCT process)

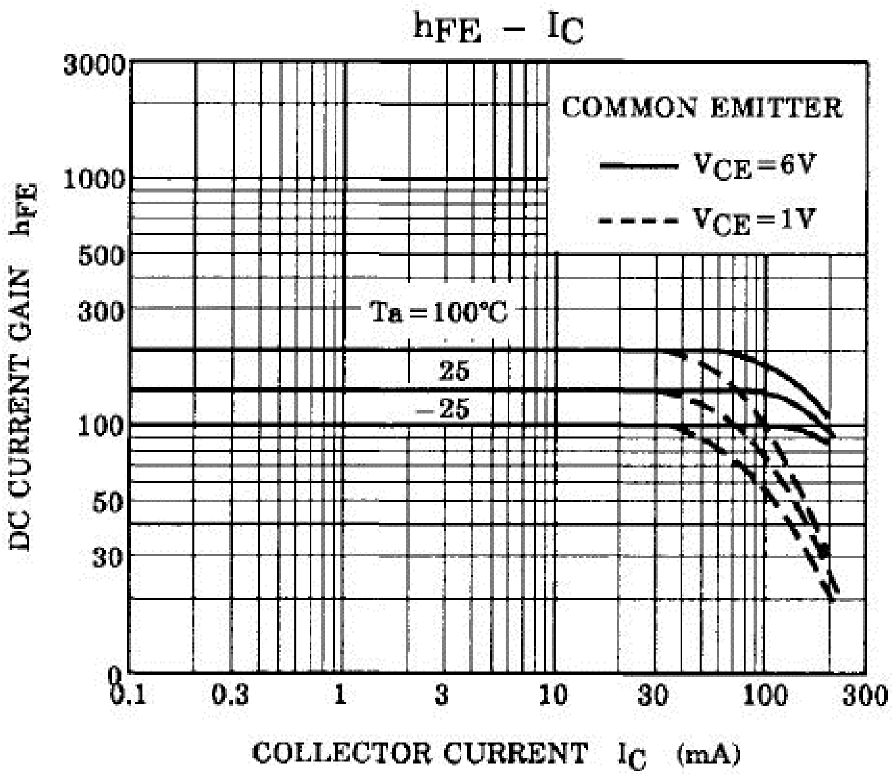
2SC1815(L)

Electrical Characteristics (Ta = 25°C)

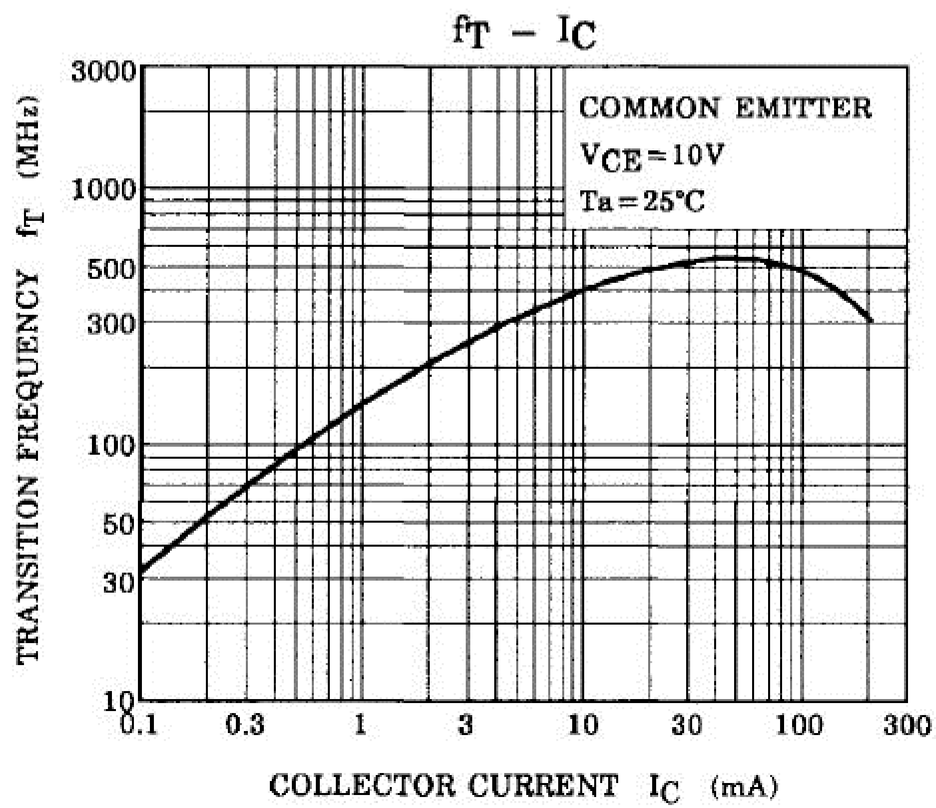
Characteristics		Symbol	Test Condition	Min	Typ.	Max	Unit
Collector cut-off current		I_{CBO}	V_{CB} = 60 V, I_E = 0	—	—	0.1	μA
Emitter cut-off current		I_{EBO}	V_{EB} = 5 V, I_C = 0	—	—	0.1	μA
DC current gain		$h_{FE\ (1)}$ (Note)	V_{CE} = 6 V, I_C = 2 mA	70	—	700	
		$h_{FE\ (2)}$	V_{CE} = 6 V, I_C = 150 mA	25	100	—	
Saturation voltage	Collector-emitter	$V_{CE\ (sat)}$	I_C = 100 mA, I_B = 10 mA	—	0.1	0.25	V
	Base-emitter	$V_{BE\ (sat)}$	I_C = 100 mA, I_B = 10 mA	—	—	1.0	
Transition frequency		f_T	V_{CE} = 10 V, I_C = 1 mA	80	—	—	MHz
Collector output capacitance		C_{ob}	V_{CB} = 10 V, I_E = 0, f = 1 MHz	—	2.0	3.5	pF
Base intrinsic resistance		$r_{bb'}$	V_{CE} = 10 V, I_E = −1 mA, f = 30 MHz	—	50	—	Ω
Noise figure		NF (1)	V_{CE} = 6 V, I_C = 0.1 mA R_G = 10 kΩ, f = 100 Hz	—	0.5	6	dB
		NF (2)	V_{CE} = 6 V, I_C = 0.1 mA R_G = 10 kΩ, f = 1 kHz	—	0.2	3	

Note: $h_{FE\ (1)}$ classification O: 70~140, Y: 120~240, GR: 200~400, BL: 350~700

Example of transistor datasheet

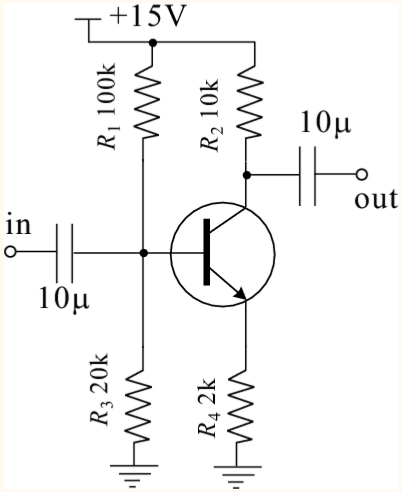


h_{fe} linear model availability in the range of J_C.

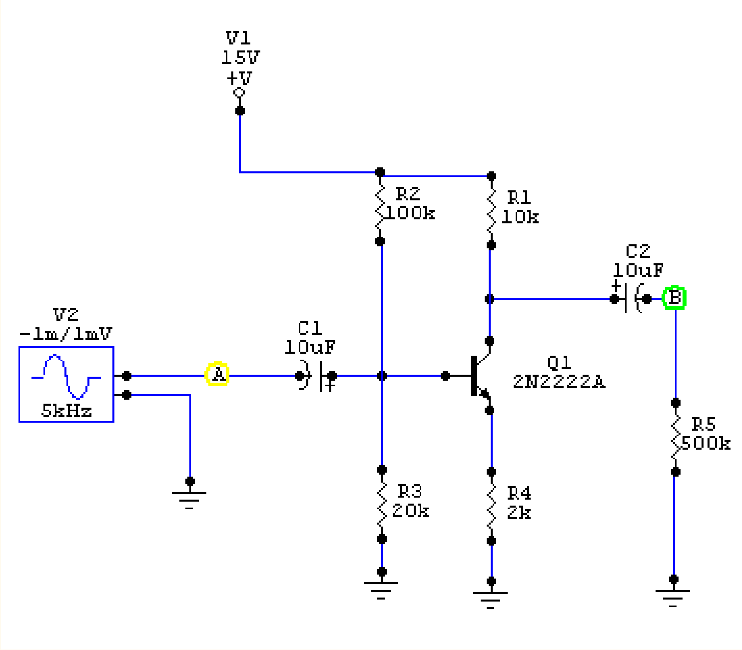


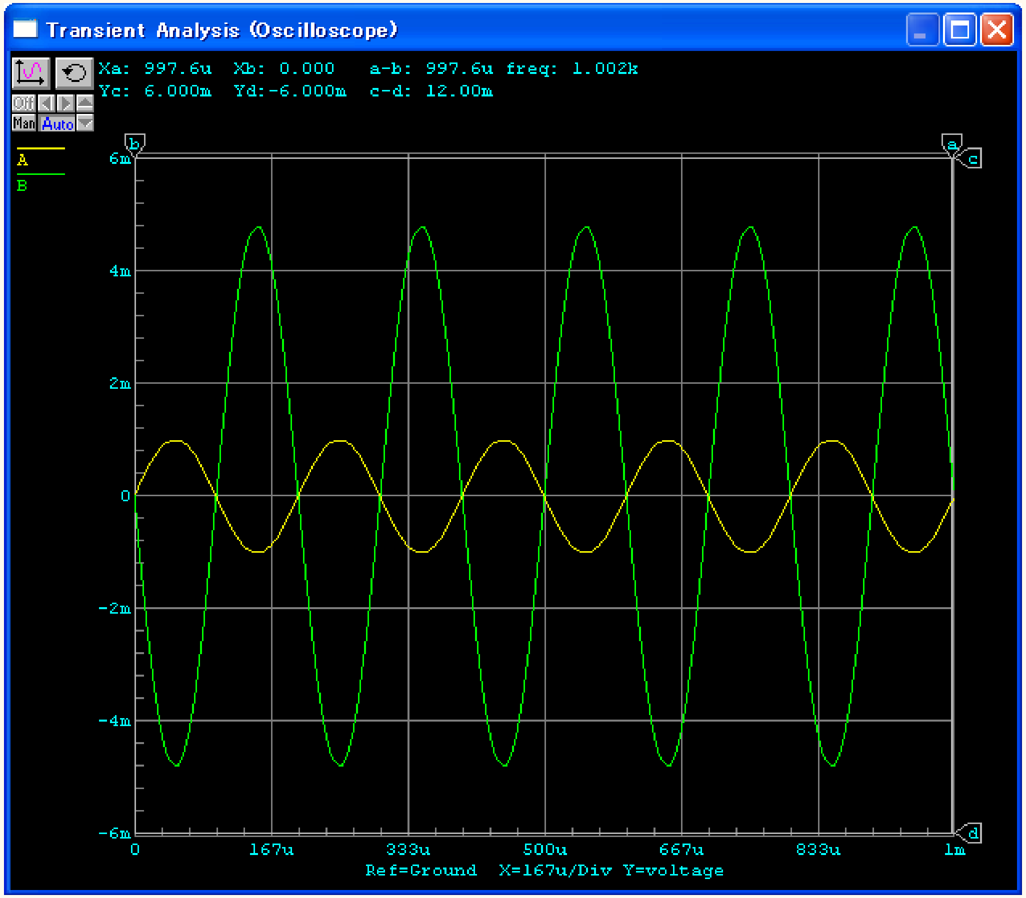
Cut-off frequency as a function of J_C

Common emitter (grounded emitter) amplifier circuit

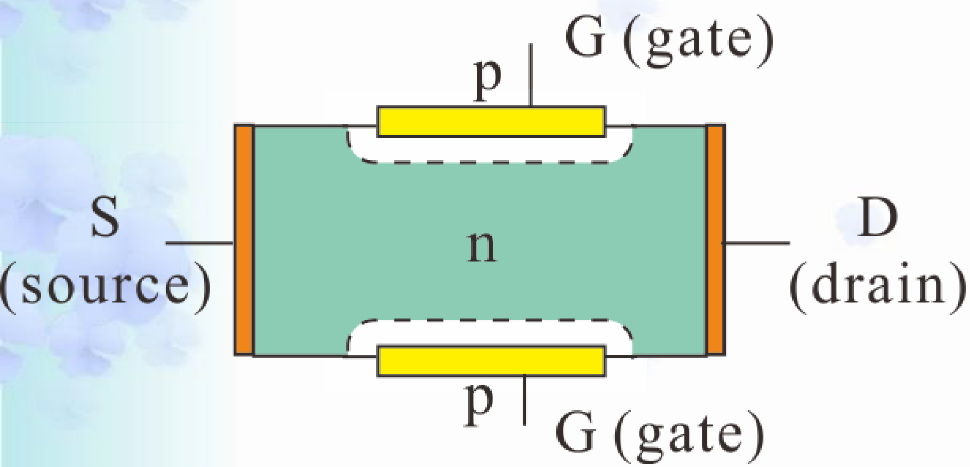


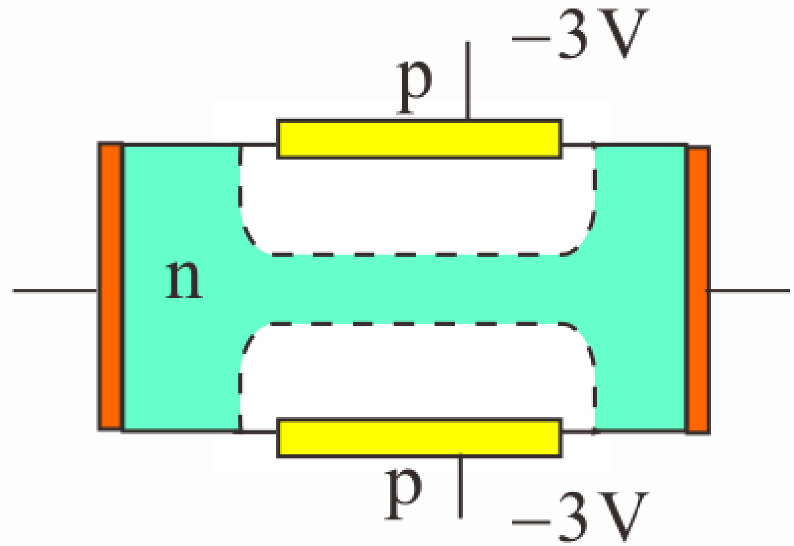
$$\Delta V_{\mathrm{C}} = R_2 \Delta J_{\mathrm{C}} \approx R_2 \Delta J_{\mathrm{E}} = R_2 \frac{\Delta V_{\mathrm{E}}}{R_4} = \frac{R_2}{R_4} \Delta V$$



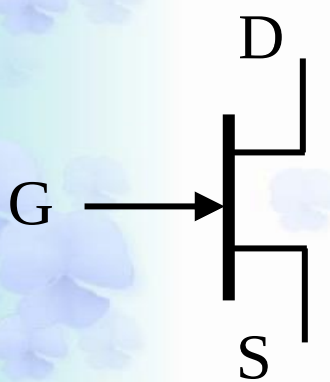


Junction FET (JFET)

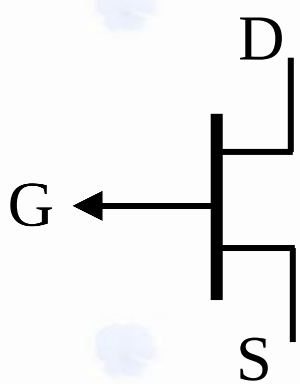




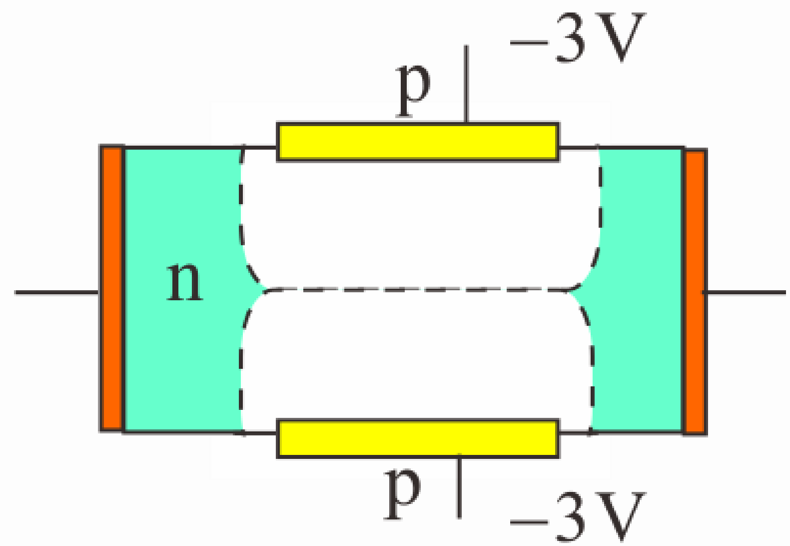
Circuit symbols



n-channel

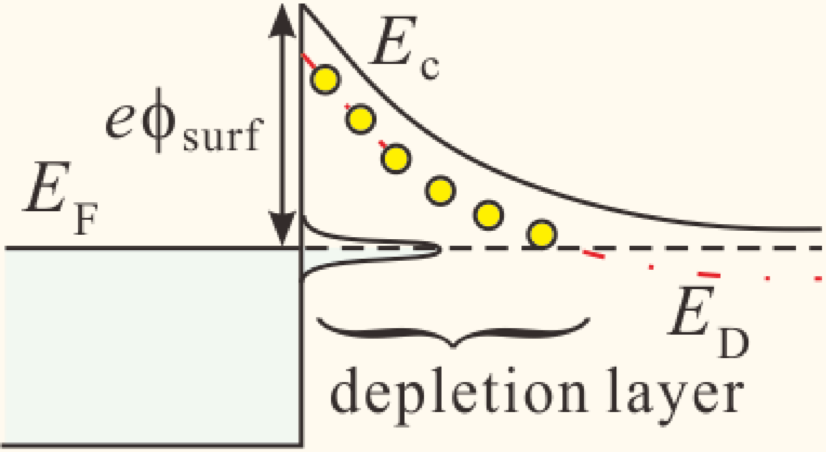


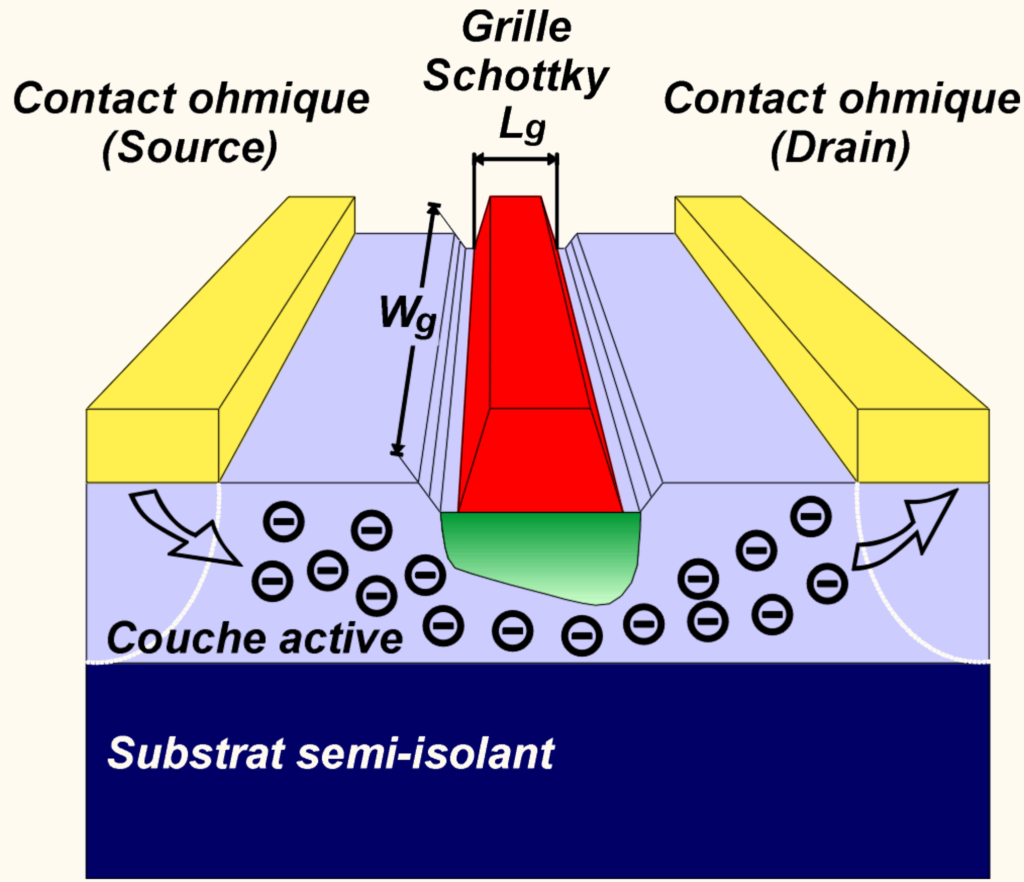
p-channel



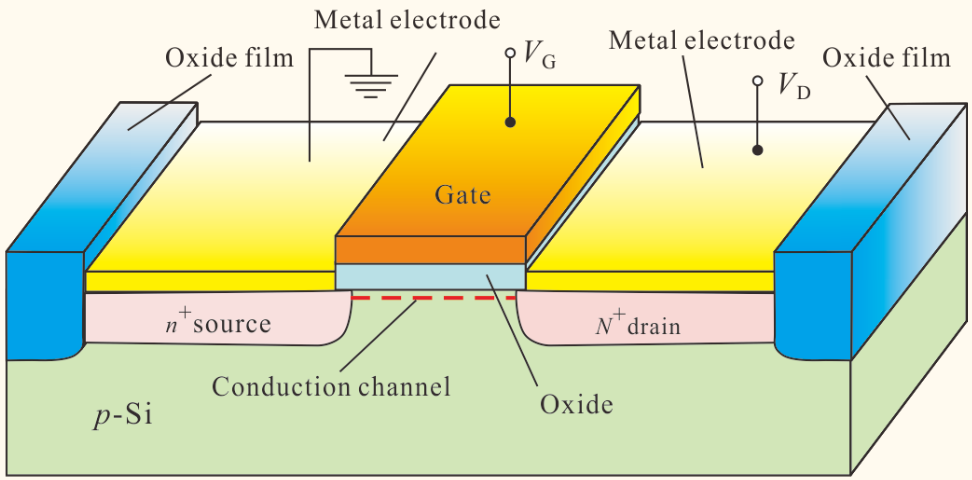
Pinch-off

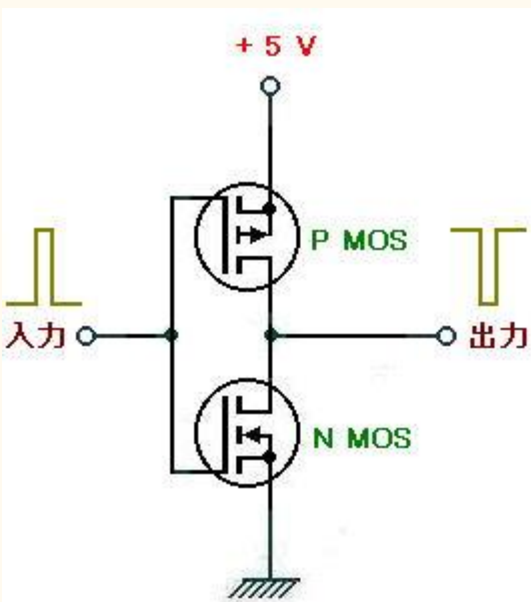
MES-FET


E_c
$e\phi_{surf}$
E_F
E_D
depletion layer


Grille
Schottky
L_g
Contact ohmique
(Source)
Contact ohmique
(Drain)
W_g
Couche active
Substrat semi-isolant

MOS-FET

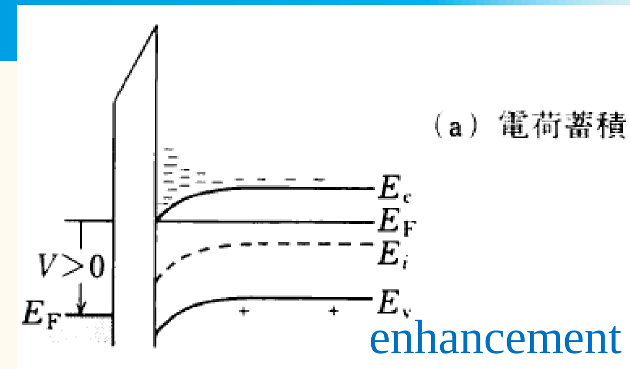


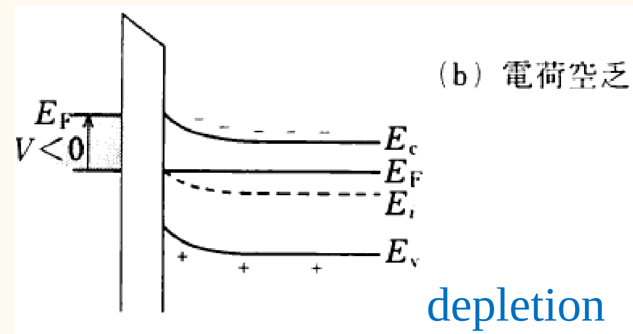


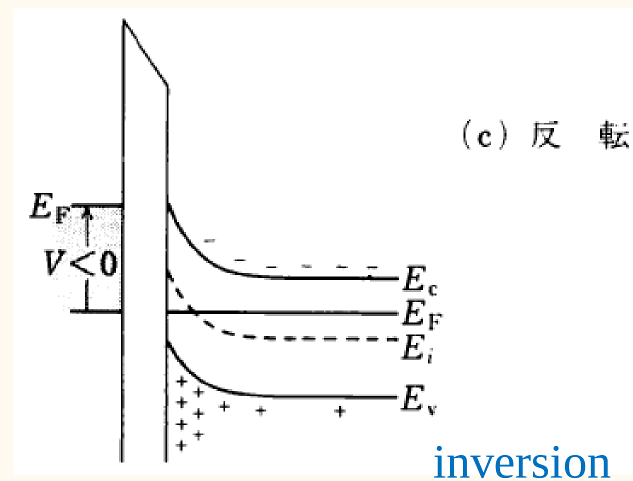
Simplified CMOS inverter circuit

Low leakage current

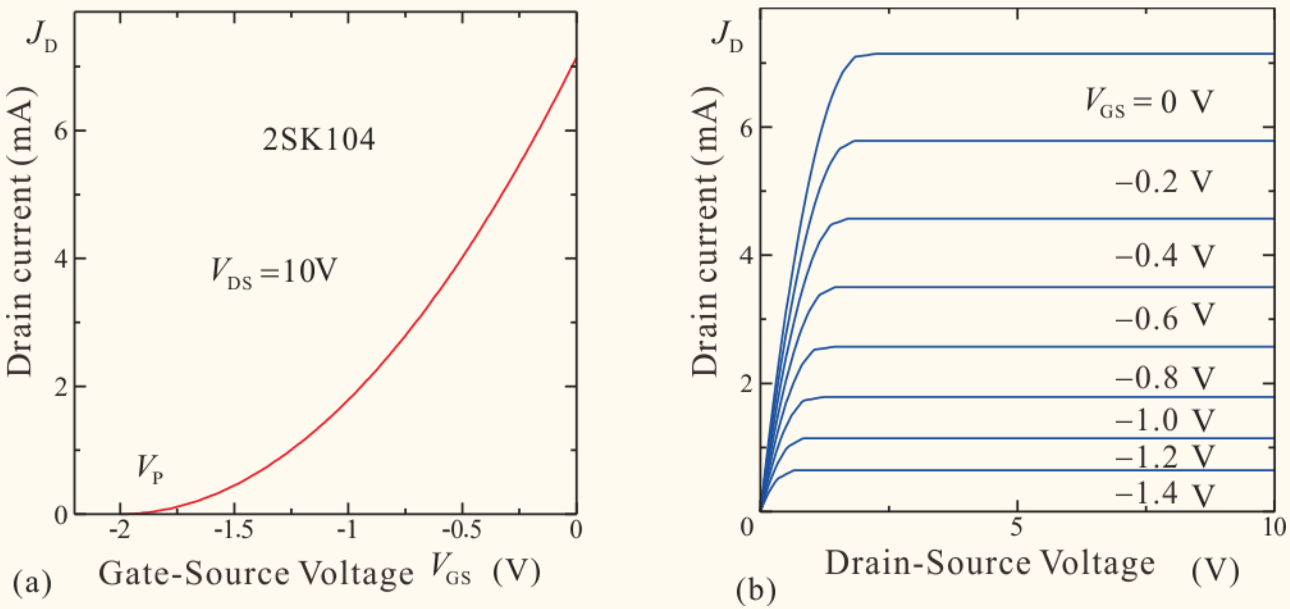
Single gate input both on/off switch







Static characteristics of FET



$$J_G \simeq 0,$$

$$J_D = f(V_G, V_D)$$

$$g_m \equiv \left(\frac{\partial J_D}{\partial V_{GS}}\right)_{V_D=\text{const.}},$$ transconductance

$$r_d \equiv \left(\frac{\partial V_D}{\partial J_D}\right)_{V_{GS}=\text{const.}}$$ Drain resistance

Locally linear approximation $$j_d = g_m v_{gs} + \frac{v_d}{r_d}$$

References

Feedback

- 土谷武士，江上正「現代制御工学」(産業図書, 2000)
- J. J. Distefano, et al. “Schaum’s outline of theory and problems of feedback and control systems” 2nd ed. (McGraw-Hill, 1990)

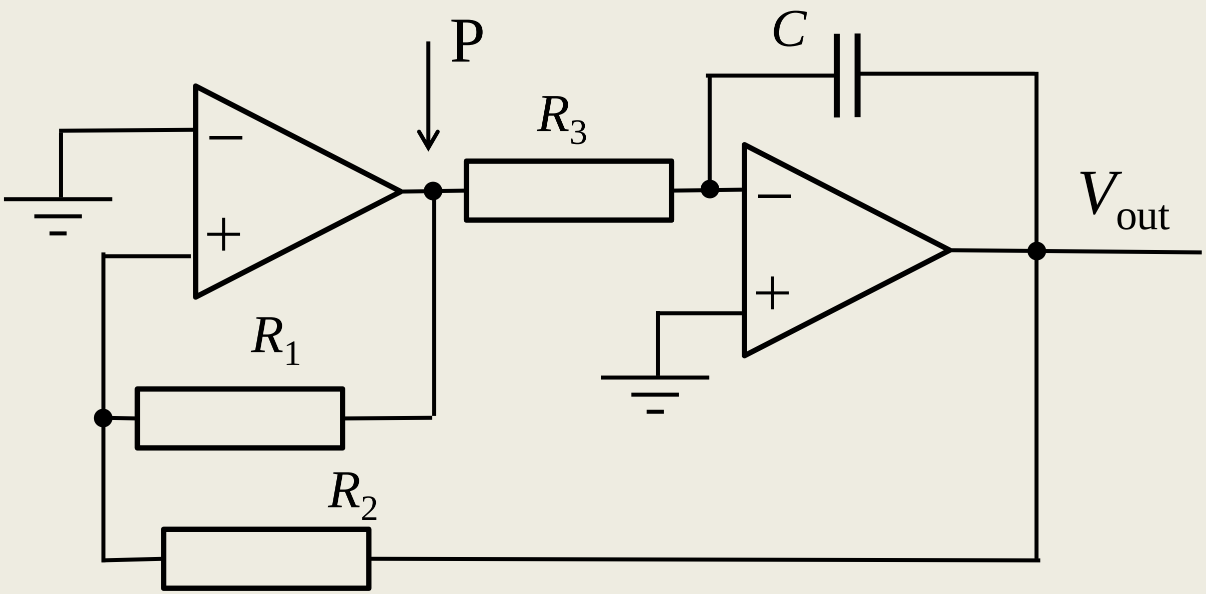
OP amp. circuit design

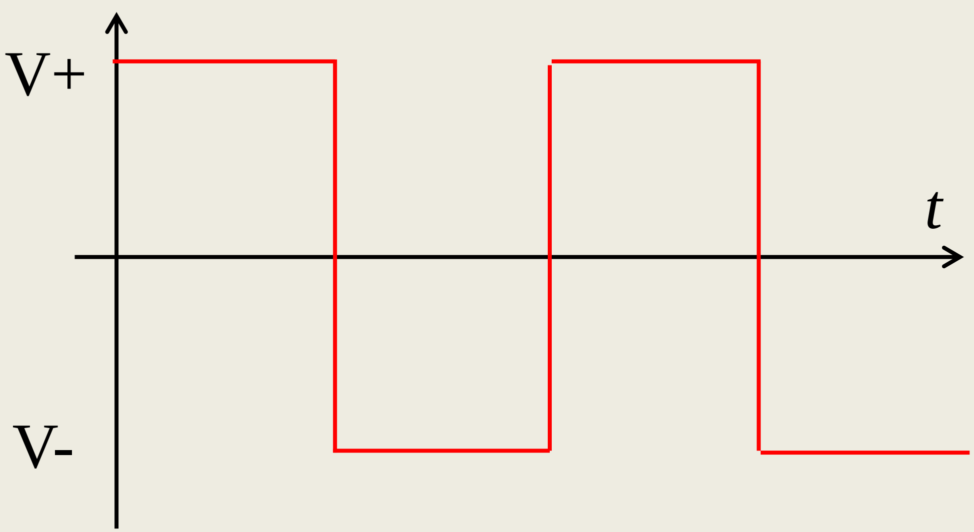
- 岡村迪夫 「OPアンプ回路の設計」 CQ出版社
- J. K. Roberge, K. H. Lundberg, “Operational Amplifiers: Theory and Practice” (MIT, 2007).
  http://web.mit.edu/klund/www/books/opamps181.pdf

BJT, FET circuits

- 松澤昭 「基礎電子回路工学」 (電気学会, 2009).
- S. M. Sze, K. K. Ng, “Physics of Semiconductor Devices” (Wiley, 2007).

Exercise C-1



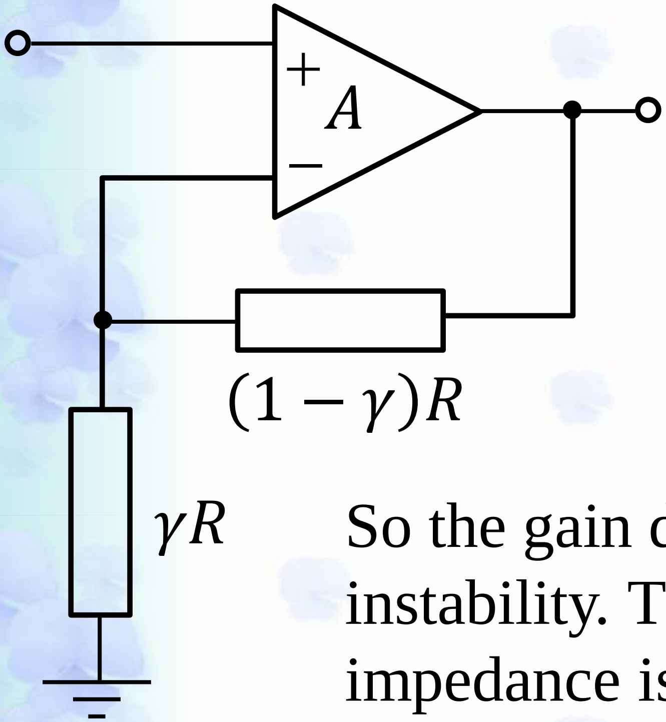


In the circuit shown in the left, at point P, a waveform in the lower panel was observed. Here V+ and V- are power source voltages for + and – respectively.

Draw a rough sketch of the waveform for V_{out}.

“Rough sketch” should contain the levels and the timing of folding points. Write a short comment why V_{out} should be in such a form.

Exercise C-2



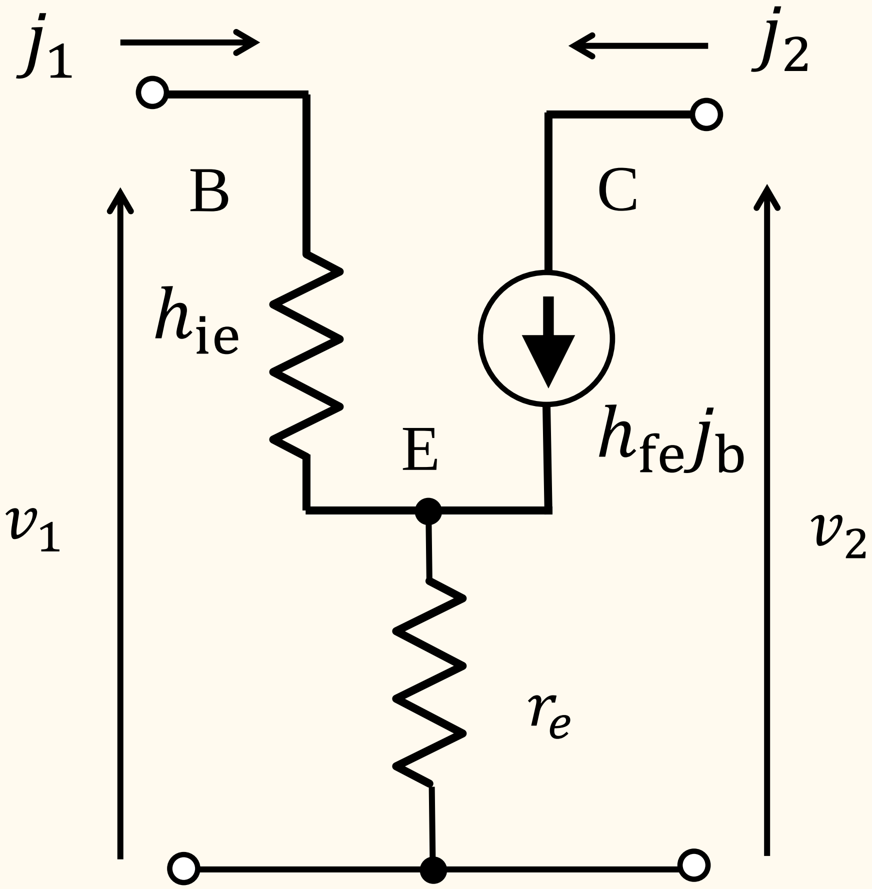
Consider a differential amplifier with the open loop gain

$$A(s) = \frac{A_0\omega_1\omega_2}{s(s+\omega_1)(s+\omega_2)}.$$

So the gain diverges with $s \to 0$ but here we ignore this instability. The input impedance is ∞, and the output impedance is 0.
It is now placed in a circuit with a feedback shown in the left.
Obtain the stability condition for γ.

(hint) Apply the Hurwitz criterion for zeros of even and odd parts of the denominator.
Or just calculate H_2.

Exercise C-3



Let us view a bipolar transistor plus an emitter resistance as a four terminal circuit as shown in the left figure.
Obtain the Y (admittance) matrix defined below for this circuit.

Calculate each element in the Y matrix for
r_e =25Ω, h_{ie} =500 Ω, h_{fe}= 200

$$\begin{pmatrix} j_1 \\ j_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$


電子回路論第7回
Electric Circuits for Physicists
東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

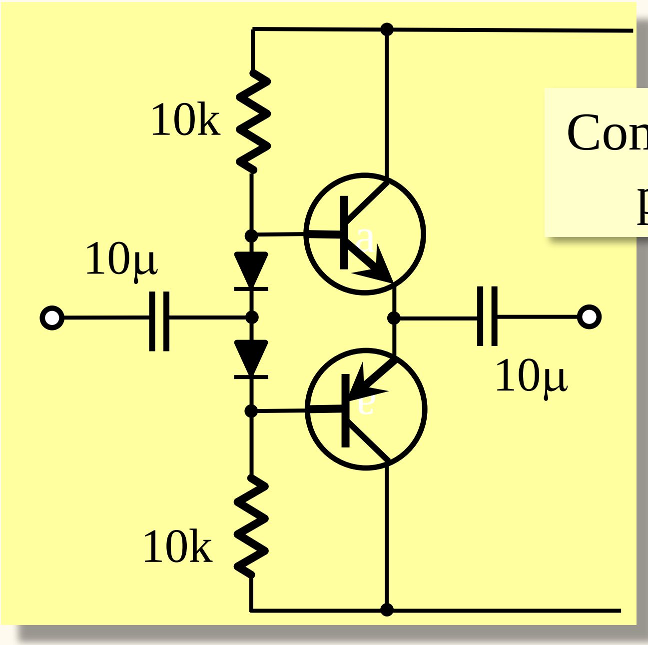
Outline

4.5 Field Effect Transistors (FETs)

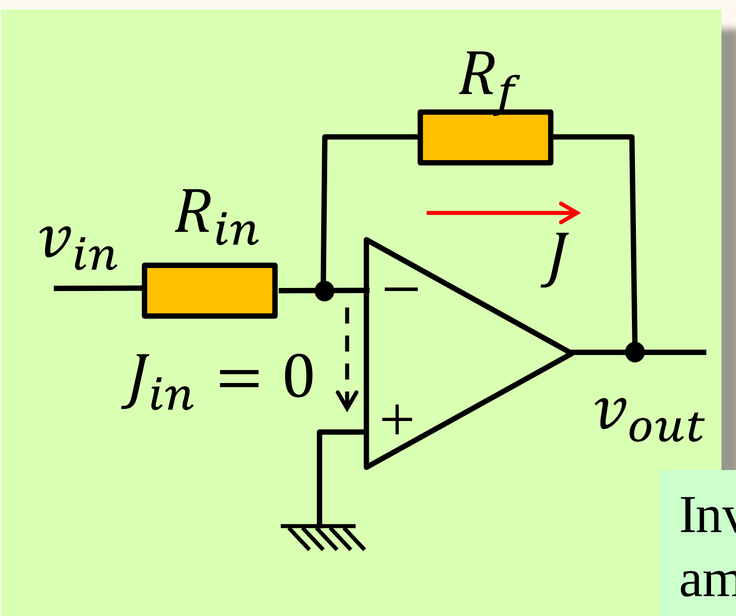
Ch.5 Distributed constant circuits

- 5.1 Transmission lines
  - 5.1.1 Coaxial cables
  - 5.1.2 Lecher lines
  - 5.1.3 Micro-strip lines
- 5.2 Wave propagation through transmission lines
  - 5.2.2 Connection and termination of transmission lines

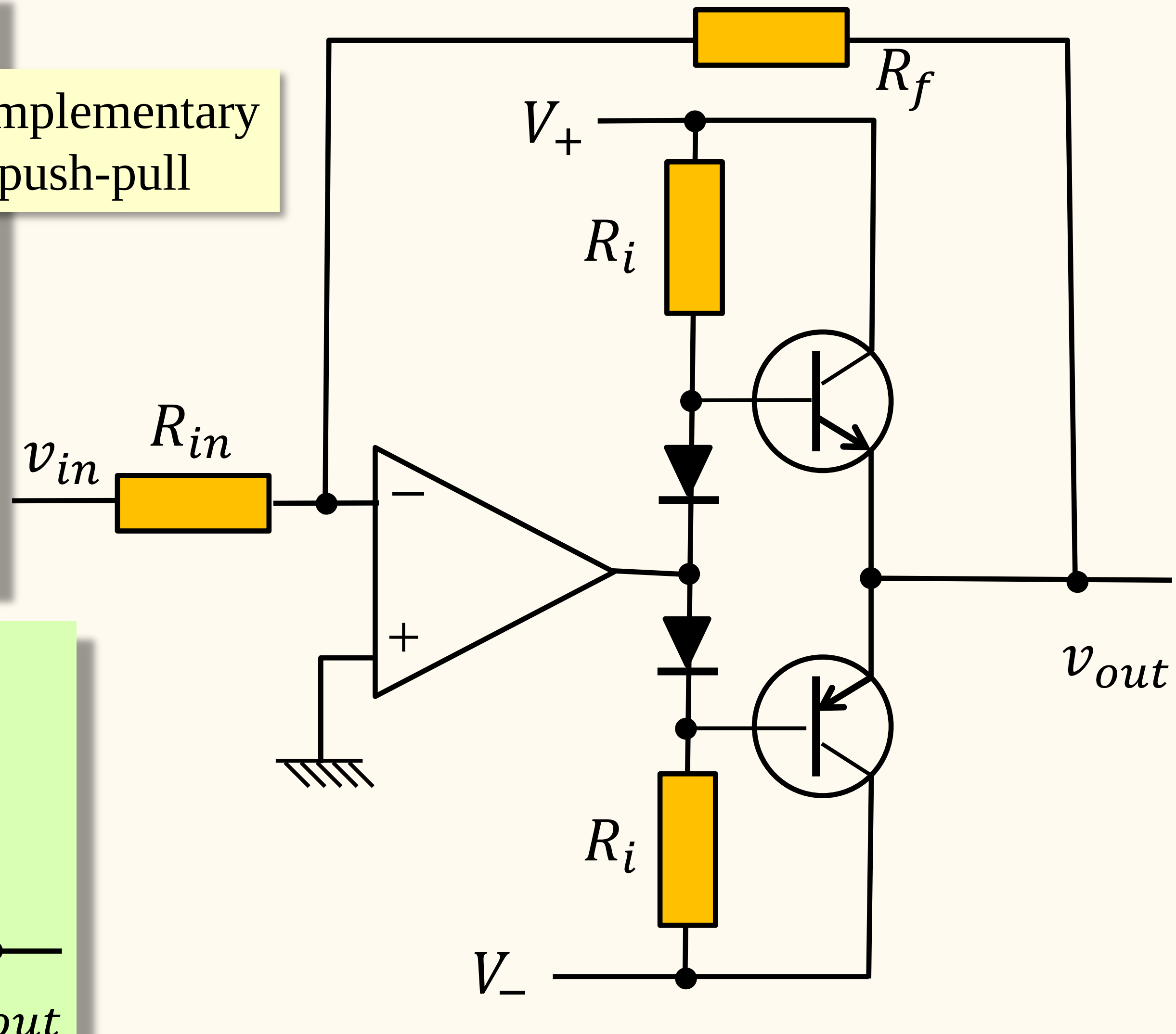
Combination of an OP-amp and discrete transistors



Complementary push-pull

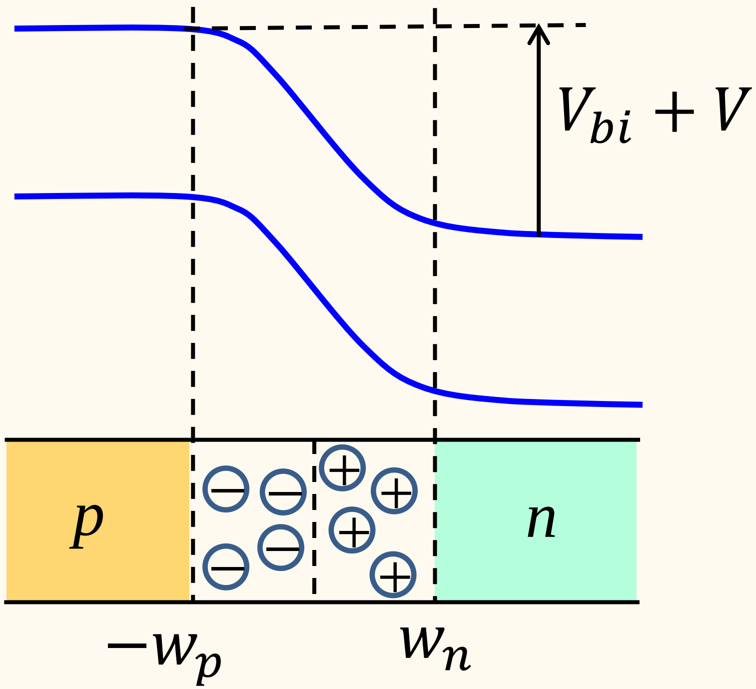


Inversion amplifier



Voltage, current booster

Depletion layer width with reverse bias voltage



Poisson equation

$$\frac{d^2\phi}{dx^2} = -aq(x) \quad (a \equiv (\epsilon\epsilon_0)^{-1})$$

$$\begin{cases} q = -eN_A & (-w_p \le x \le 0), \\ q = eN_D & (0 \le x \le w_n) \end{cases}$$

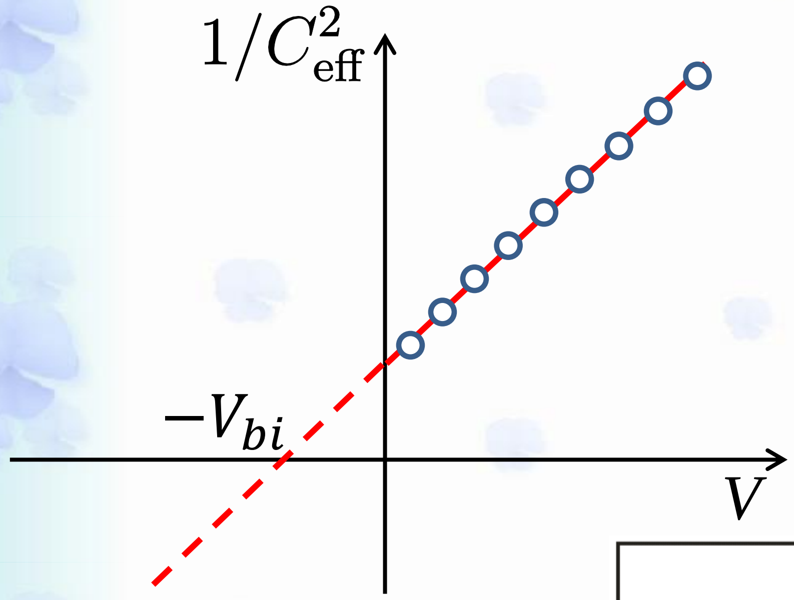
$$\phi(-\infty) = 0$$

$$\phi(-w_p) = 0, \quad \left.\frac{d\phi}{dx}\right|_{-w_p} = 0,$$

$$\phi(w_n) = V + V_{\mathrm{bi}}, \quad \left.\frac{d\phi}{dx}\right|_{w_n} = 0$$

$$\phi(x) = \begin{cases} (aeN_A/2)(x + w_p)^2 & (-w_p \le x \le 0), \\ V + V_{\mathrm{bi}} - (aeN_D/2)(x - w_n)^2 & (0 \le x \le w_n) \end{cases}$$

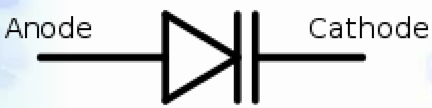
Effective capacitance and reverse bias voltage



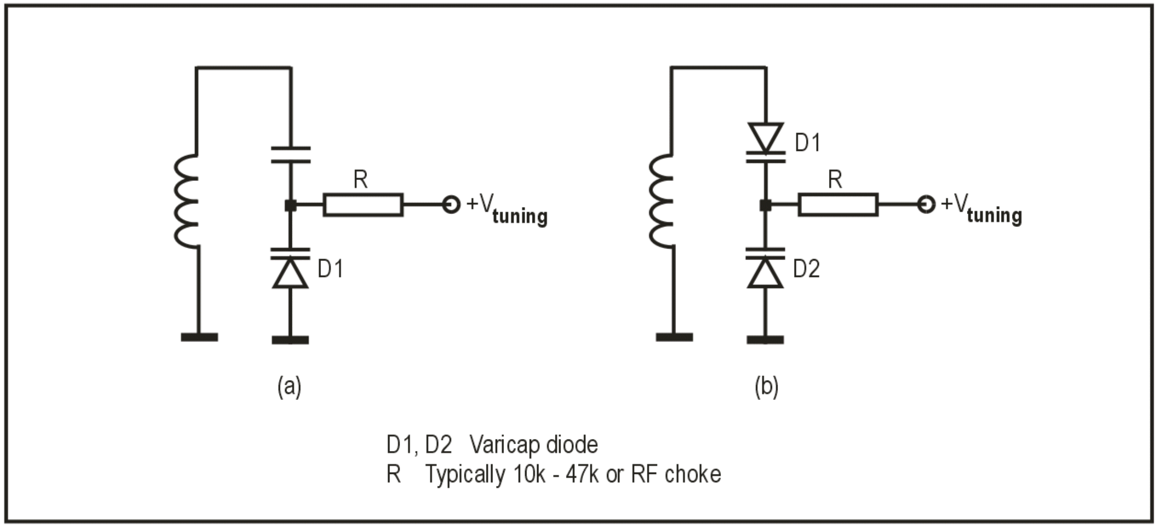
$$\frac{1}{C_{\text{eff}}^2} = \frac{2}{\epsilon\epsilon_0 e N_{\text{D}}}(V + V_{\text{bi}})$$

Doping profiler

Varicap diode



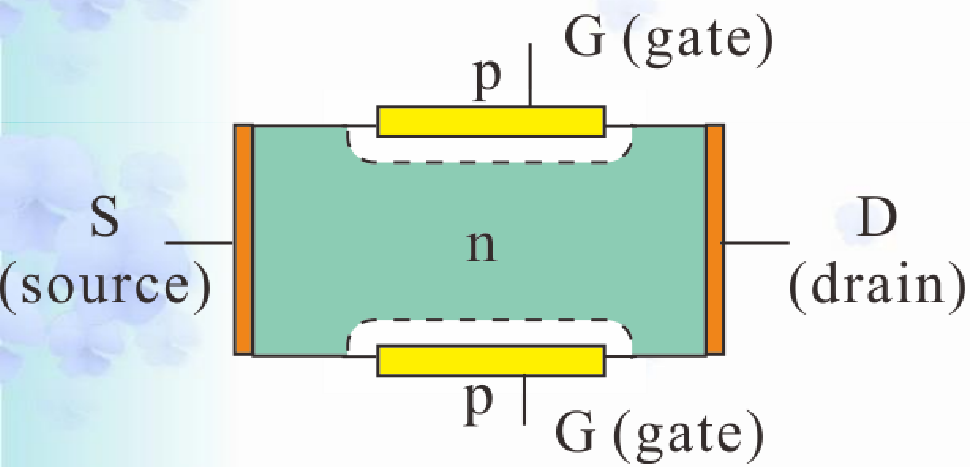
KB505

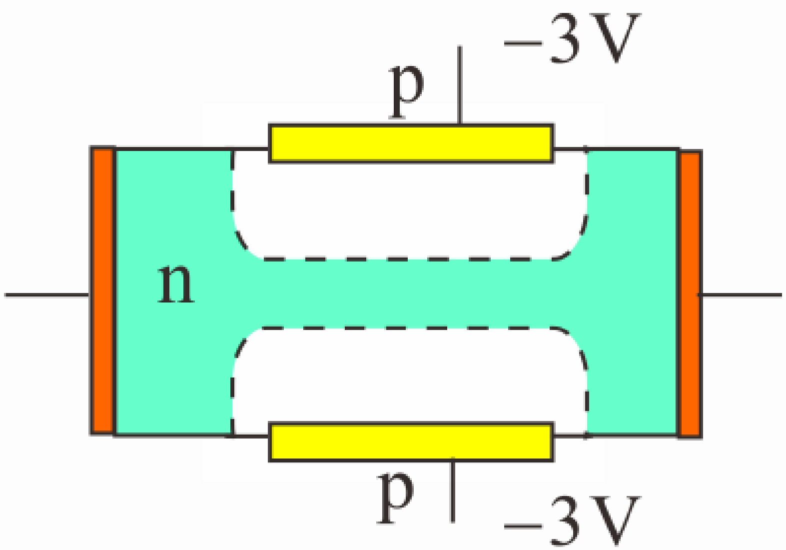


Frequency modulation
Phase lock loop

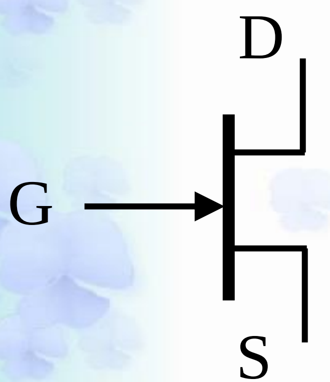
4.4 Field effect transistor (FET)

Junction FET (JFET)

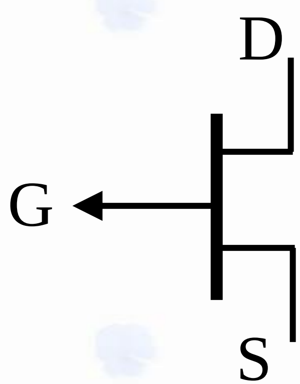




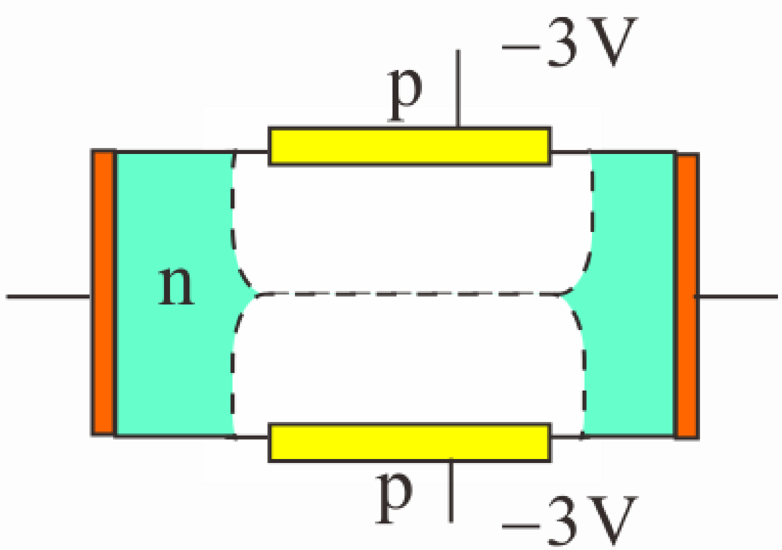
Circuit symbols



n-channel

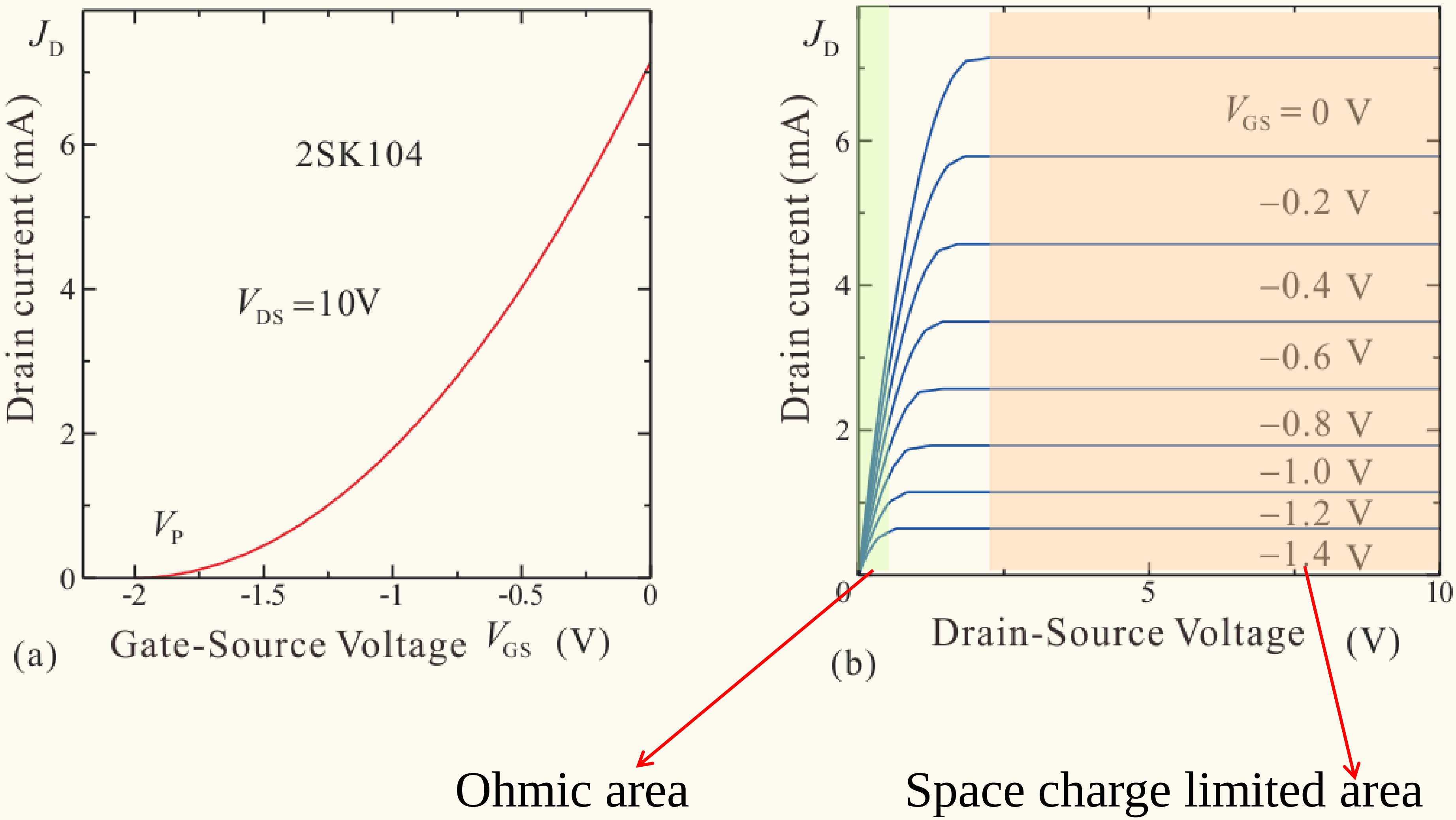


p-channel

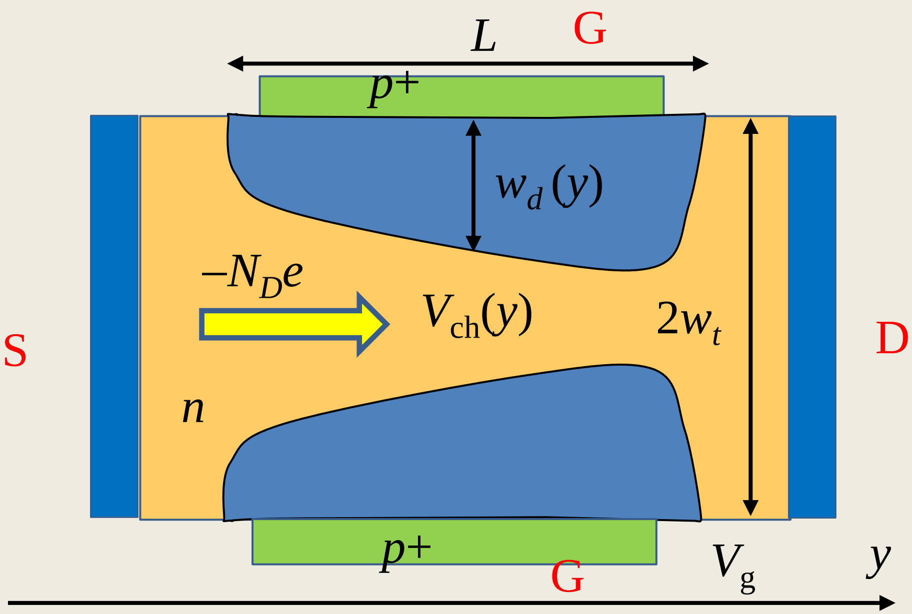


Pinch-off

Static characteristics of FET



Space-charge limitation of source-drain current



$$V(y) = V_g + V_{vi} - V_{ch}(y)$$

$$w_d(y) = \sqrt{\frac{2\epsilon\epsilon_0 V(y)}{eN_D}}$$

$$J_{ch} = eN_D\mu_n \frac{dV_{ch}}{dy} \cdot 2[w_t - w_d(y)]W$$

conductivity

electric field

channel width

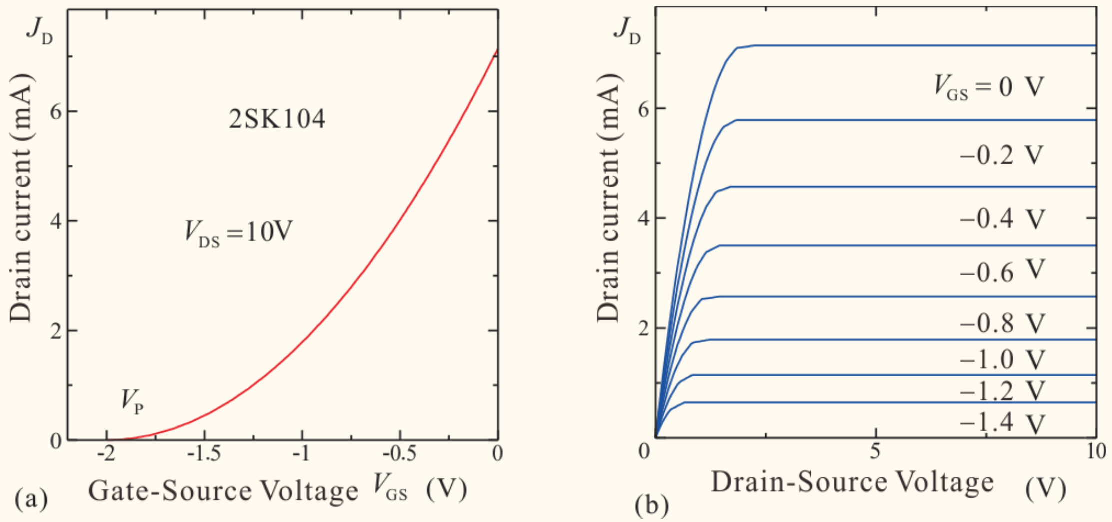
$$J_{ch}L = \int_0^L J_{ch}dy = 2eN_D\mu_n W \int_0^L (w_t - w_d)\frac{dV}{dy}dy = 2w_t eN_D\mu_n W \int_{V_0}^{V_L} \left(1 - \frac{w_d}{w_t}\right) dV$$

pinch off (internal) voltage: $w_d(V_c) = w_t \quad V_c = \frac{eN_D w_t^2}{2\epsilon\epsilon_0}$

$$J_{ch} = \frac{2N_D e\mu_n W w_t}{L}\left[V_L - V_0 + \frac{2}{3\sqrt{V_c}}(V(V_0)^{3/2} - V(V_L)^{3/2})\right]$$

Only valid for $w_d < w_t/2$.

Static characteristics of FET



$$J_{\mathrm{G}} \simeq 0,$$
$$J_{\mathrm{D}} = f(V_{\mathrm{G}}, V_{\mathrm{D}})$$

$$g_m \equiv \left(\frac{\partial J_{\mathrm{D}}}{\partial V_{\mathrm{GS}}}\right)_{V_{\mathrm{D}}=\mathrm{const.}}$$ transconductance

Low bias current:
small power consumption

$$r_d \equiv \left(\frac{\partial V_{\mathrm{D}}}{\partial J_{\mathrm{D}}}\right)_{V_{\mathrm{GS}}=\mathrm{const.}}$$ Drain resistance

Locally linear approximation

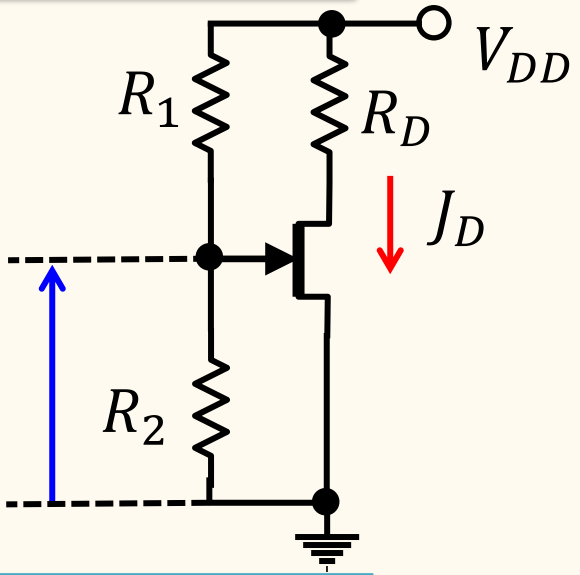
$$j_d = g_m v_{gs} + \frac{v_d}{r_d}$$

$$v_d = -r_d g_m v_{gs} + r_d j_d$$

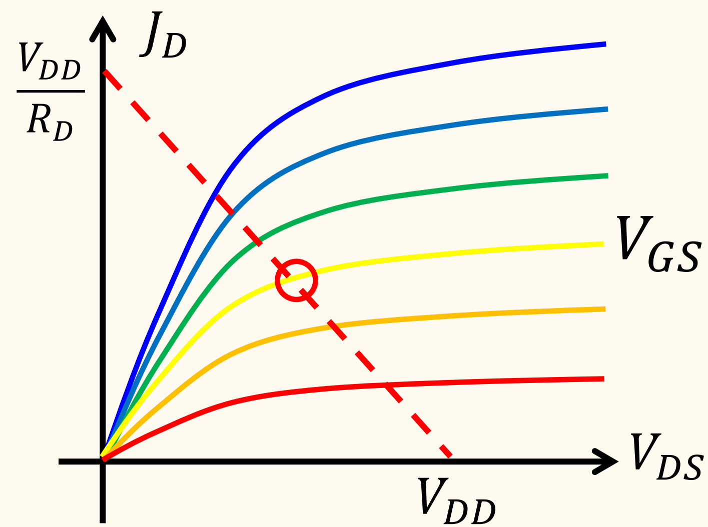
Amplification factor (voltage gain) $\equiv \mu$

Biasing circuits for FETs

Fixed bias circuit

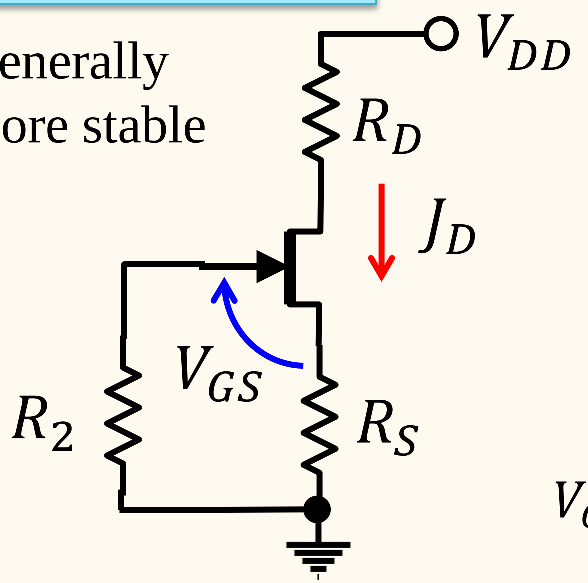


$$V_{\mathrm{GS}} = \frac{R_2}{R_1 + R_2} V_{\mathrm{DD}}, \quad V_{\mathrm{DS}} = V_{\mathrm{DD}} - R_{\mathrm{D}} J_{\mathrm{D}}$$

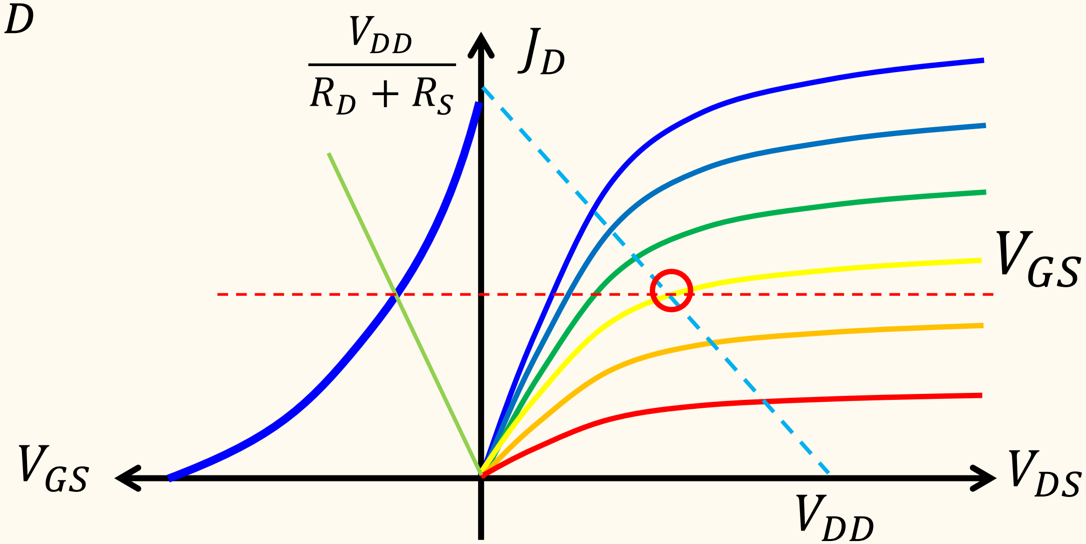


Self-biasing circuit

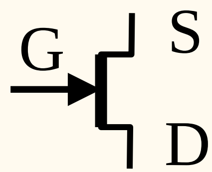
Generally more stable



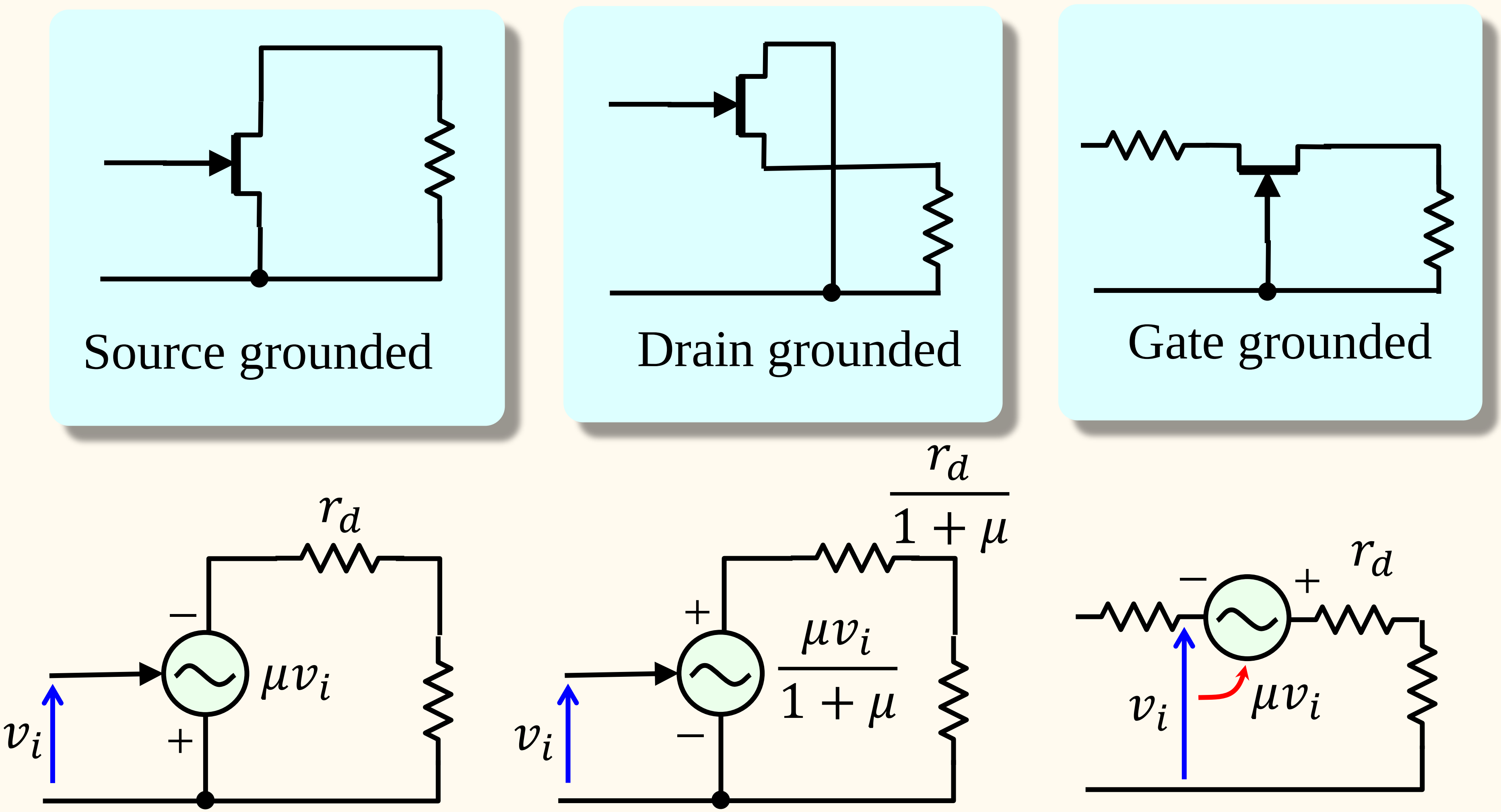
$$V_{\mathrm{GS}} = -R_{\mathrm{S}} J_{\mathrm{D}}, \quad V_{\mathrm{DS}} = V_{\mathrm{DD}} - (R_{\mathrm{D}} + R_{\mathrm{S}}) J_{\mathrm{D}}$$



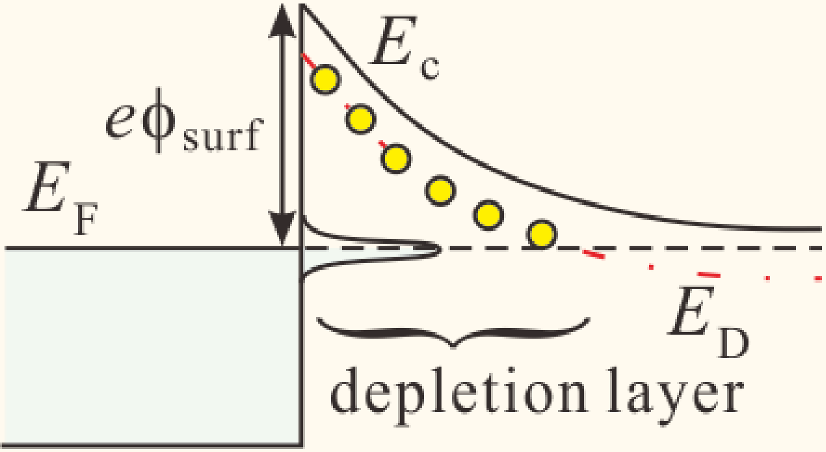
Equivalent signal circuits for FET



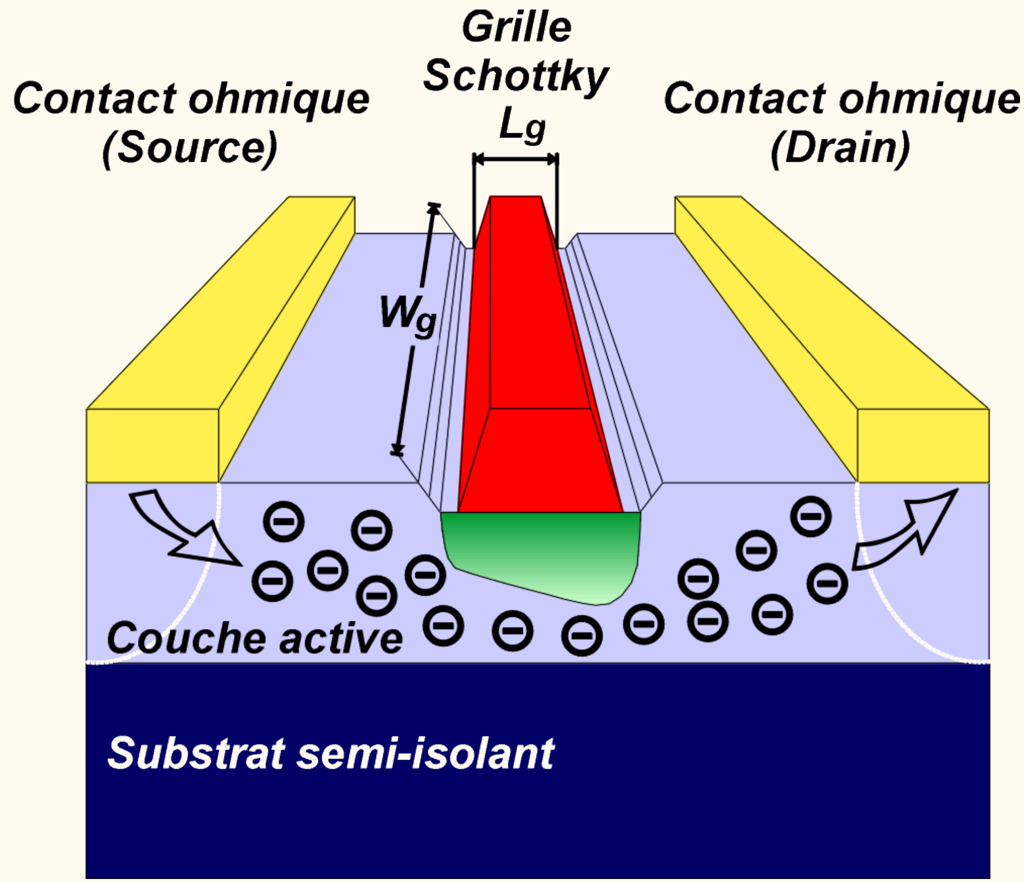
$$\mu = r_d g_m$$



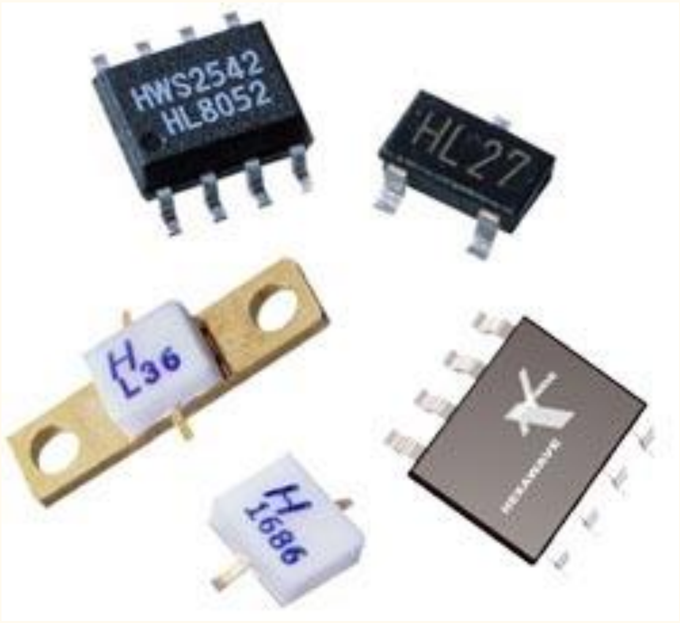
MES-FET



E_c
$e\phi_{surf}$
E_F
E_D
depletion layer

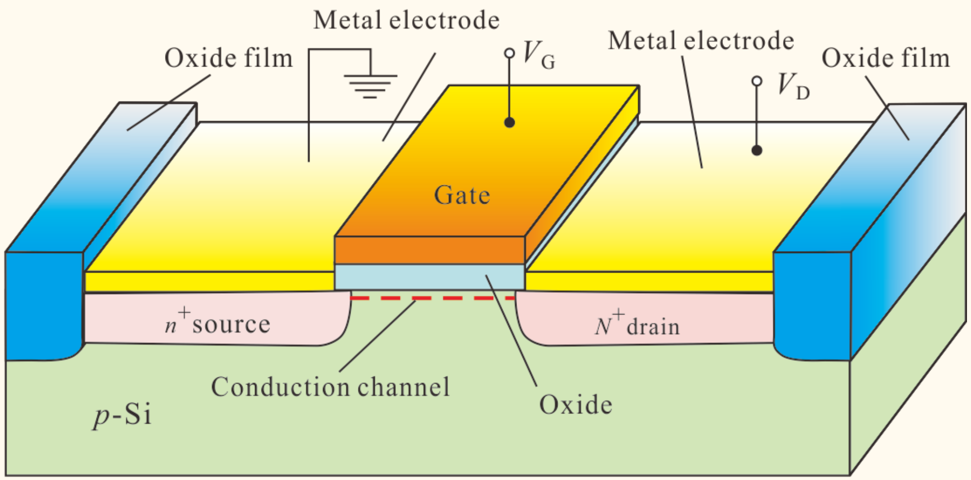


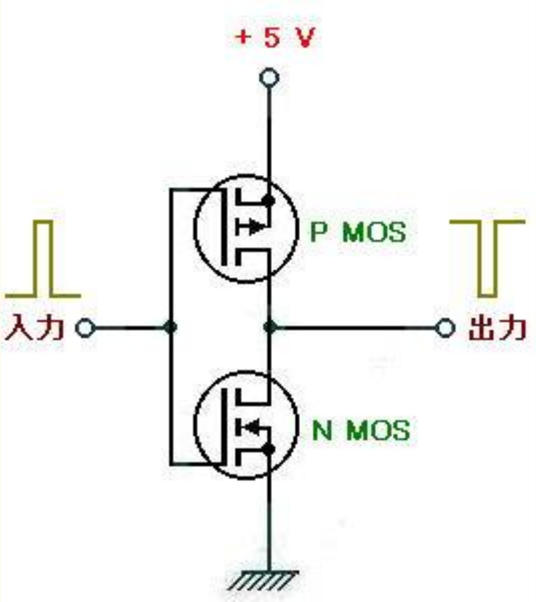
Grille
Schottky
L_g
Contact ohmique
(Source)
Contact ohmique
(Drain)
W_g
Couche active
Substrat semi-isolant



HWS2542
HL8052
HL27

MOS-FET

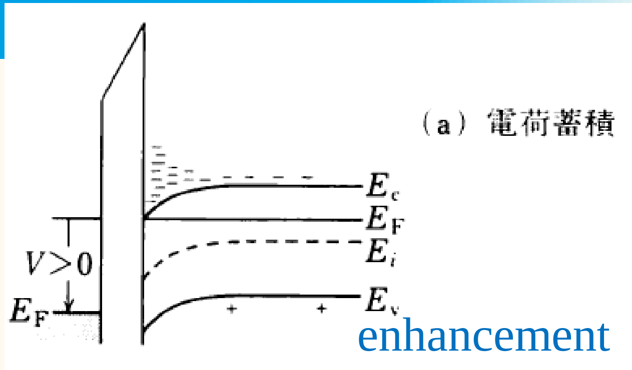


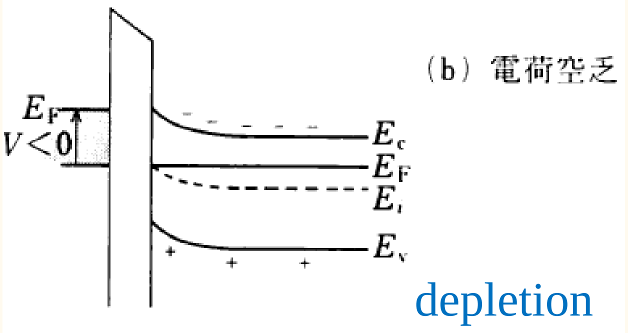


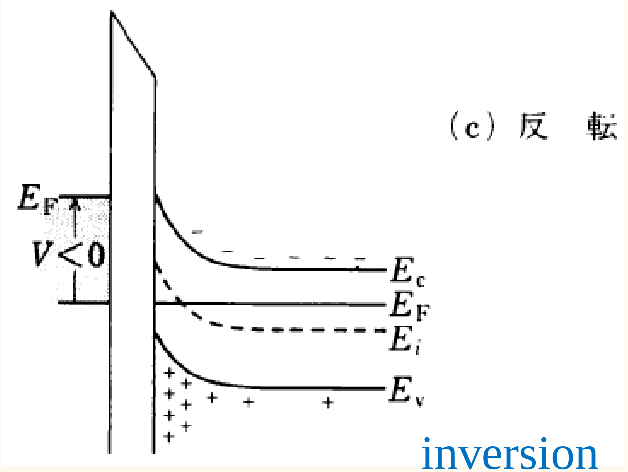
Simplified CMOS inverter circuit

Low leakage current

Single gate input both on/off switch

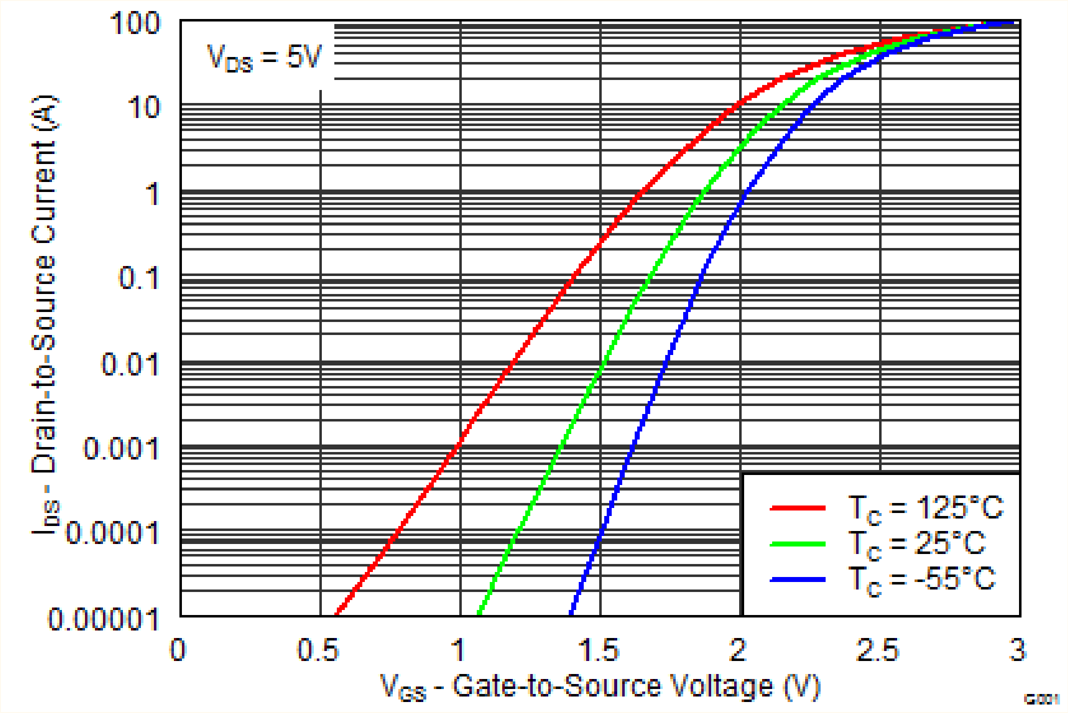






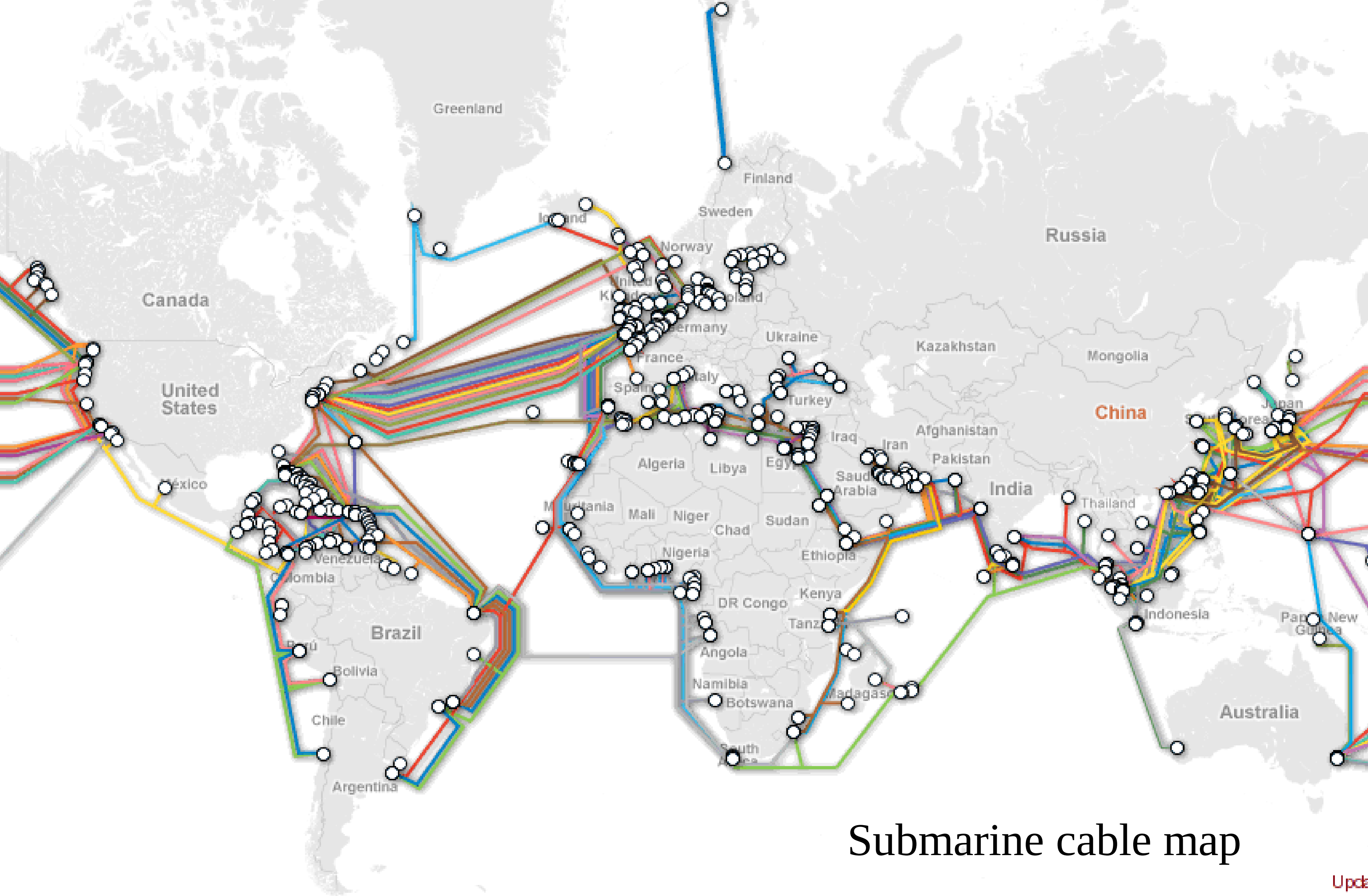
MOSFET switching characteristics

From datasheet CSD87381P power MOSFET (Texas instr.).



More than 7 orders change in J_D within 3 V change of V_{GS}.

Ch.5 Distributed constant circuits



Submarine cable map

Distributed constant circuit concept

1. In what case we need to consider distributed constant circuits?

Characteristic sizes of devices $\gtrsim$ wavelength of electromagnetic signal

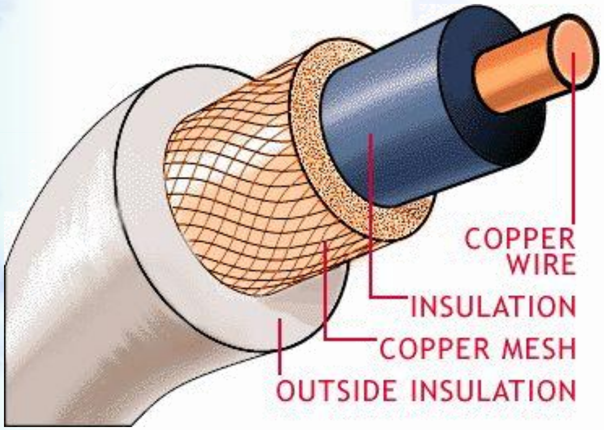
2. A typical scheme to make the shift for distributed circuit

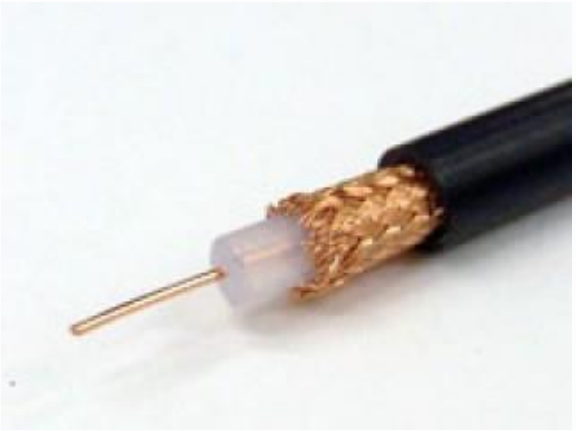
Lumped constant circuit → 1. Connection of unit circuits 2. Taking the infinitesimal limit → Distributed constant circuit

3. Distributed constant circuits : transmission lines

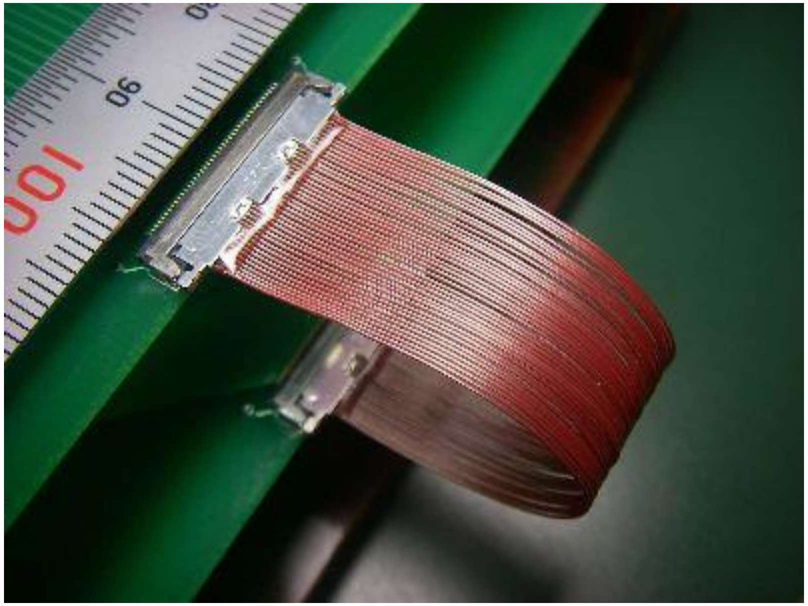
Coaxial cables, Lecher lines, micro-strip lines, waveguides, optical fibers

5.1.1 Coaxial cable

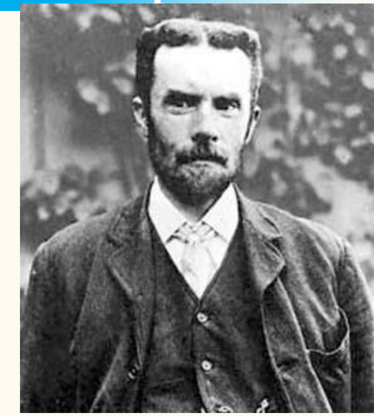




Thin coaxial cable AWG50 (ϕ25μm)

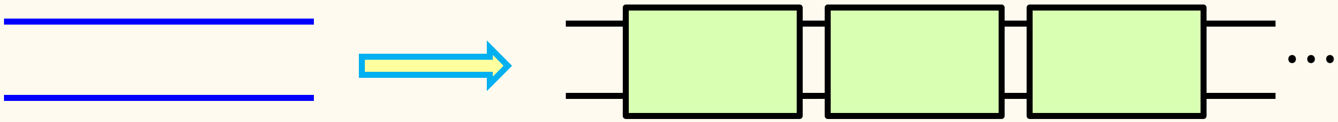


Transmission line as a series of infinitesimal terminal-pairs

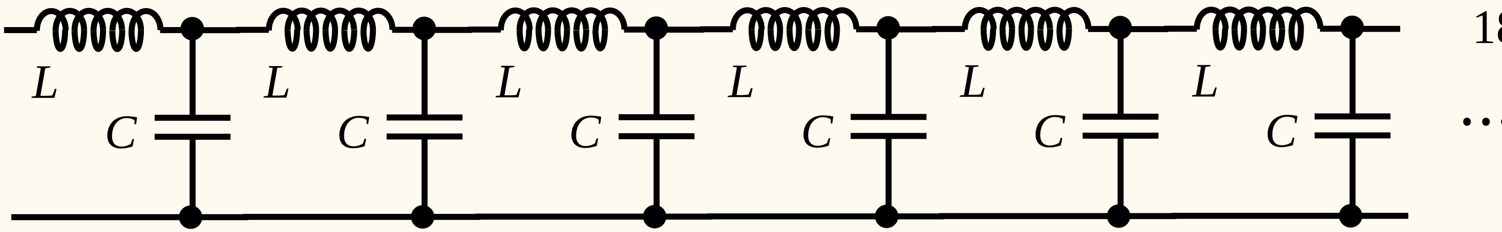


Oliver Heaviside
1850- 1925

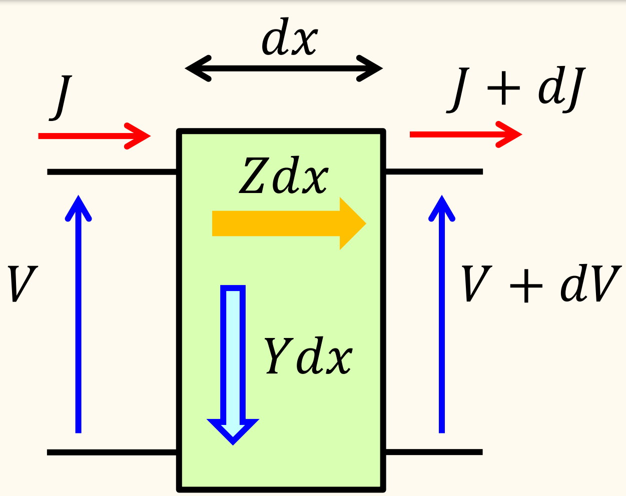
Transmission line → divide into four terminal circuits



Each unit should have delay. Ignore energy dissipation.



Then take the infinitesimal limit



Width → 0, Number → ∞

$$dV = -JZdx, \quad dJ = -VYdx$$

$$\begin{cases} \dfrac{d^2J}{dx^2} = YZJ, \\ \dfrac{d^2V}{dx^2} = YZV \end{cases}$$

Telegraphic equation

Characteristic impedance

$$\kappa \equiv \sqrt{YZ} \qquad \text{(dimension: } L^{-1}\text{)}$$

$$J(x,t) = J(0,t)\exp(\pm\kappa x), \quad V(x,t) = V(0,t)\exp(\pm\kappa x)$$

−: Progressive, +: Retrograde

$$\frac{V}{J} = \mp\frac{Z}{\kappa} = \mp\sqrt{\frac{Z}{Y}}$$

Characteristic impedance

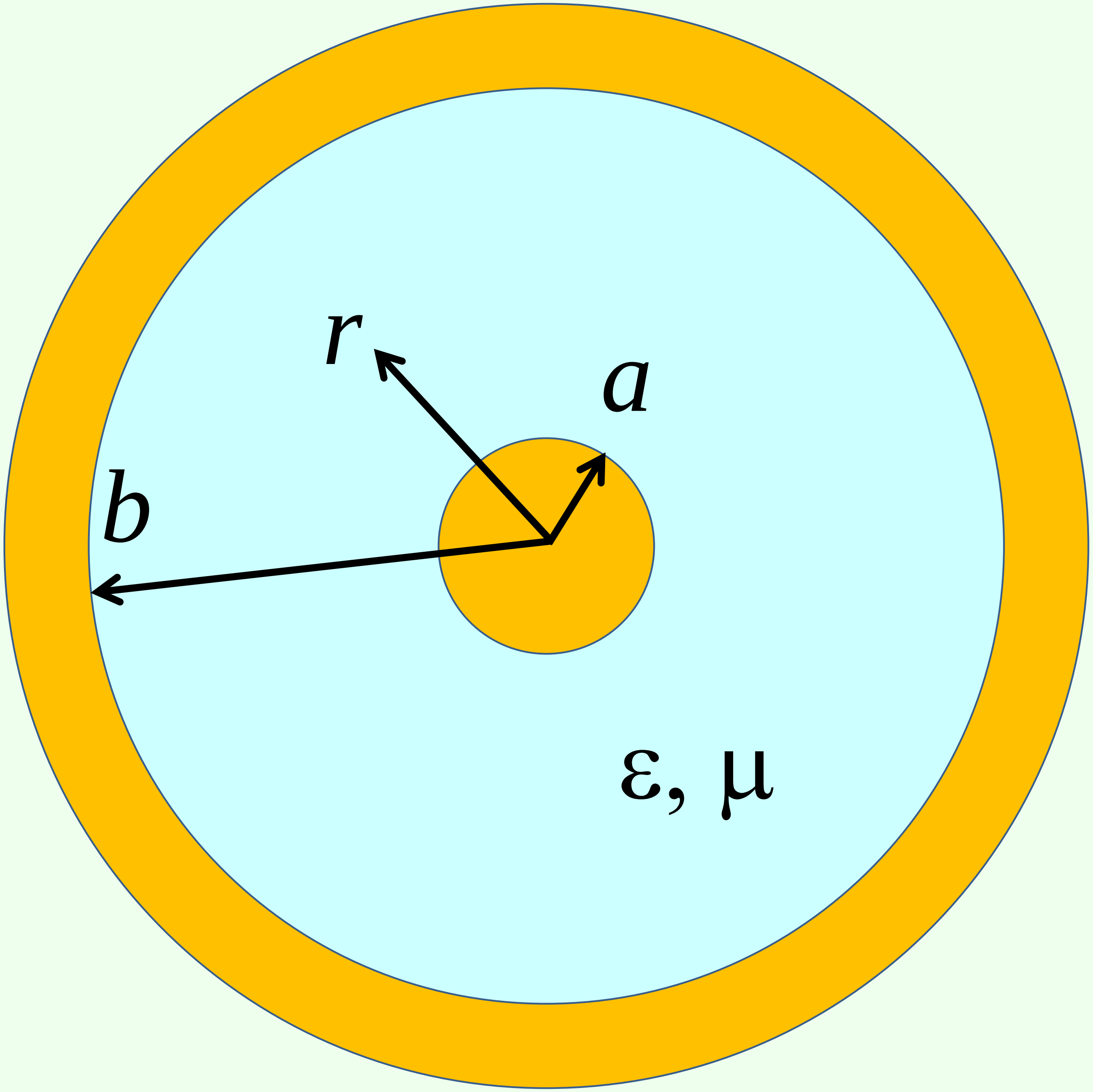
Pure reactance $Y = i\omega C,\ Z = i\omega L$ For L and C model

$$\kappa = \sqrt{-\omega^2 LC} = i\frac{\omega}{\omega_0}, \quad \omega_0 \equiv \frac{1}{\sqrt{LC}}$$

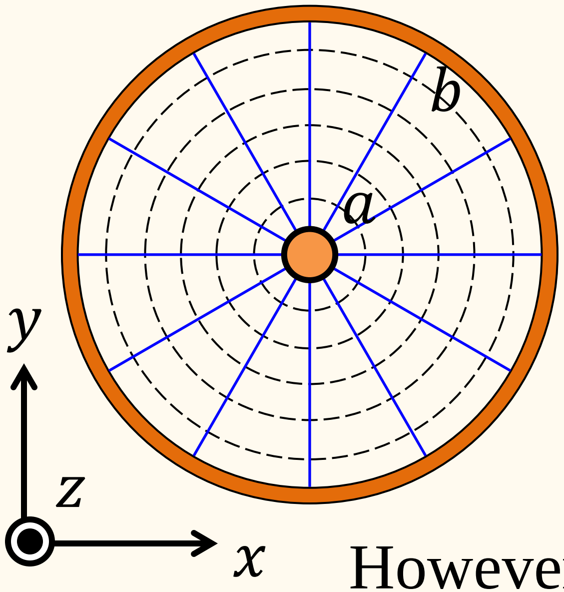
(dimension: velocity)

$$Z_0 = \sqrt{\frac{L}{C}}$$

Coaxial cable setup



Maxwell theory



$$E = E_0(x,y)e^{i\omega t-\gamma z}, \quad H = H_0(x,y)e^{i\omega t-\gamma z}$$

From Maxwell equations

$$(\omega^2\epsilon\mu+\gamma^2)\begin{pmatrix}E_x\\E_y\end{pmatrix} = \begin{pmatrix}-\gamma\partial_x & -i\omega\mu\partial_y\\ -\gamma\partial_y & i\omega\mu\partial_x\end{pmatrix}\begin{pmatrix}E_z\\H_z\end{pmatrix},$$

$$(\omega^2\epsilon\mu+\gamma^2)\begin{pmatrix}H_x\\H_y\end{pmatrix} = \begin{pmatrix}i\omega\mu\partial_y & -\gamma\partial_x\\ -i\omega\mu\partial_x & -\gamma\partial_y\end{pmatrix}\begin{pmatrix}E_z\\H_z\end{pmatrix}.$$

However in TEM (transverse electric and magnetic) mode:

$E_z = H_z = 0$ *i.e.*, the RHSs are zero.

For the fields along *x* and *y* to survive, $\omega^2\epsilon\mu+\gamma^2=0 \quad \therefore \gamma = \pm i\omega\sqrt{\epsilon\mu}$

Propagation velocity $v = \dfrac{\omega}{\omega\sqrt{\epsilon\mu}} = \dfrac{1}{\sqrt{\epsilon\mu}}$

In such a case, from Maxwell equations: $\mathrm{rot}_{xy}\boldsymbol{H}=0, \ \mathrm{rot}_{xy}\boldsymbol{E}=0$

→ Potentials are conceivable for ***H*** and ***E***.

Maxwell theory

$$\boldsymbol{E} = \nabla_{xy}\mathcal{U}/\sqrt{\epsilon}, \quad \boldsymbol{H} = \nabla_{xy}\mathcal{V}/\sqrt{\mu}$$

$$\frac{\partial\mathcal{U}}{\partial x} = \frac{\partial\mathcal{V}}{\partial y}, \quad \frac{\partial\mathcal{U}}{\partial y} = -\frac{\partial\mathcal{V}}{\partial x}$$ Cauchy-Riemann theorem

Characteristic impedance: $Z_0 = \dfrac{\mathcal{U}_a - \mathcal{U}_b}{J\sqrt{\epsilon}}$

If we can express *V* and *J* in the form of distributed constant circuit model (*L* and *C* model), the equivalence is certified.

Capacitance part

$$V = \frac{q}{\epsilon}\int_a^b \frac{dr}{2\pi r} = \frac{q}{2\pi\epsilon}\log\frac{b}{a} = \frac{q}{C}$$

$$\therefore C = \frac{2\pi\epsilon}{\log(b/a)}$$

Maxwell theory

Inductance part

Core current J, shield current $-J$

$$H(r) = \frac{J}{2\pi r}, \quad B(r) = \frac{\mu J}{2\pi r}$$

Flux per length: $\Phi = \int_a^b dr B(r) = \frac{\mu J}{2\pi} \log \frac{b}{a}$

Self inductance per length: $L = \frac{\mu}{2\pi} \log(b/a)$

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}} \log\left(\frac{b}{a}\right)$$

cf. Characteristic impedance of vacuum $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 376\Omega$

Coaxial cable 2

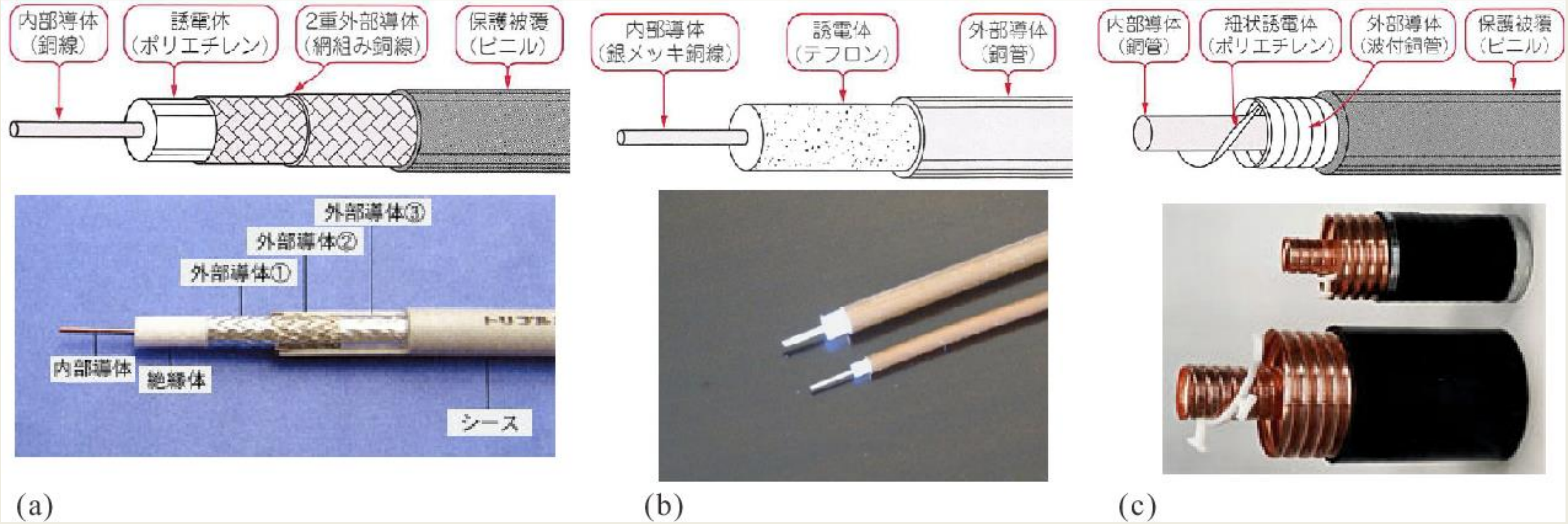


図12 同軸ケーブルの型名（JIS C3501）

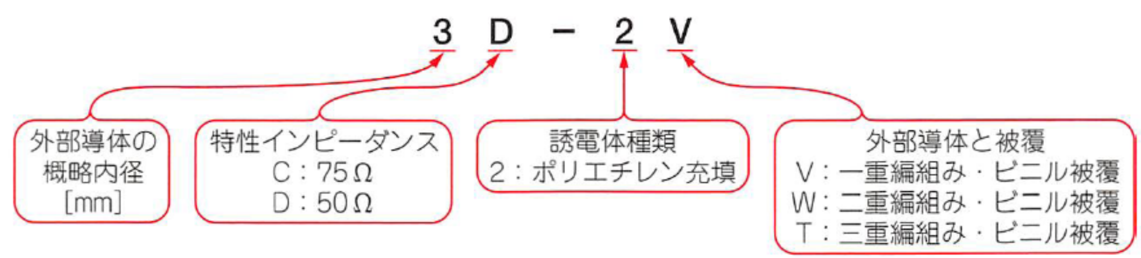
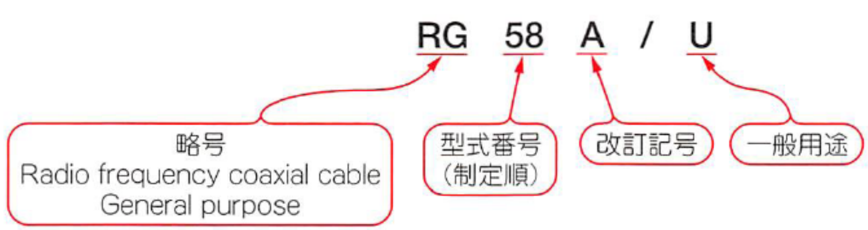
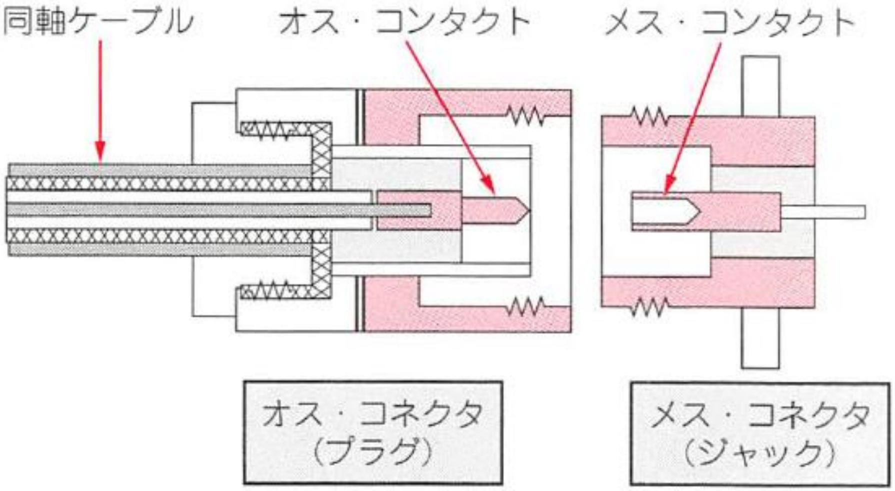


図13 MIL規格での同軸ケーブル型名の例



Coaxial connectors

図22 同軸コネクタの構造(概念図)

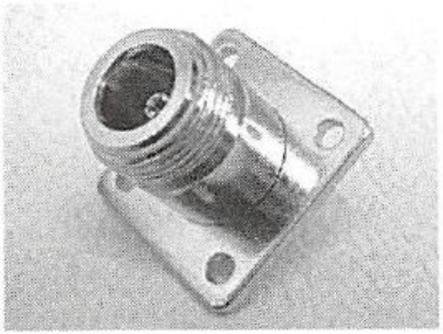


代表的な同軸コネクタの最高使用周波数例

形　式	外部導体内径	最高使用周波数
BNC	約 7 mm	2 ~ 4 GHz
N	約 7 mm	10 ~ 18 GHz
7 mm	7 mm	~ 18 GHz
SMA	4.15 mm	18 GHz
3.5 mm	3.5 mm	26.5 GHz
K	2.92 mm	40 GHz
2.4 mm	2.4 mm	50 GHz
V	1.85 mm	65 GHz
W	1.1 mm	110 GHz
1.0 mm	1.0 mm	110 GHz

Coaxial connectors

写真2 N型コネクタ

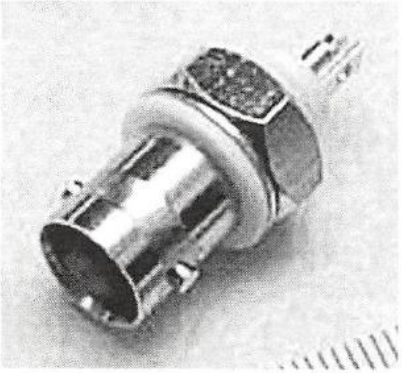


(a) フランジ付きジャック

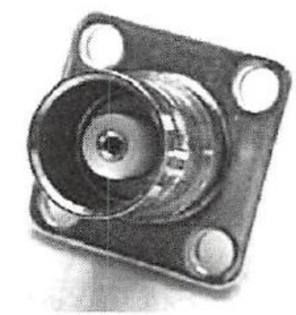
(b) プラグ

(c) プラグ[(b)を分解]

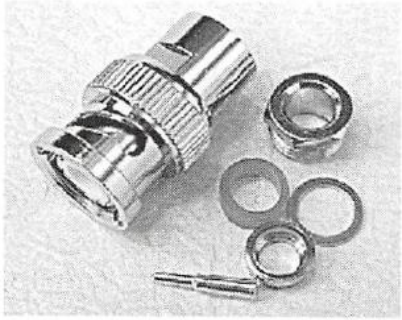
写真3 BNC型コネクタ



(a) 絶縁型ジャック
(高周波に向かない)



(b) フランジ付きジャック



(c) プラグ

Coaxial connectors 2

SMA-type

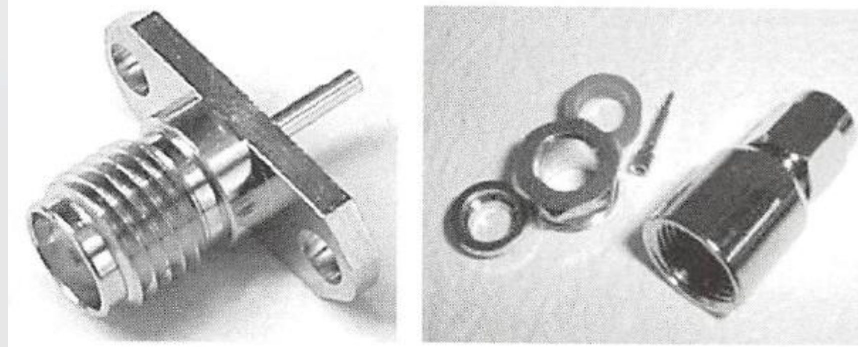


写真4 SMA型コネクタ

(a) ジャック (b) プラグ

jack plug

K-type

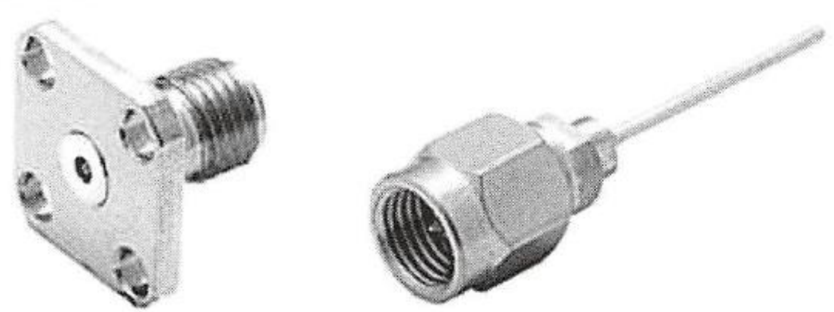


写真6 K型コネクタ

(a) ジャック (b) プラグ

V-type

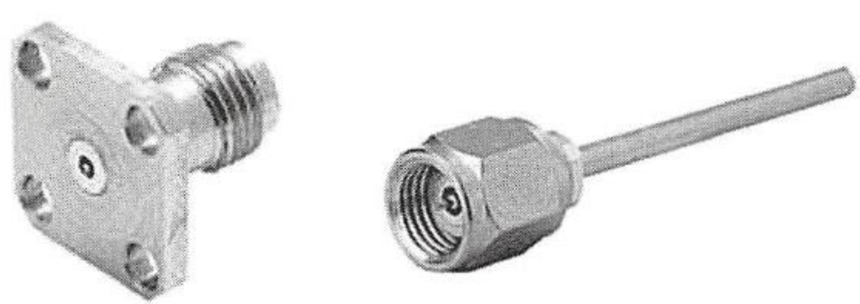


写真7 V型コネクタ

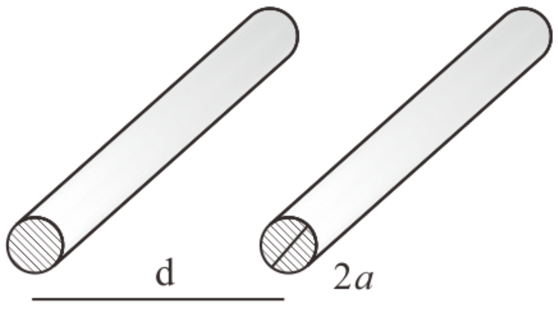
(a) ジャック (b) プラグ

LEMO cables and connectors

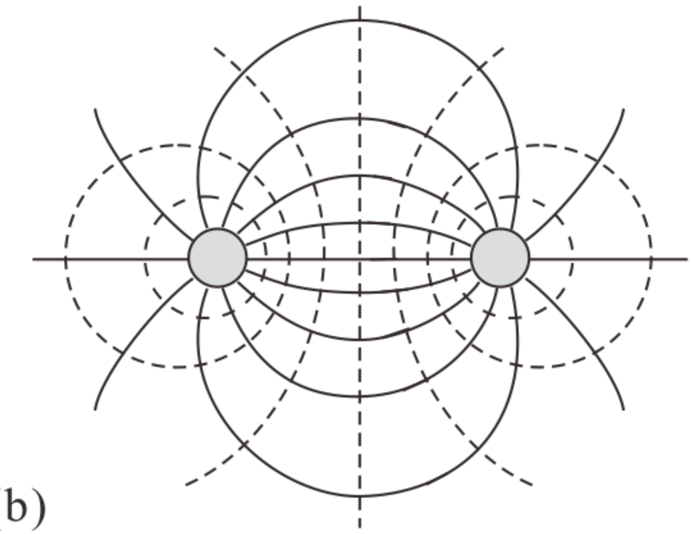
http://www.lemo.com/

High-energy physics experiment, etc.

Lecher line



(a)

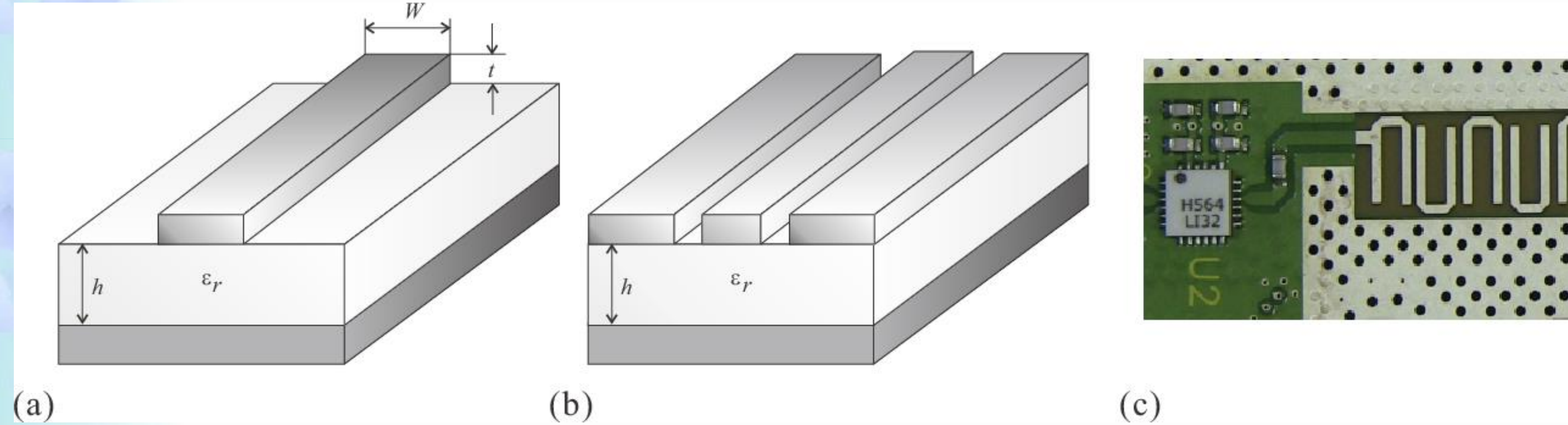


(b)

(c)

$$\phi_1 = -\phi_2 = \frac{J\sqrt{\mu}}{2\pi}\log\frac{d}{a} \qquad Z_0 = \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\pi}\log\frac{d}{a}$$

Micro strip line



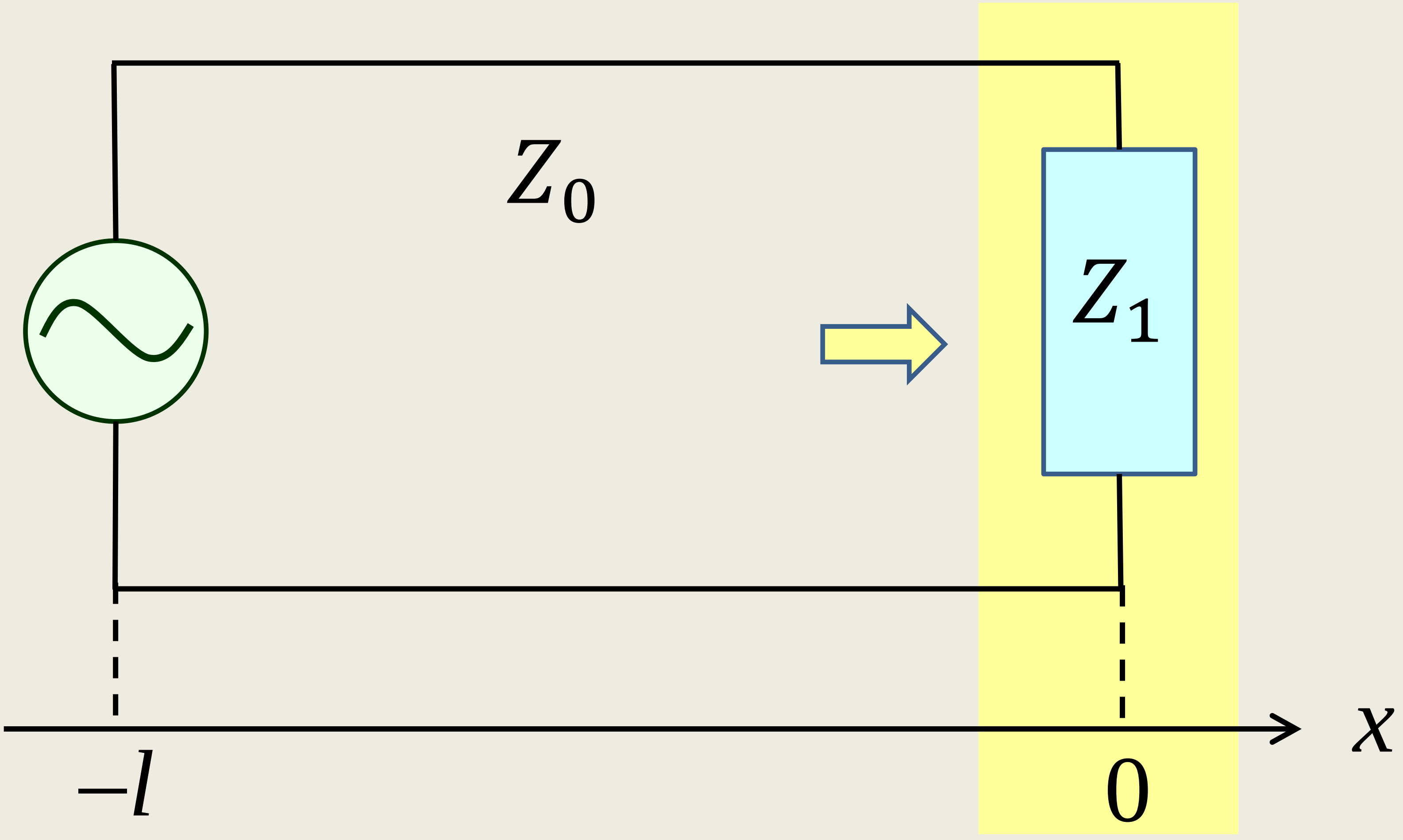
Wide (W/h>3.3) strip

$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{2\sqrt{\epsilon_r}}\left\{\frac{W}{2h} + \frac{1}{\pi}\log 4 + \frac{\epsilon_r+1}{2\pi\epsilon_r}\log\left[\frac{\pi e}{2}\left(\frac{W}{2h}+0.94\right)\right]\frac{\epsilon_r-1}{2\pi\epsilon_r^2}\log\frac{e\pi^2}{16}\right\}^{-1}$$

Narrow (W/h<3.3) strip

$$Z(W,h,\epsilon_r) = \frac{Z_{F0}}{\pi\sqrt{2(\epsilon_r+1)}}\left\{\log\left[\frac{4h}{W} + \sqrt{\left(\frac{4h}{W}\right)^2+2}\right] - \frac{1}{2}\frac{\epsilon_r-1}{\epsilon_r+1}\left(\log\frac{\pi}{2} + \frac{1}{\epsilon_r}\log\frac{4}{\pi}\right)\right\}$$

Connection and termination



Connection and termination

At $x = 0$:
$$\begin{cases} J = J_+ + J_- \quad \text{(definition right positive)} \\ \quad \text{progressive} \quad \text{retrograde} \\ \\ V = V_+ + V_- = Z_0(J_+ - J_-) \end{cases}$$

$$Z_1 = \frac{V}{J} = \frac{J_+ - J_-}{J_+ + J_-} Z_0$$

Reflection coefficient: $$r = \frac{V_-}{V_+} = -\frac{J_-}{J_+} = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$Z_1 = Z_0$: no reflection, i.e., impedance matching

$Z_1 = +\infty$ (open circuit end) : $r = 1$, i.e., free end

$Z_1 = 0$ (short circuit end) : $r = -1$, i.e., fixed end

Connection and termination

Finite reflection → Standing wave

Voltage-Standing Wave Ratio (VSWR): $= \dfrac{1+|r|}{1-|r|}$

At $x = -l$

$$\left.\begin{aligned} V &= V_{+0}\exp(\kappa l) + V_{-0}\exp(-\kappa l) = [J_{+0}\exp(\kappa l) - J_{-0}\exp(-\kappa l)]Z_0 \\ J &= J_{+0}\exp(\kappa l) + J_{-0}\exp(-\kappa l) \end{aligned}\right\}$$

$$Z_l = \frac{V}{J} = \frac{J_{+0}e^{\kappa l} - J_{-0}e^{-\kappa l}}{J_{+0}e^{\kappa l} + J_{-0}e^{-\kappa l}} Z_0$$

Reflection coefficient:

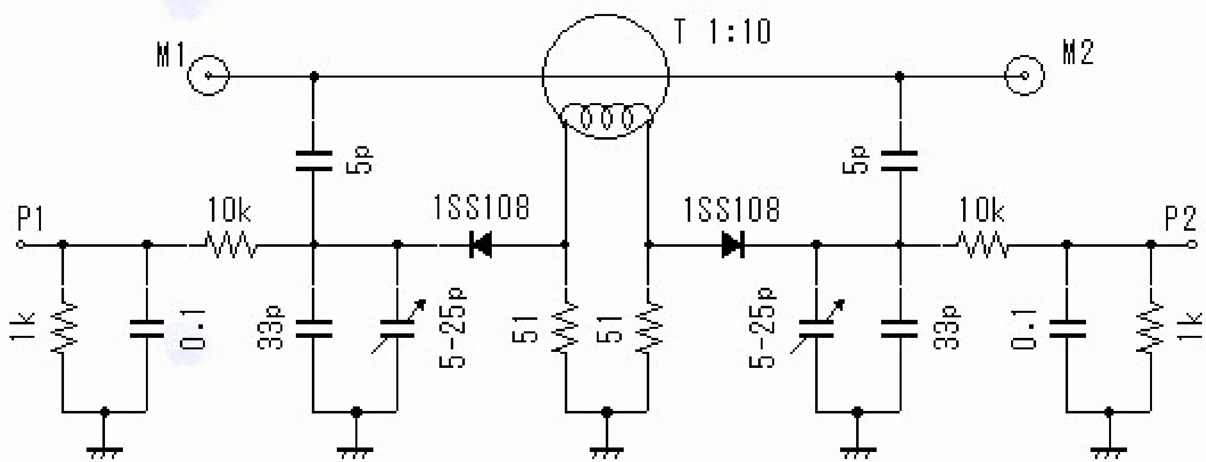
$$r_l = \frac{V_-}{V_+} = \frac{V_{-0}e^{-\kappa l}}{V_{+0}e^{\kappa l}} = r\exp(-2\kappa l)$$

SWR measurement

SWR Meters:

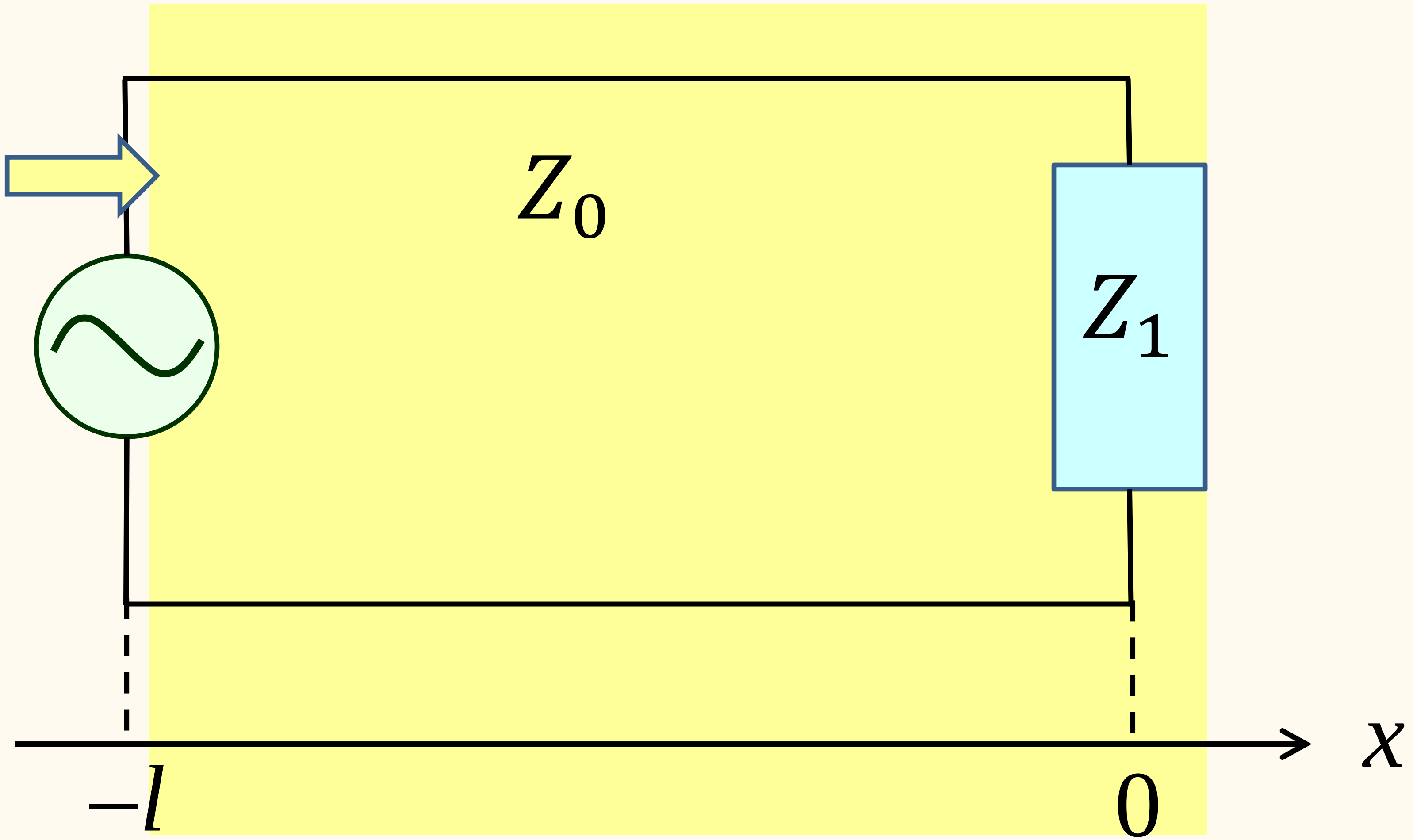
Desktop types

Cross-meter



Handy type

Connection and termination



Connection and termination

Transmission line connection.
Characteristic impedance Z_0, $Z_0{}'$

At the connection point, only the local relation between V and J affects the reflection coefficient.

The local impedance from the left hand side is $Z_0{}'$.

$$r = \frac{Z_0' - Z_0}{Z_0' + Z_0}$$

電子回路論第 8 回

Electric Circuits for Physicists

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Outline

5.1 Transmission lines

TEM mode Lecher line

Micro-strip line

TE, TM mode Waveguide

Optical fiber

5.2 Propagation in transmission lines

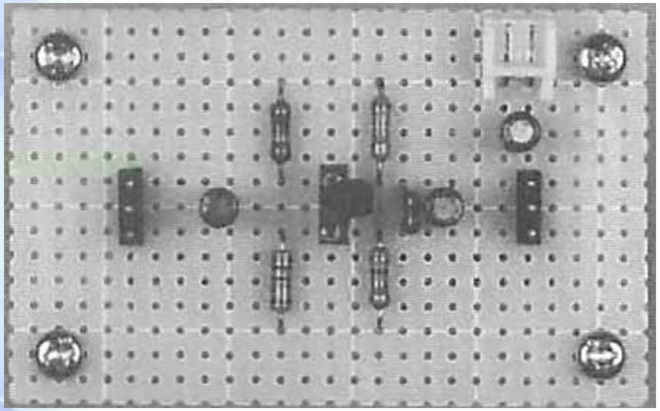
Termination and connection

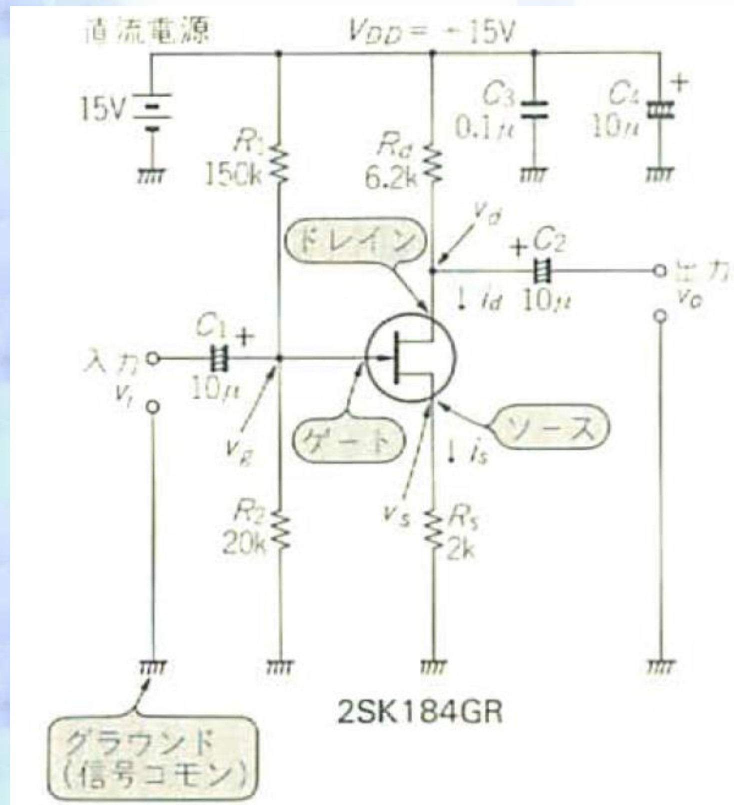
Smith chart

Scattering matrix

Impedance matching

Comment: bias + signal superposition





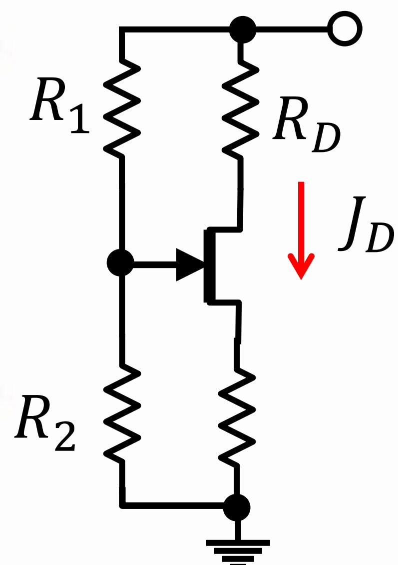
For bias (dc) circuits

All the capacitors can be viewed as break line.

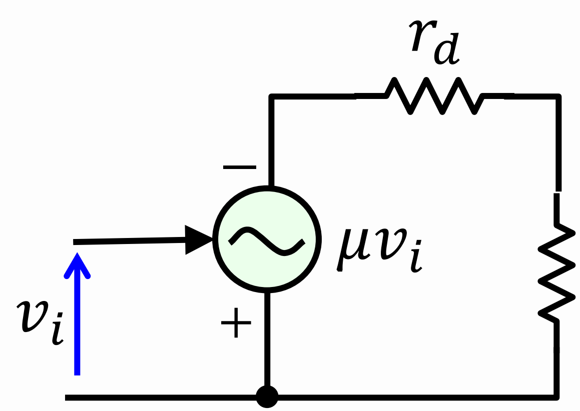
For small amplitude (high-frequency) circuits

All the capacitors can be viewed as short circuits.

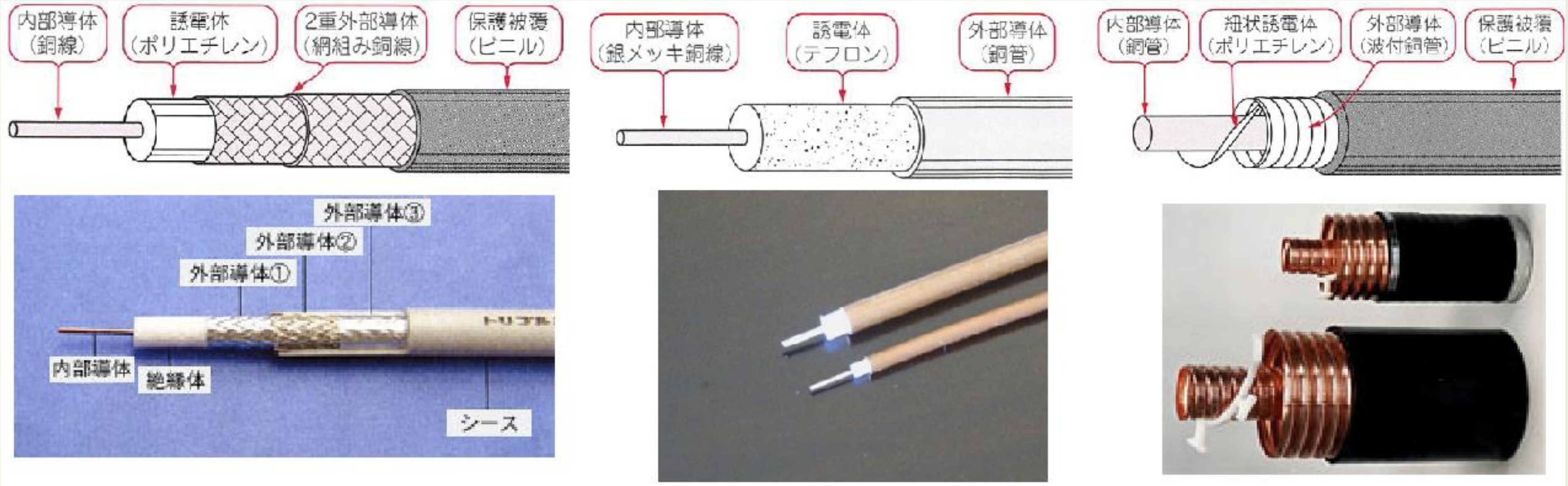
Self-bias



Source-grounded



Coaxial cable 2



(a) (b) (c)

図12 同軸ケーブルの型名（JIS C3501）

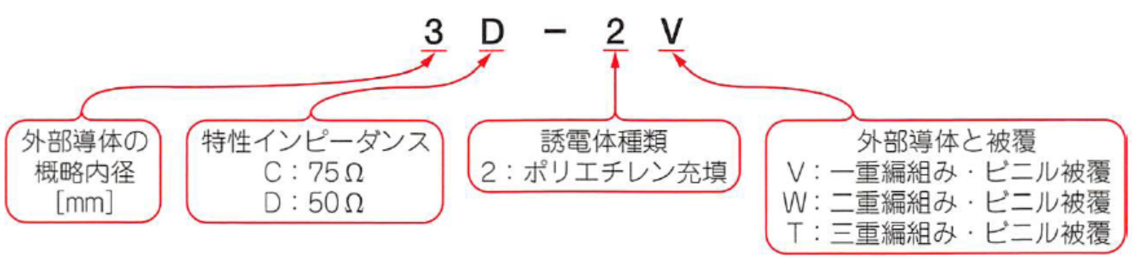
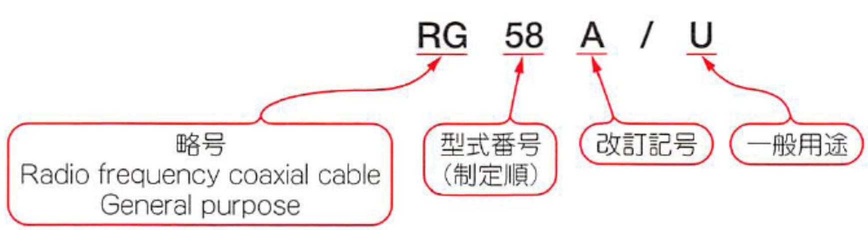
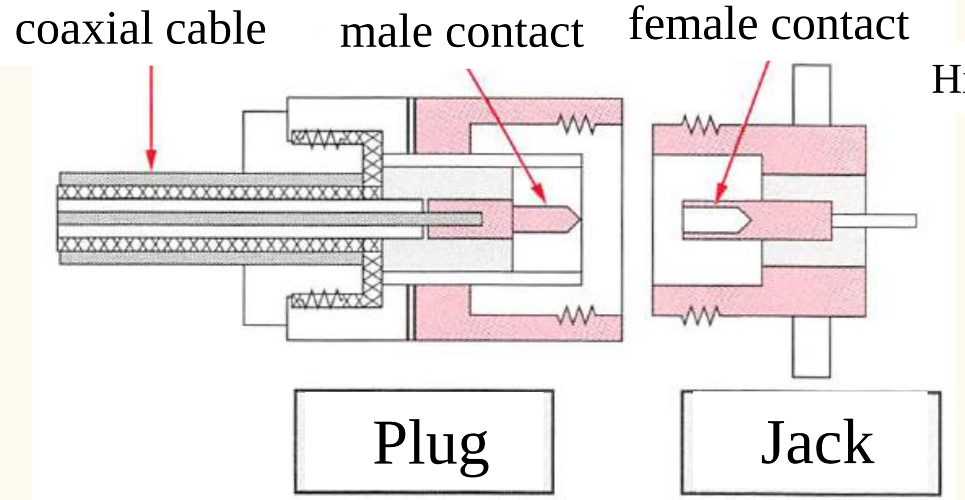


図13 MIL規格での同軸ケーブル型名の例



Coaxial connectors

図22 coaxial connector (schematic)

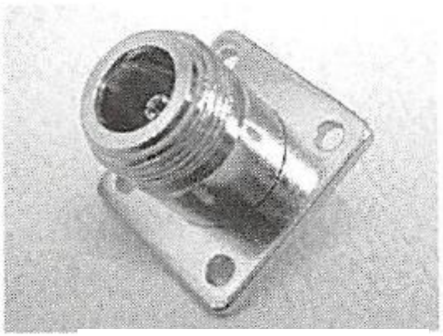


Highest available frequencies for coaxial connectors

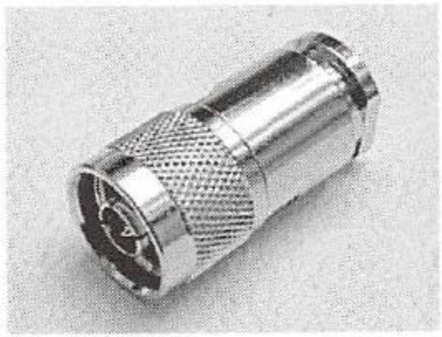
type	outer diam.	highest freq.
BNC	約 7 mm	2 ～ 4 GHz
N	約 7 mm	10 ～ 18 GHz
7 mm	7 mm	～ 18 GHz
SMA	4.15 mm	18 GHz
3.5 mm	3.5 mm	26.5 GHz
K	2.92 mm	40 GHz
2.4 mm	2.4 mm	50 GHz
V	1.85 mm	65 GHz
W	1.1 mm	110 GHz
1.0 mm	1.0 mm	110 GHz

Coaxial connectors

N-type connectors



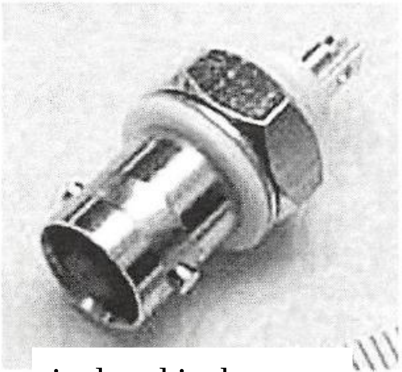
(a) jack with flange



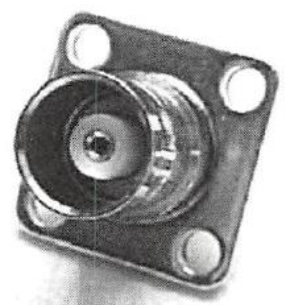
(b) plug

(c) plug [disassembled (b)]

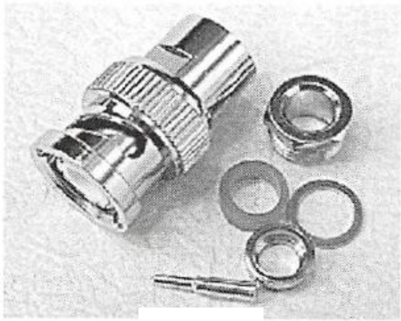
BNC-type connectors



isolated jack
(not for high freq.)



jack with flange

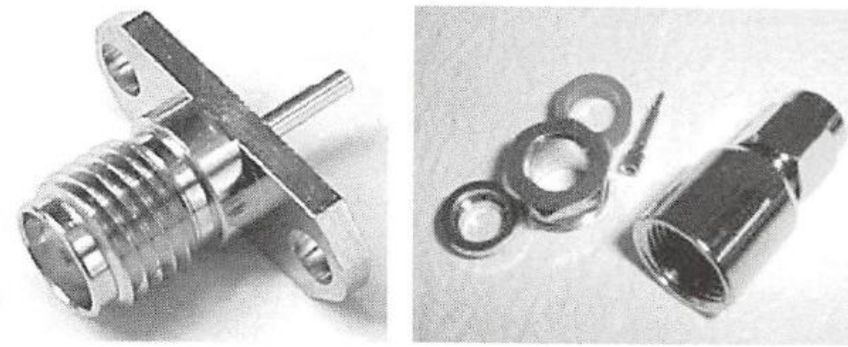


plug

Coaxial connectors 2

SMA-type

写真4 SMA型コネクタ

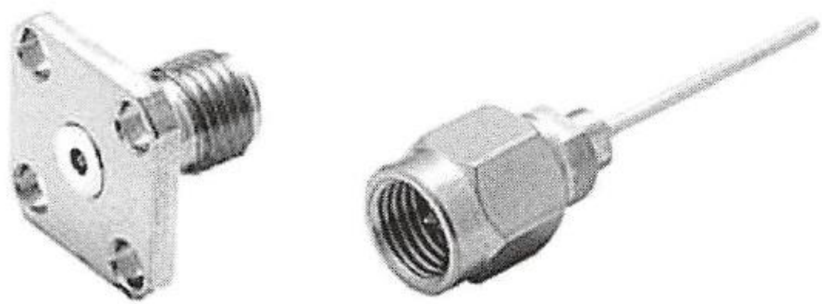


(a) ジャック (b) プラグ

jack plug

K-type

写真6 K型コネクタ

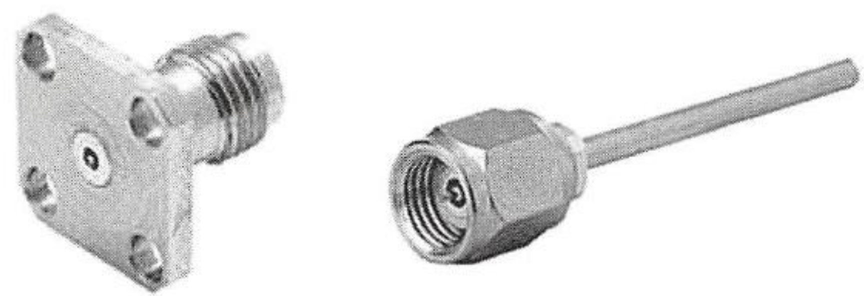


(a) ジャック (b) プラグ

jack plug

V-type

写真7 V型コネクタ



(a) ジャック (b) プラグ

jack plug

LEMO cables and connectors

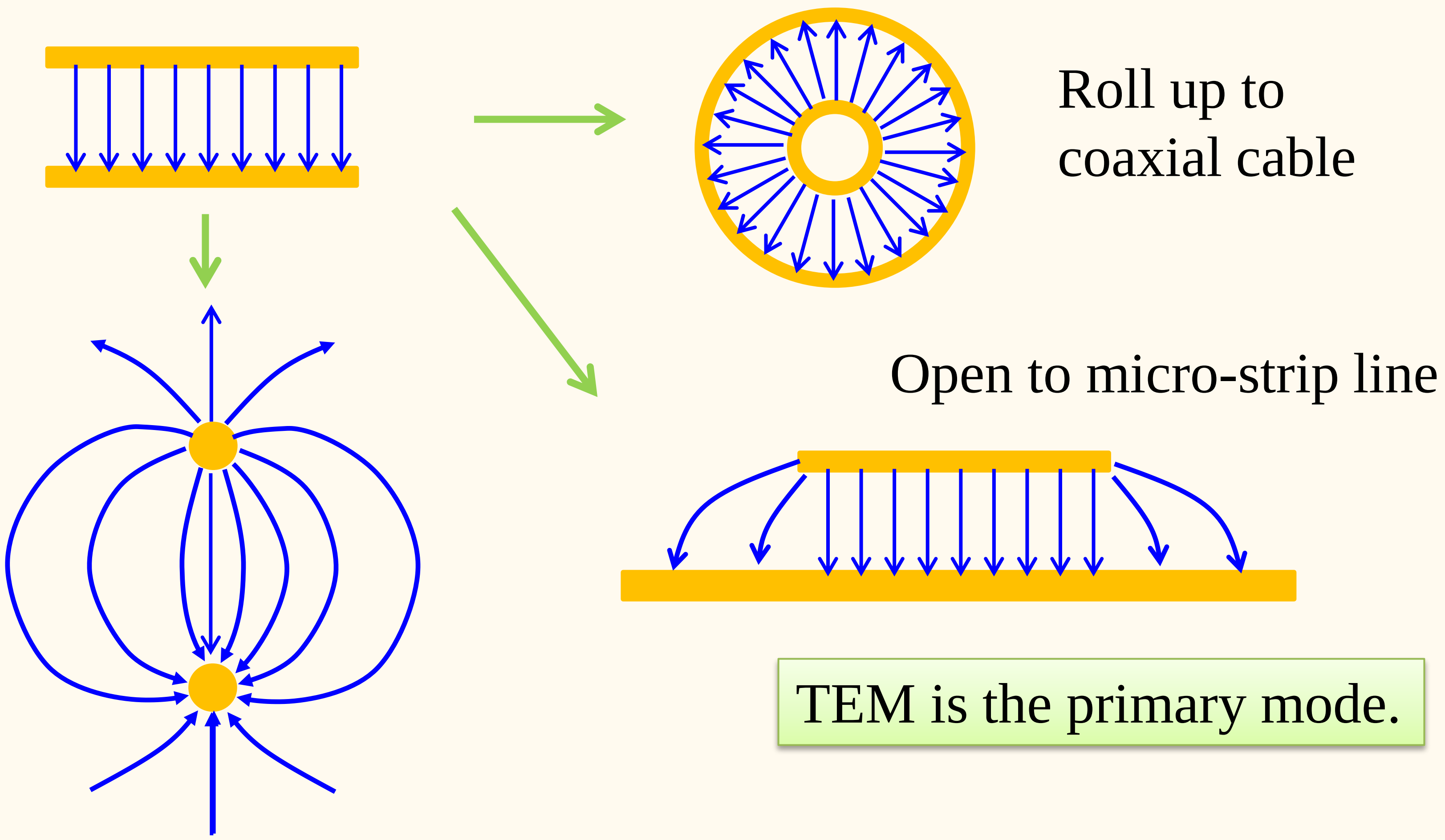
http://www.lemo.com/

High-energy physics experiment, etc.

Transmission lines with TEM mode

Transmission lines with two conductors are "families".

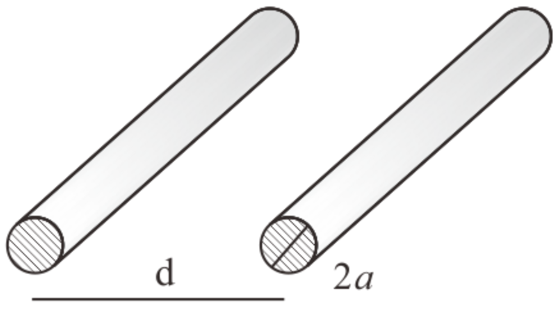
Electromagnetic field confinement with parallel-plate capacitor



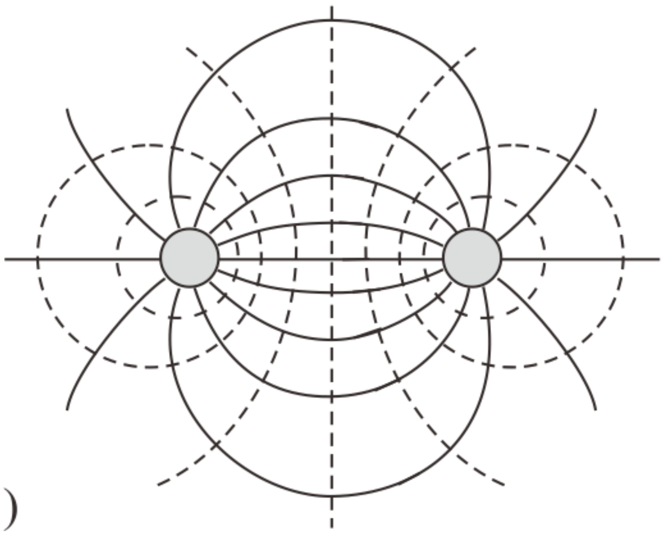
TEM is the primary mode.

Shrink to dipole (Lecher line)

Lecher line



(a)

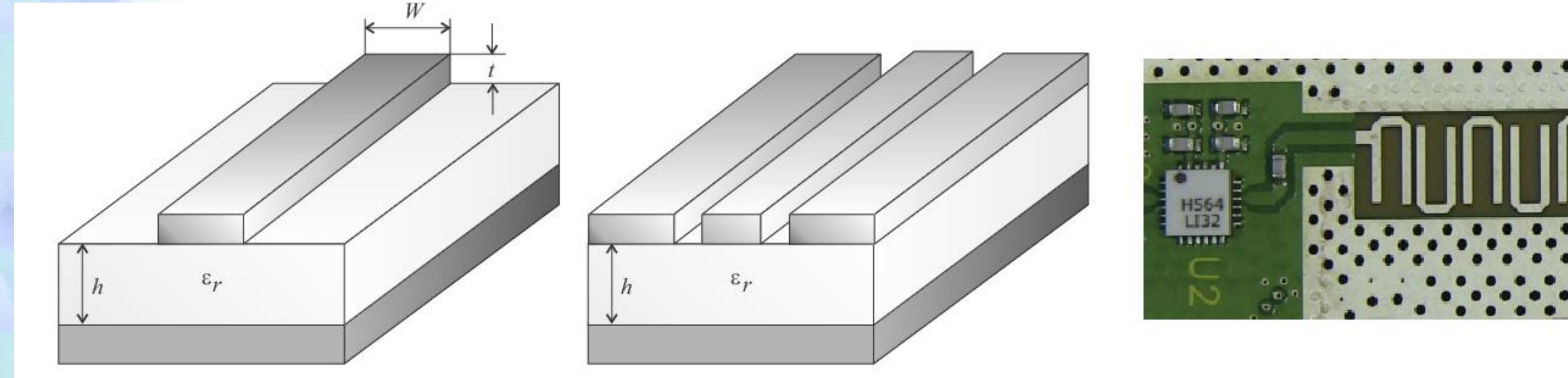


(b)

(c)

$$\phi_1 = -\phi_2 = \frac{J\sqrt{\mu}}{2\pi} \log \frac{d}{a} \qquad Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\pi} \log \frac{d}{a}$$

Micro strip line



(a) (b) (c)

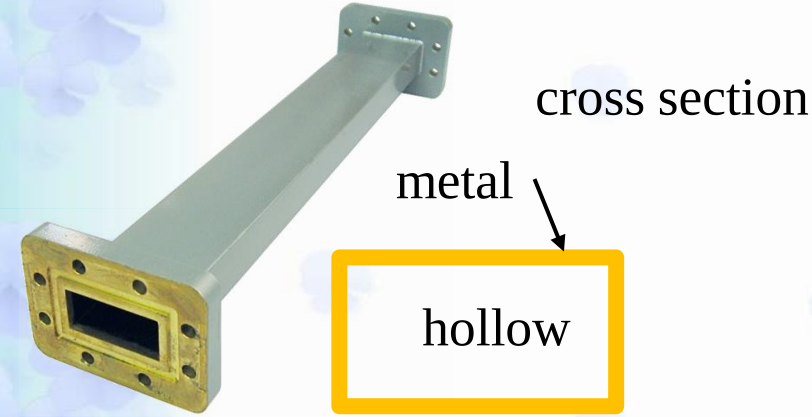
Wide (W/h>3.3) strip

$$Z(W,h,\epsilon_r)=\frac{Z_{F0}}{2\sqrt{\epsilon_r}}\left\{\frac{W}{2h}+\frac{1}{\pi}\log 4+\frac{\epsilon_r+1}{2\pi\epsilon_r}\log\left[\frac{\pi e}{2}\left(\frac{W}{2h}+0.94\right)\right]\frac{\epsilon_r-1}{2\pi\epsilon_r^2}\log\frac{e\pi^2}{16}\right\}^{-1}$$

Narrow (W/h<3.3) strip

$$Z(W,h,\epsilon_r)=\frac{Z_{F0}}{\pi\sqrt{2(\epsilon_r+1)}}\left\{\log\left[\frac{4h}{W}+\sqrt{\left(\frac{4h}{W}\right)^2+2}\right]-\frac{1}{2}\frac{\epsilon_r-1}{\epsilon_r+1}\left(\log\frac{\pi}{2}+\frac{1}{\epsilon_r}\log\frac{4}{\pi}\right)\right\}$$

Waveguide



Electromagnetic field is confined into a simply-connected space.

⇓

TEM mode cannot exist.

Maxwell equations give

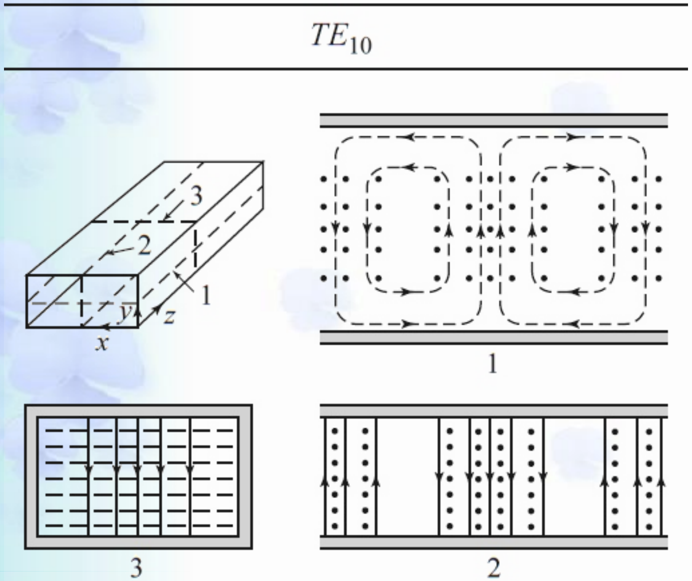
$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]E_z=-(\omega^2\epsilon\mu+\gamma^2)E_z,$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]H_z=-(\omega^2\epsilon\mu+\gamma^2)H_z.$$

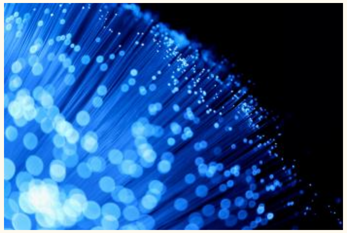
Helmholtz equation

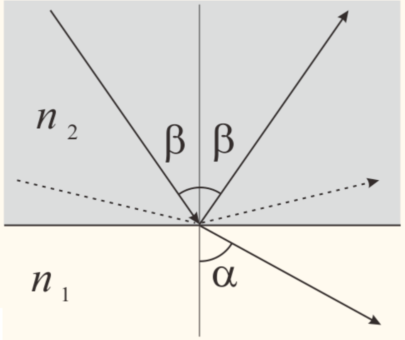
$E_z = 0$: TE mode,

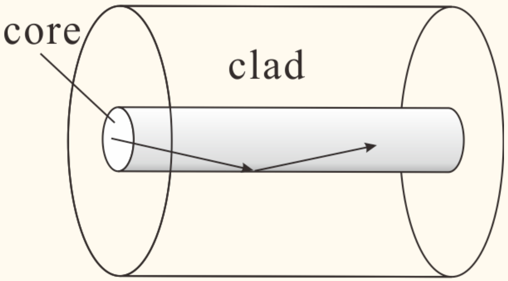
$H_z = 0$: TM mode



Optical fiber

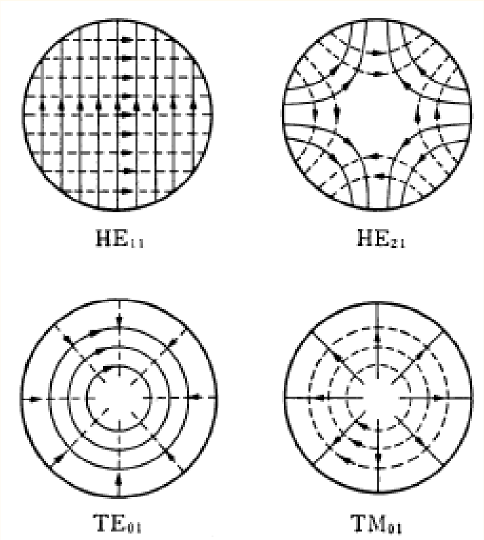


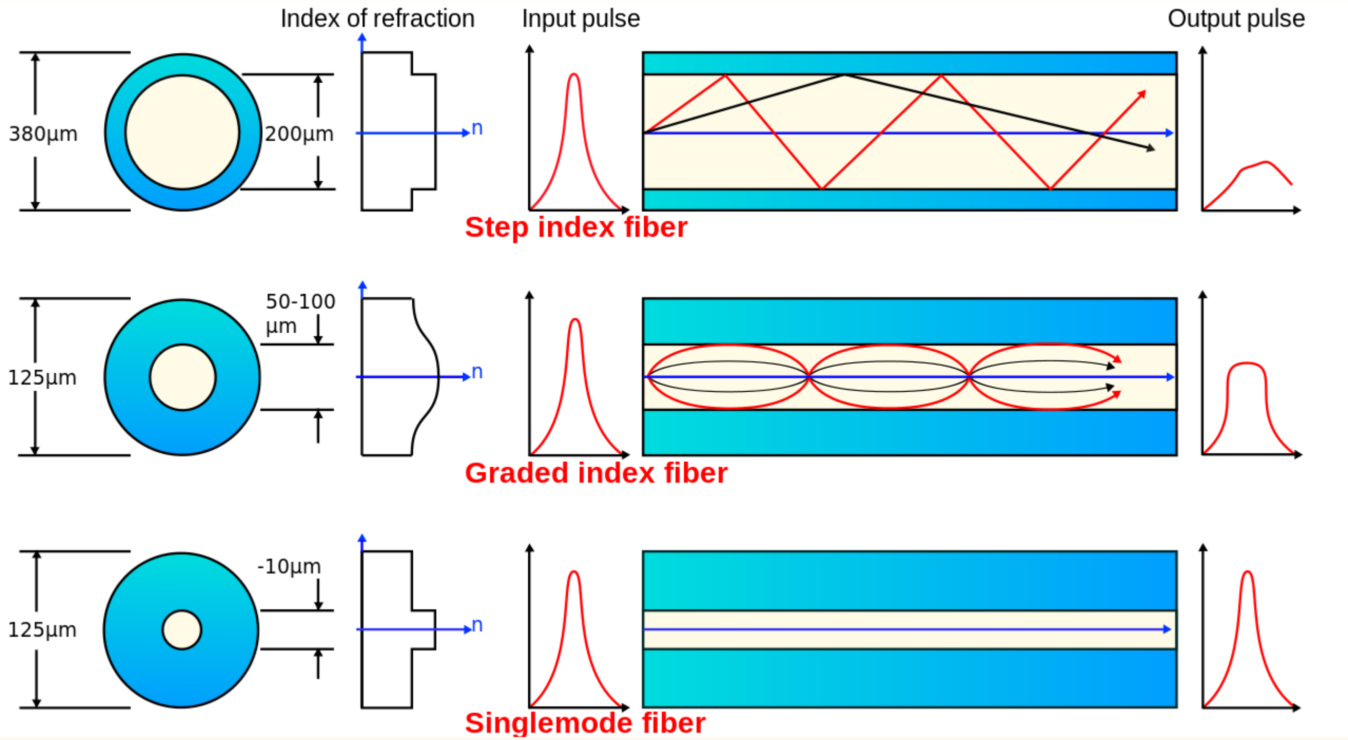




step-type optical fiber

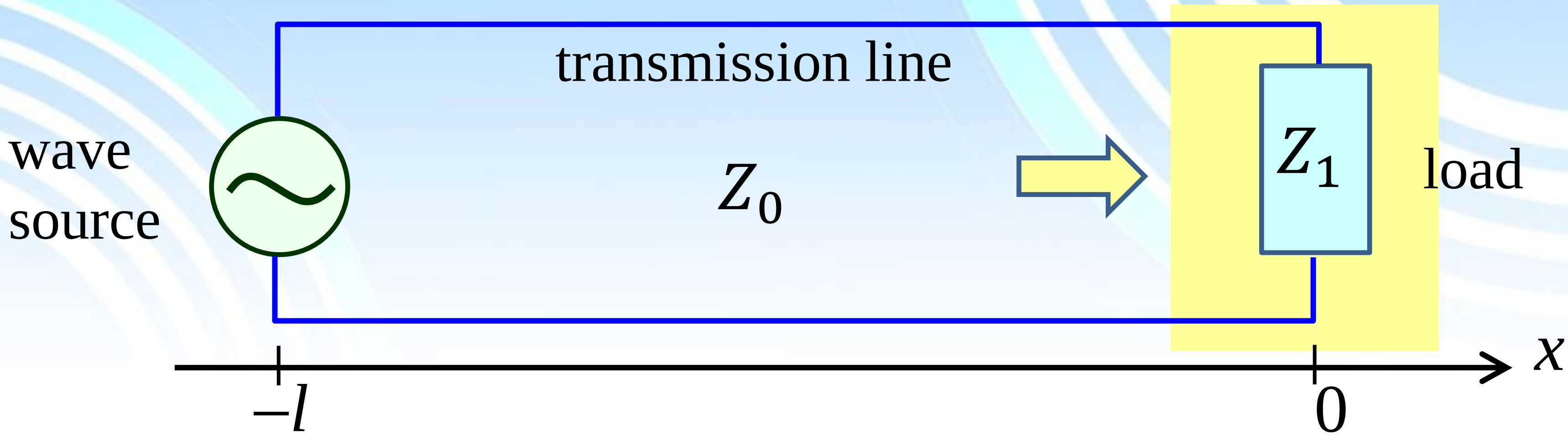
Difference in dielectric constant





no dispersion

Termination of transmission line



Termination of a transmission line with length l and characteristic impedance Z_0 at $x = 0$ with a resistor Z_1.

At $x = 0$:

$$J = J_+ + J_- \quad \text{(definition right positive)}$$

(J_+: progressive, J_-: retrograde)

$$V = V_+ + V_- = Z_0(J_+ - J_-)$$

Comment: Sign of Ohm's law in transmission lines reflects direction of waves (depends on the definitions).

Termination of transmission line

$$\pm 2Z_0 J_\pm = 2V_\pm = J \pm Z_0 V$$

synthesized impedance: $Z_1 = \dfrac{V}{J} = \dfrac{J_+ - J_-}{J_+ + J_-} Z_0$

reflection coefficient: $r = \dfrac{V_-}{V_+} = -\dfrac{J_-}{J_+} = \dfrac{Z_1 - Z_0}{Z_1 + Z_0}$

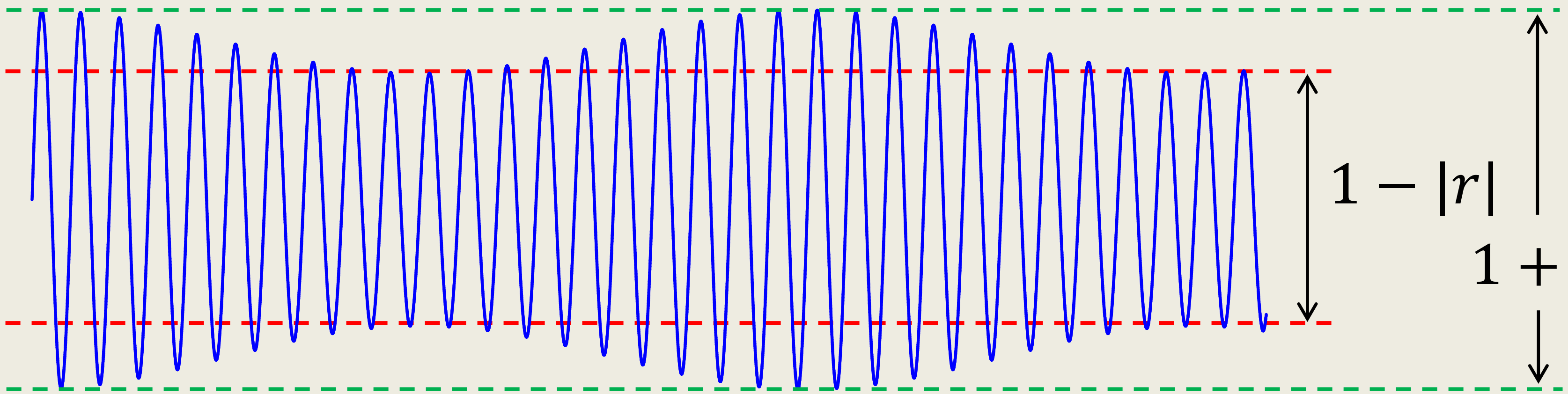
$Z_1 = Z_0$: no reflection, i.e., impedance matching

$Z_1 = +\infty$ (open circuit end) : $r = 1$, i.e., free end

$Z_1 = 0$ (short circuit end) : $r = -1$, i.e., fixed end

Connection and termination

Finite reflection → Standing wave



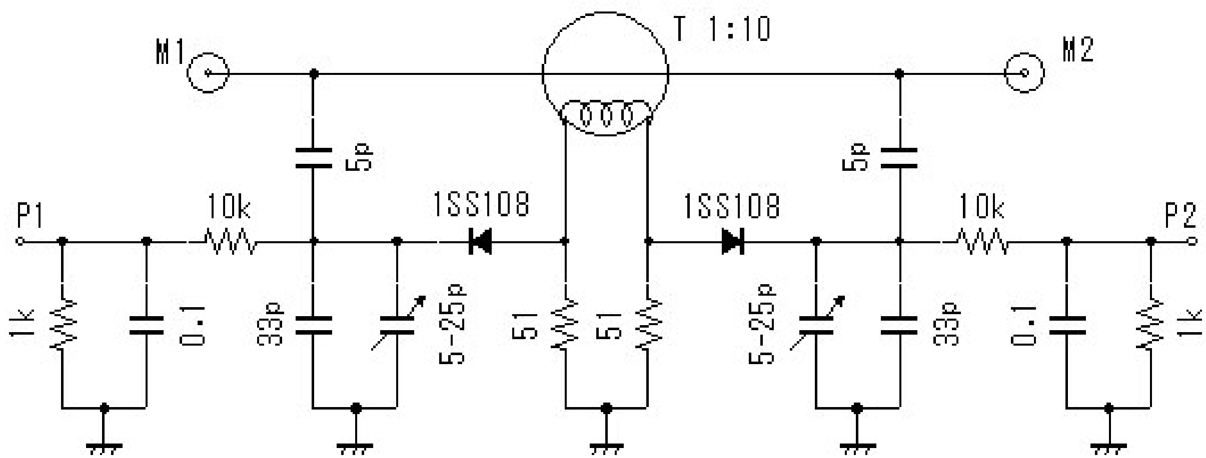
Voltage-Standing Wave Ratio (VSWR): $= \dfrac{1+|r|}{1-|r|}$

SWR Meters:

desktop types

cross-meter

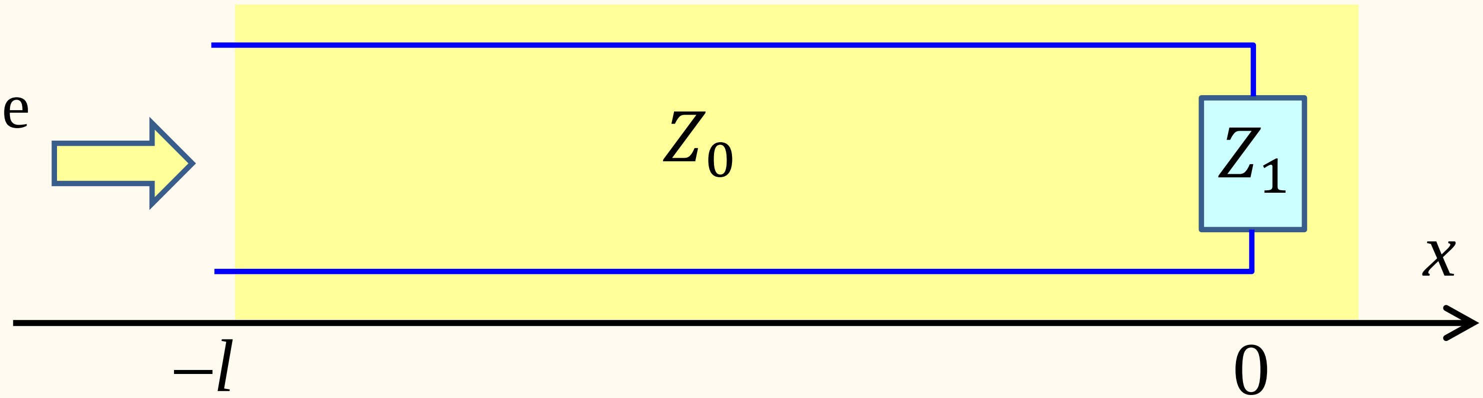
handy type



directional coupler

Synthesized impedance

total impedance from this side



At $x = -l$

$$\left.\begin{aligned} V &= V_{+0}\exp(\kappa l) + V_{-0}\exp(-\kappa l) = [J_{+0}\exp(\kappa l) - J_{-0}\exp(-\kappa l)]Z_0 \\ J &= J_{+0}\exp(\kappa l) + J_{-0}\exp(-\kappa l) \end{aligned}\right\}$$

$$Z_l = \frac{V}{J} = \frac{J_{+0}e^{\kappa l} - J_{-0}e^{-\kappa l}}{J_{+0}e^{\kappa l} + J_{-0}e^{-\kappa l}} Z_0$$

Reflection coefficient: $r_l = \dfrac{V_-}{V_+} = \dfrac{V_{-0}e^{-\kappa l}}{V_{+0}e^{\kappa l}} = r\exp(-2\kappa l)$

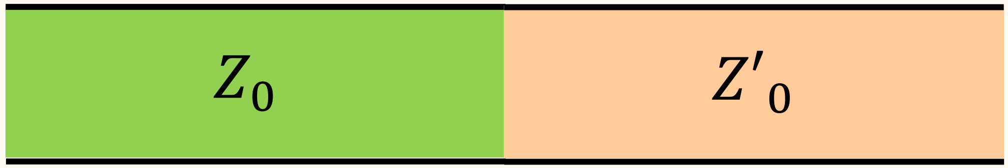
Connection and termination

Transmission line connection.
Characteristic impedance Z_0, ${Z_0}'$

At the connection point, only the local relation between V and J affects the reflection coefficient.

The local impedance from the left hand side is ${Z_0}'$.

$$r = \frac{Z_0' - Z_0}{Z_0' + Z_0}$$



5.2.3 Smith chart, Immittance chart

End impedance Z_1: Normalized end impedance $Z_n \equiv Z_1/Z_0$

$$Z_n = x + iy, \quad r = u + iw \quad (x, y, u, w \in \mathbb{R})$$

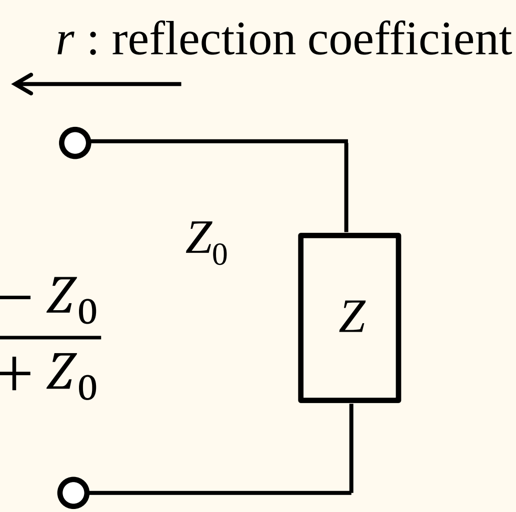
$$u + iw = r = \frac{Z_n - 1}{Z_n + 1} = \frac{(x-1) + iy}{(x+1) + iy}$$

$$\left.\begin{aligned} \text{real:} \quad & x - 1 = (x+1)u - yw \\ \text{imaginary:} \quad & y = yu + w(x+1) \end{aligned}\right\}$$

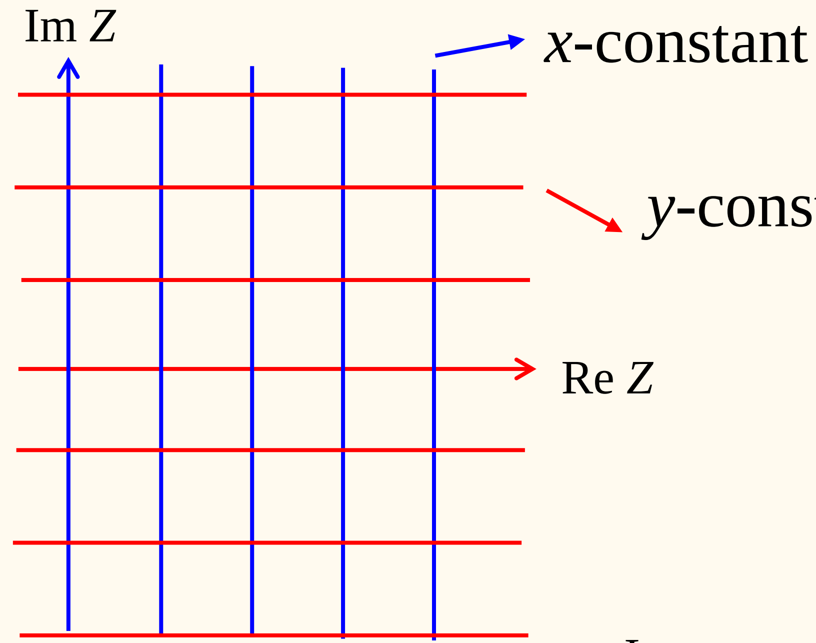
x: constant → $$\left(u - \frac{x}{x+1}\right)^2 + w^2 = \frac{1}{(x+1)^2}$$ constant resistance circle

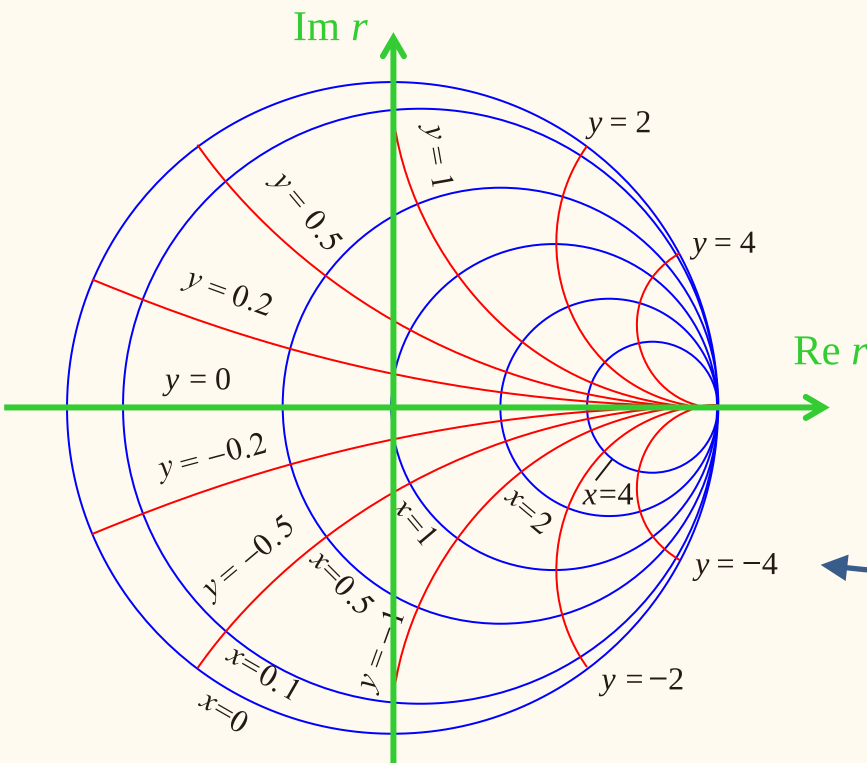
y: constant → $$(u-1)^2 + \left(w - \frac{1}{y}\right)^2 = \frac{1}{y^2}$$ constant reactance circle

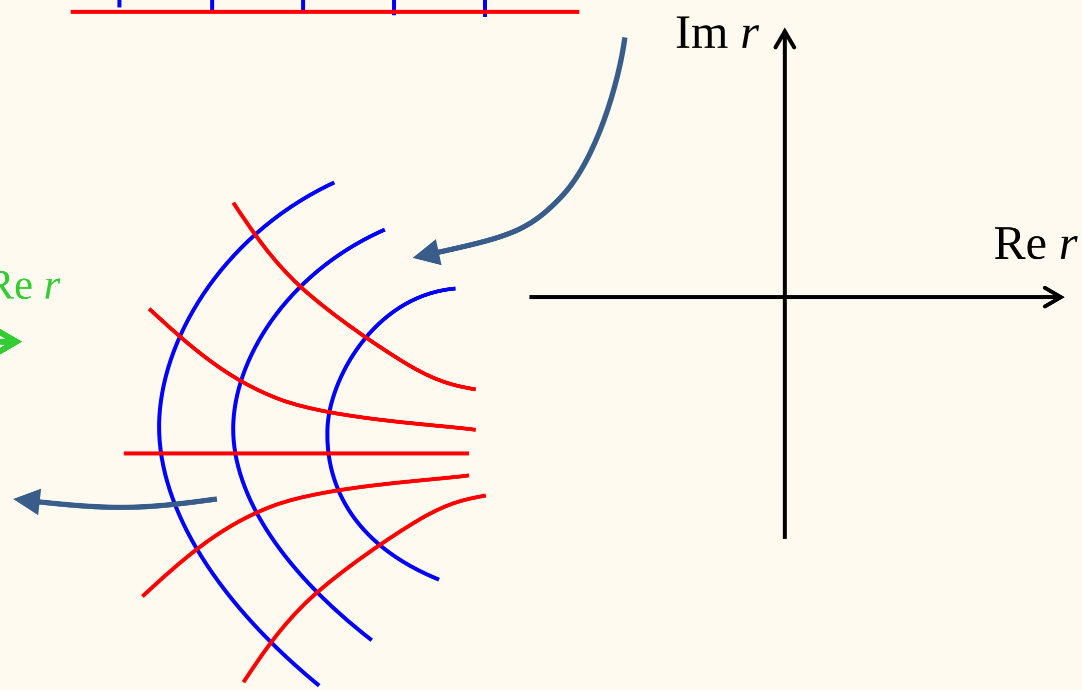
5.2.3 Smith chart, Immittance chart



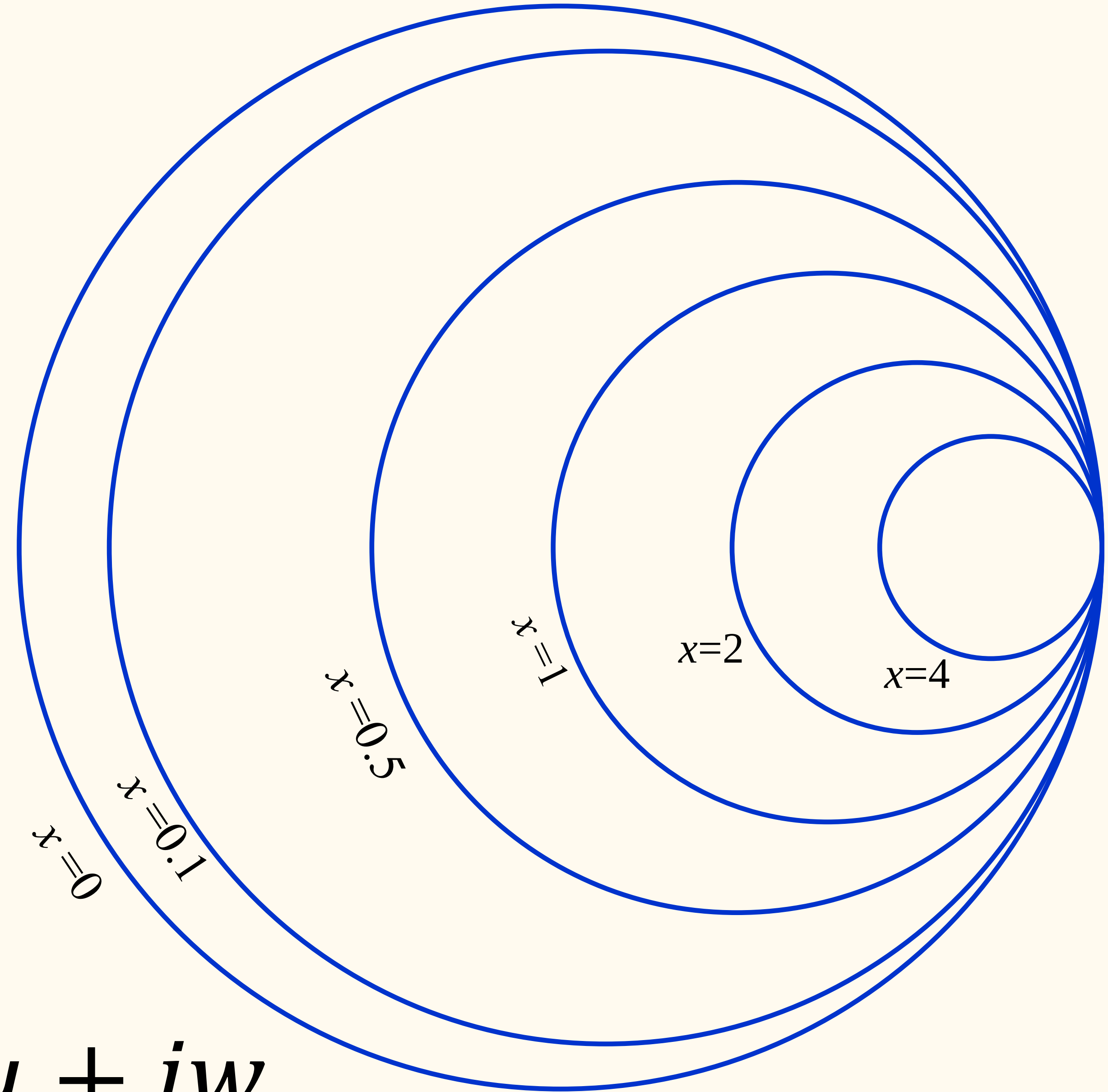
$$r = \frac{V_-}{V_+} = \frac{Z - Z_0}{Z + Z_0}$$





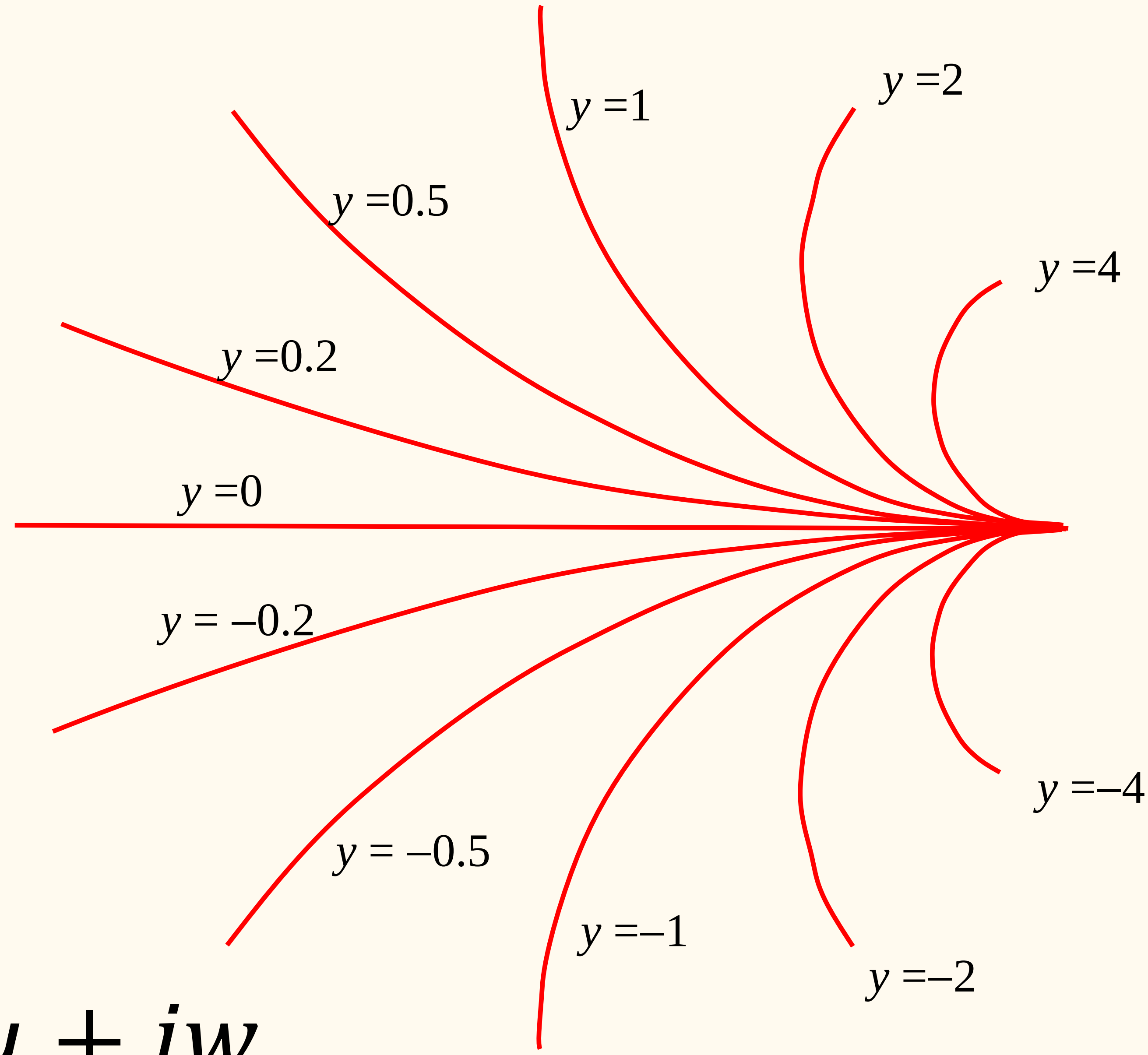


5.2.3 Smith chart, Immittance chart



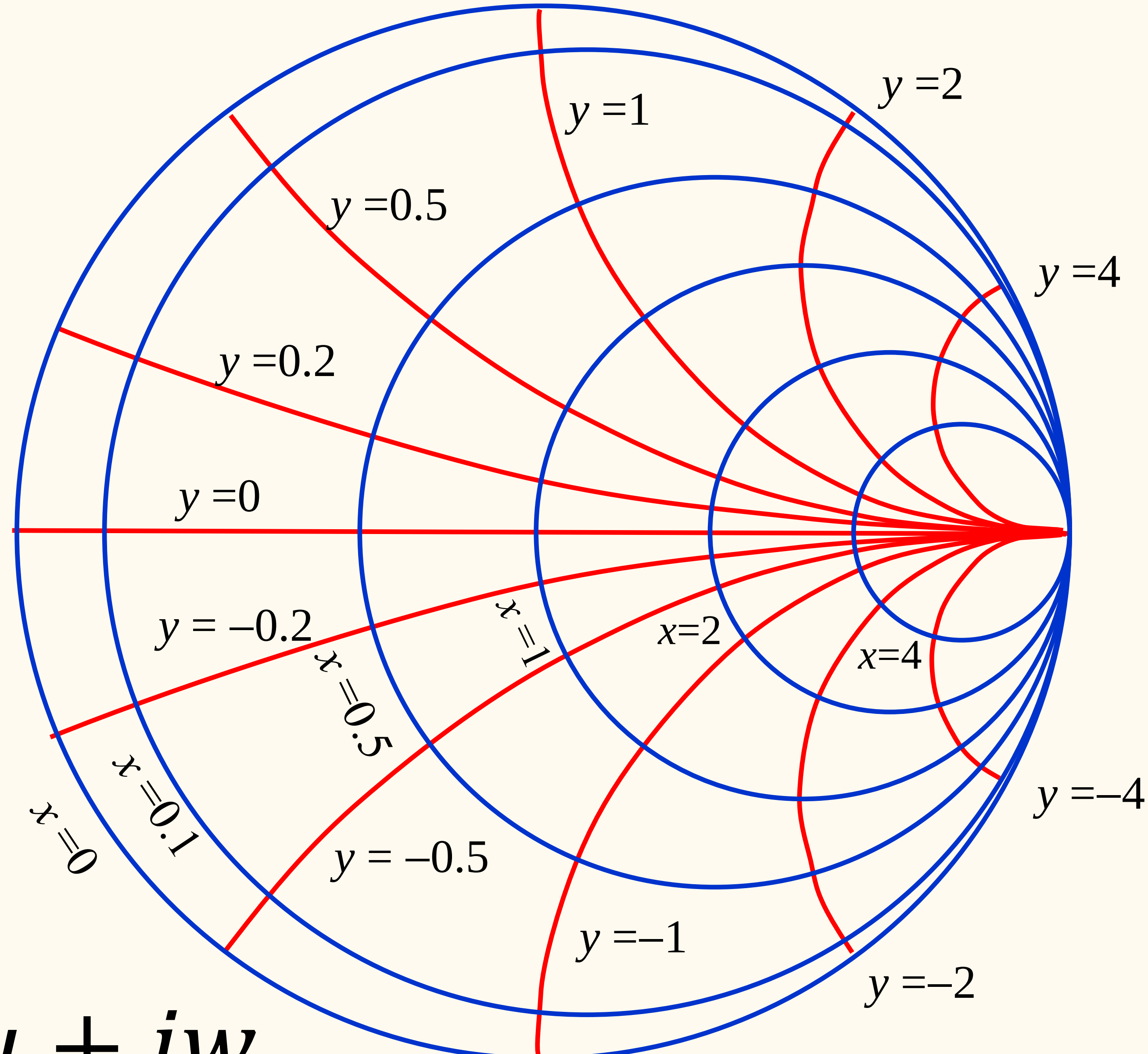
$$r = u + iw$$

5.2.3 Smith chart, Immittance chart



$$r = u + iw$$

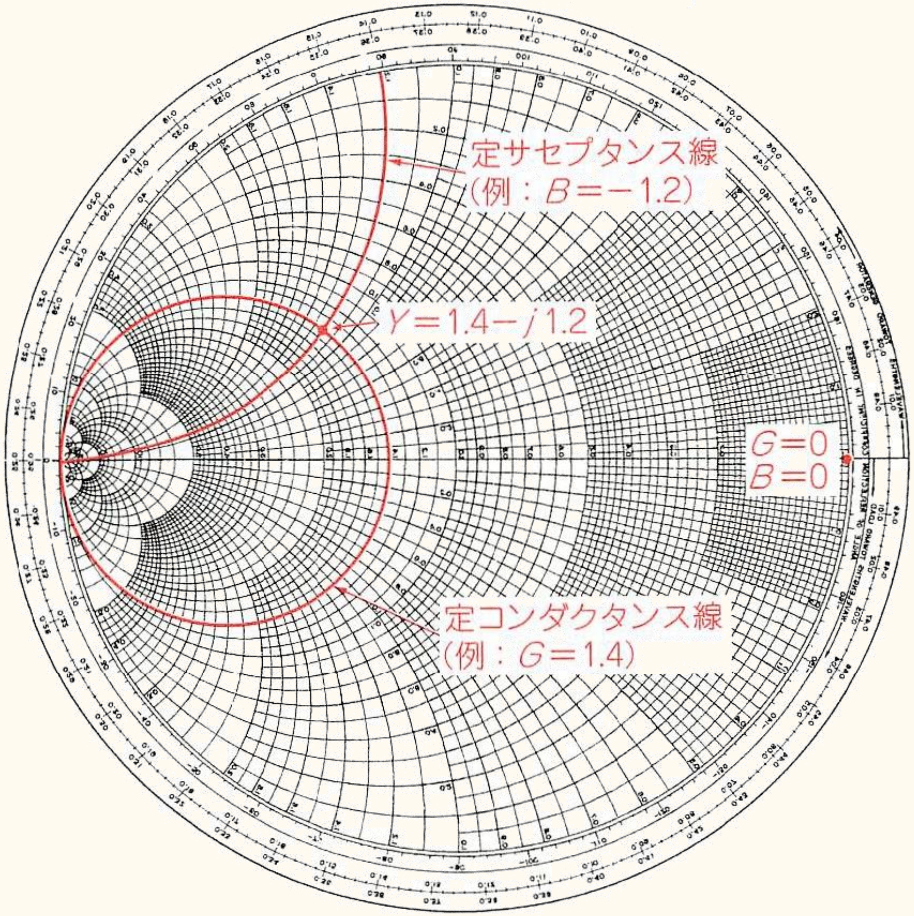
5.2.3 Smith chart, Immittance chart



$$r = u + iw$$

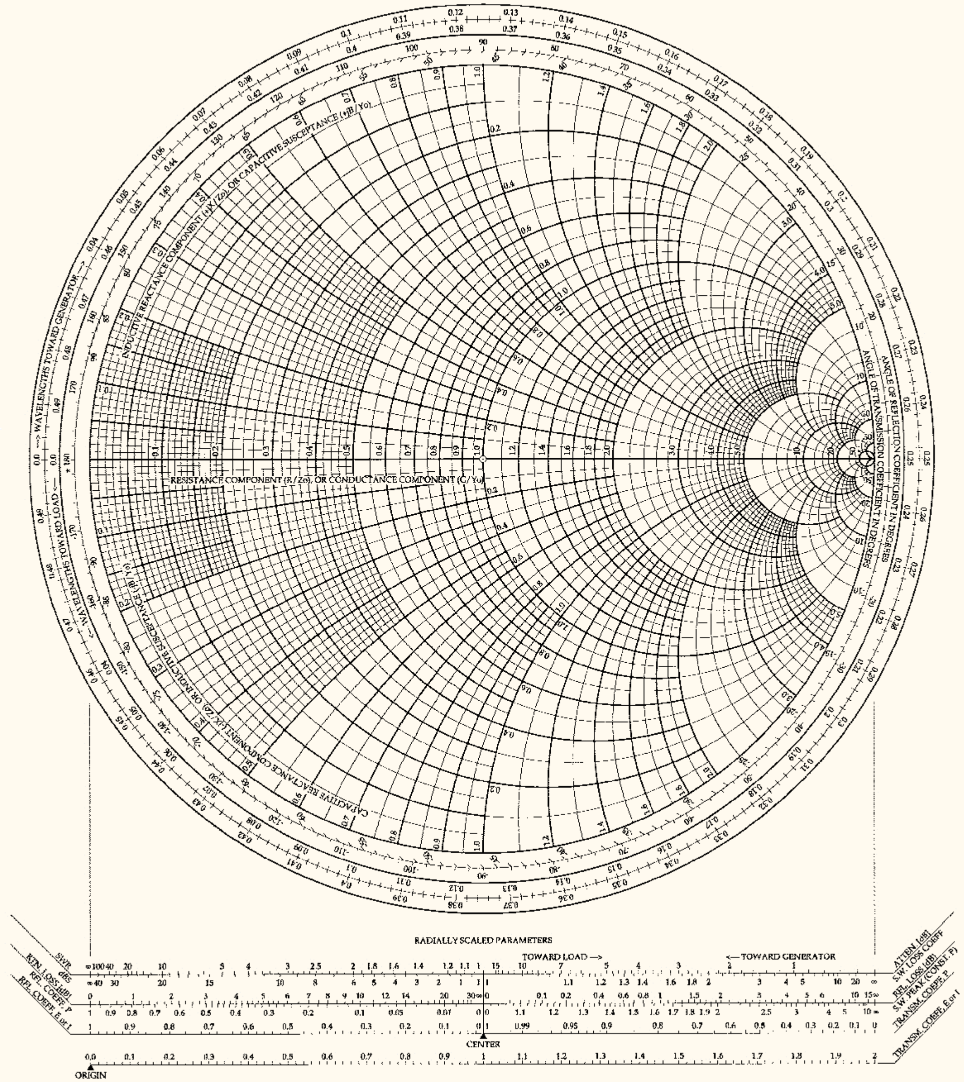
Smith chart

5.2.3 Smith chart, Immittance chart



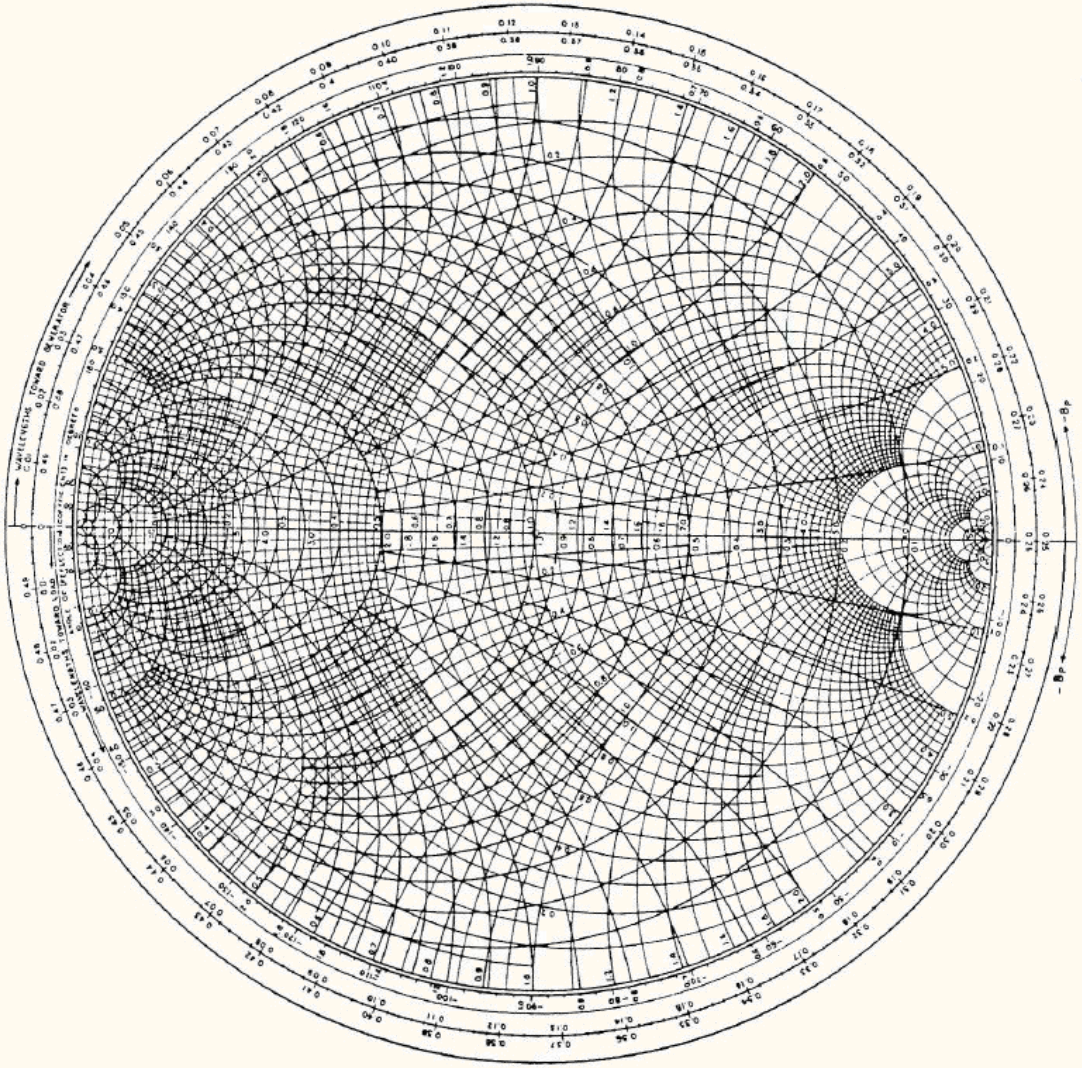
Admittance chart

$$r = u + iw$$



Smith chart

5.2.3 Smith chart, Immittance chart



Immittance chart

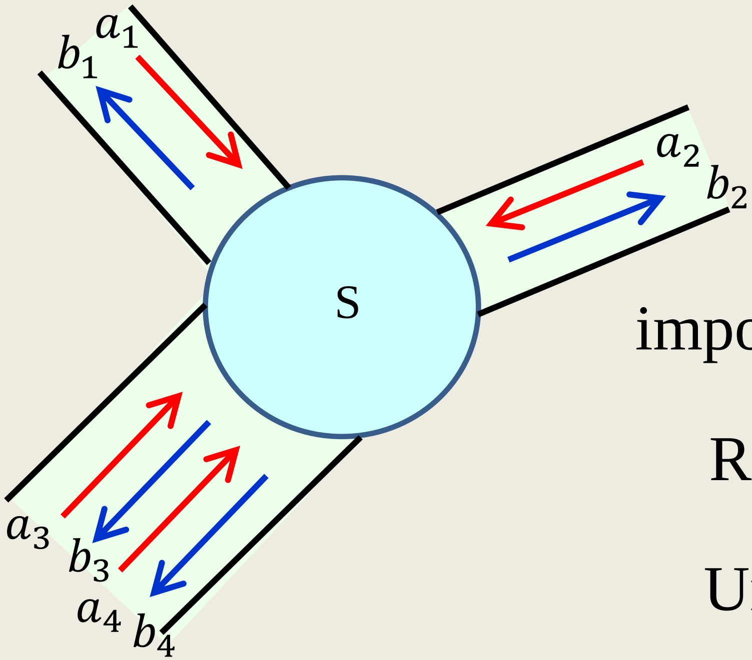
$$r = u + iw$$

5.3 Scattering (S) matrix (S parameters)

How to treat multipoint (crossing point) systematically?

Transmission lines: wave propagating modes → Channels

Take $|a_i|^2$, $|b_i|^2$ to be powers (energy flow).



output S-matrix input

$$\begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & & & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix}$$

$$\boldsymbol{b} = \mathrm{S}\boldsymbol{a}$$

important properties:

Reciprocity $$S_{ij} = S_{ji}$$

Unitarity $$\sum_j S_{ji} S_{jk}^* = \delta_{ik}$$

(In case, no dissipation, no amplification)

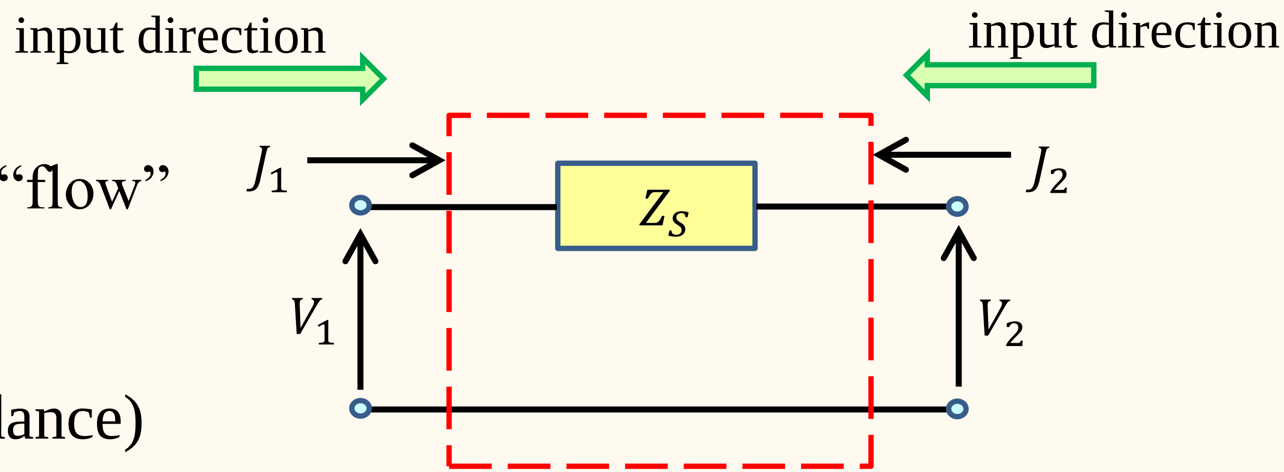
5.3 S matrix (S parameters)

Propagation with no dissipation

$$\begin{cases} a_n = \dfrac{V_{n+}}{\sqrt{Z_{0n}}} = J_{n+}\sqrt{Z_{0n}}, & \text{incident power wave} \\ b_n = \dfrac{V_{n-}}{\sqrt{Z_{0n}}} = J_{n-}\sqrt{Z_{0n}} & \text{reflected (transmitted) power wave} \end{cases}$$

$$|a_n|^2 = \frac{|V_{n+}|^2}{Z_{0n}} = |J_{n+}|^2 Z_{0n}$$

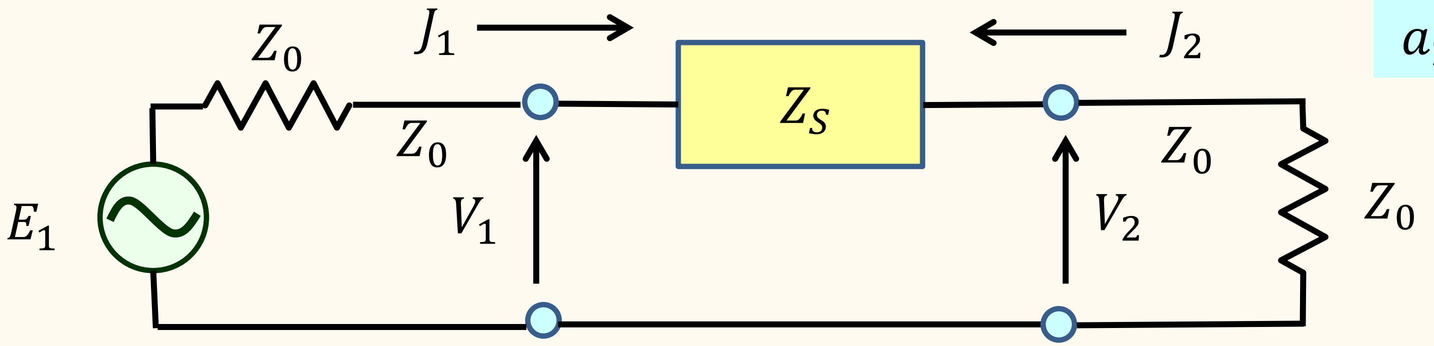
Simplest example: series impedance Z_S



Take voltage as the "flow" quantity.
(assume common characteristic impedance)

5.3 S-matrix (S-parameters)

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad \begin{pmatrix} V_{1-} \\ V_{2-} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_{1+} \\ V_{2+} \end{pmatrix}$$



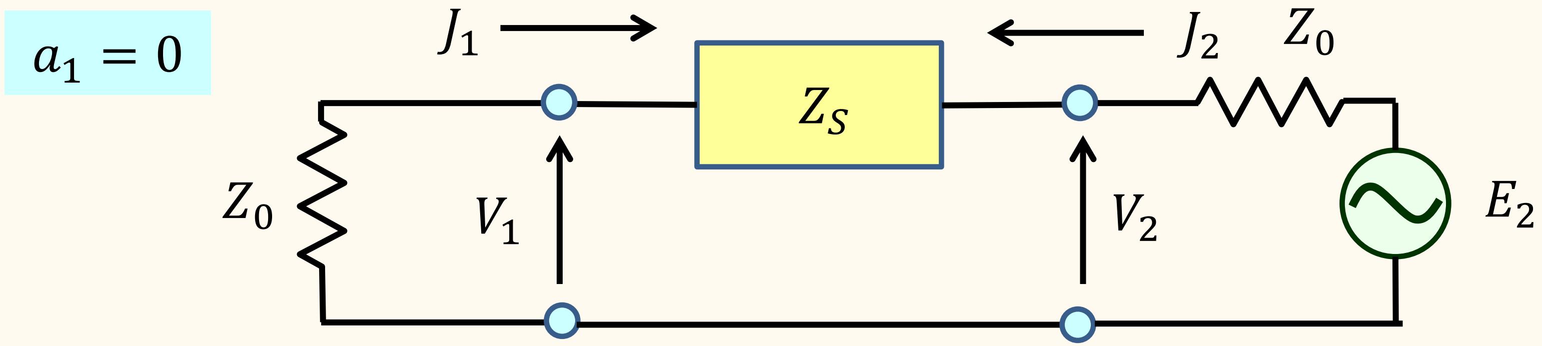
$a_2 = 0$

Terminate 2 with $Z_0 \to a_2 = 0$

$$S_{11} = \frac{V_{1-}}{V_{1+}} = \frac{V_1 - Z_0 J_1}{V_1 + Z_0 J_1} = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{(Z_S + Z_0) - Z_0}{(Z_S + Z_0) + Z_0} = \frac{Z_S}{Z_S + 2Z_0}$$

$$S_{21} = \frac{V_{2-}}{V_{1+}} = \frac{V_2 - Z_0 J_2}{V_1 + Z_0 J_1} = \frac{Z_0 J_1 + Z_0 J_1}{(Z_S + Z_0) J_1 + Z_0 J_1} = \frac{2Z_0}{Z_S + 2Z_0} \; (J_2 = -J_1)$$

5.3 S-matrix (S-parameters)



Terminate 1 with $Z_0 \to a_1 = 0$ (should be symmetric)

$$S_{12} = \frac{2V_1}{V_2 + Z_0 J_2} = \frac{2Z_0 J_2}{(Z_S + Z_0)J_2 + Z_0 J_2} = \frac{2Z_0}{Z_S + 2Z_0}$$

$$S_{22} = \frac{Z_S}{Z_S + 2Z_0}$$

Generally

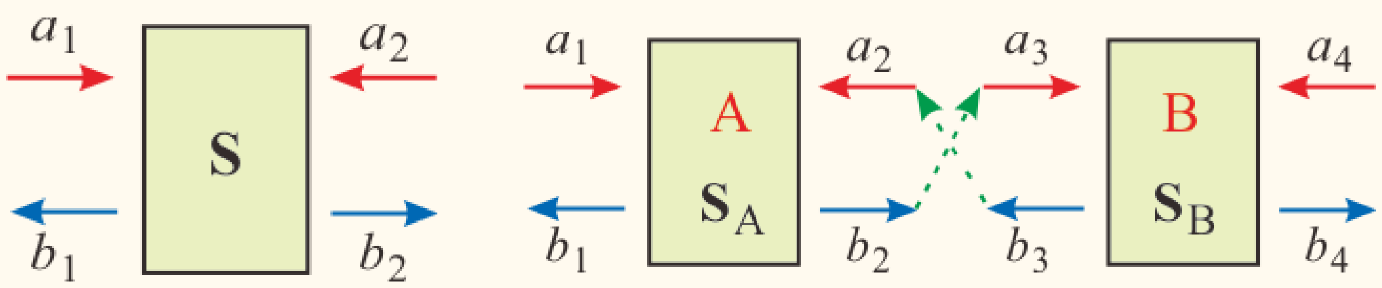
$$\mathrm{S} = \frac{1}{\det Z} \times \begin{pmatrix} (Z_{11} - Z_0)(Z_{22} + Z_0) - Z_{12}Z_{21} & 2Z_0 Z_{12} \\ 2Z_0 Z_{21} & (Z_{11} + Z_0)(Z_{22} - Z_0) - Z_{12}Z_{21} \end{pmatrix}$$

Cascade connection of S-matrices

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \mathbf{S} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_\mathrm{L} & t_\mathrm{R} \\ t_\mathrm{L} & r_\mathrm{R} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$r_{L,R}$, $t_{L,R}$: complex reflection, transmission coefficients satisfying

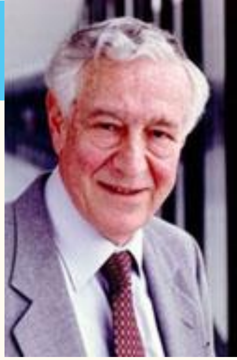
$$T_\mathrm{L,R} = |t_\mathrm{L,r}|^2 = 1 - R_\mathrm{L,R} = 1 - |r_\mathrm{L,R}|^2$$



$$\mathbf{S}_\mathrm{AB} = \begin{pmatrix} r_\mathrm{L}^\mathrm{AB} & t_\mathrm{R}^\mathrm{AB} \\ t_\mathrm{L}^\mathrm{AB} & r_\mathrm{R}^\mathrm{AB} \end{pmatrix} = \begin{pmatrix} r_\mathrm{L}^\mathrm{A} + t_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} t_\mathrm{L}^\mathrm{A} & t_\mathrm{R}^\mathrm{A} \left(I - r_\mathrm{L}^\mathrm{B} r_\mathrm{R}^\mathrm{A}\right)^{-1} t_\mathrm{R}^\mathrm{B} \\ t_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} t_\mathrm{L}^\mathrm{A} & r_\mathrm{R}^\mathrm{B} + t_\mathrm{L}^\mathrm{B} \left(I - r_\mathrm{R}^\mathrm{A} r_\mathrm{L}^\mathrm{B}\right)^{-1} r_\mathrm{R}^\mathrm{A} t_\mathrm{R}^\mathrm{B} \end{pmatrix}$$

$$(I - r_R^A r_L^B)^{-1} = I + r_R^A r_L^B + (r_R^A r_L^B)^2 + \cdots$$

Conduction channels in quantum transport



Rolf Landauer

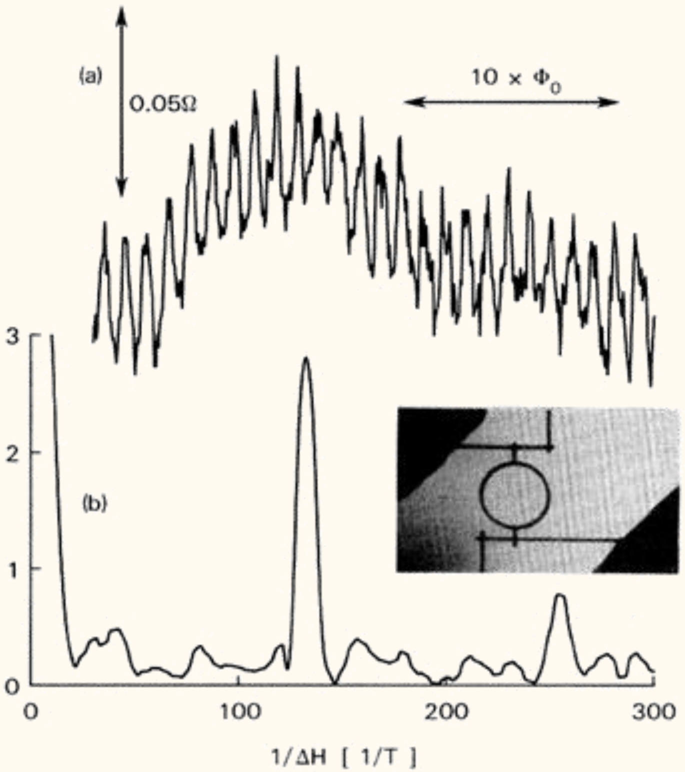
Electron (quantum mechanical) waves also have propagating modes in solids.

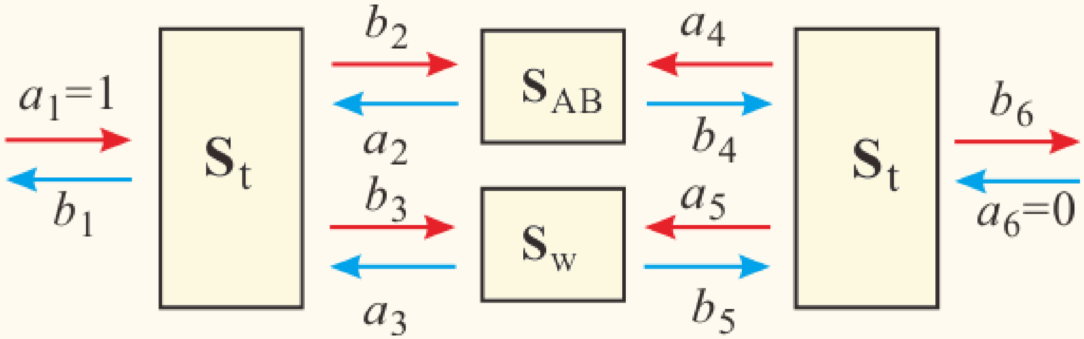
→ Conduction channel

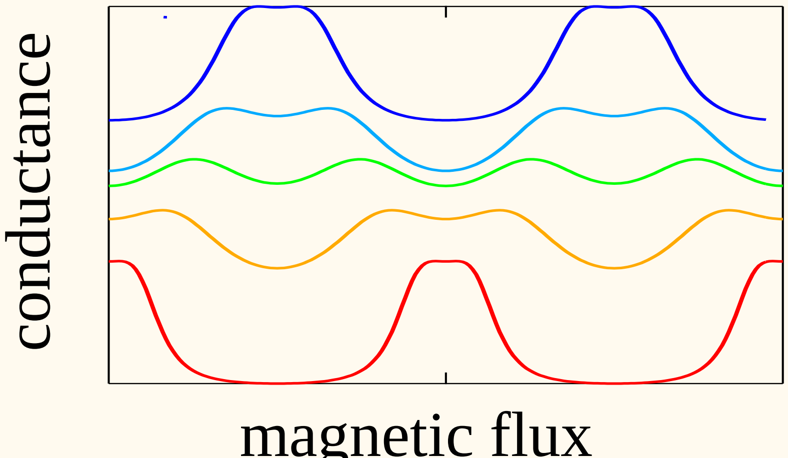
Landauer eq.:

the conductance of a single perfect quantum channel is $\dfrac{e^2}{h}$

AB ring S-matrix model







S-parameter representation of high-frequency devices

DATA SHEET

NEC

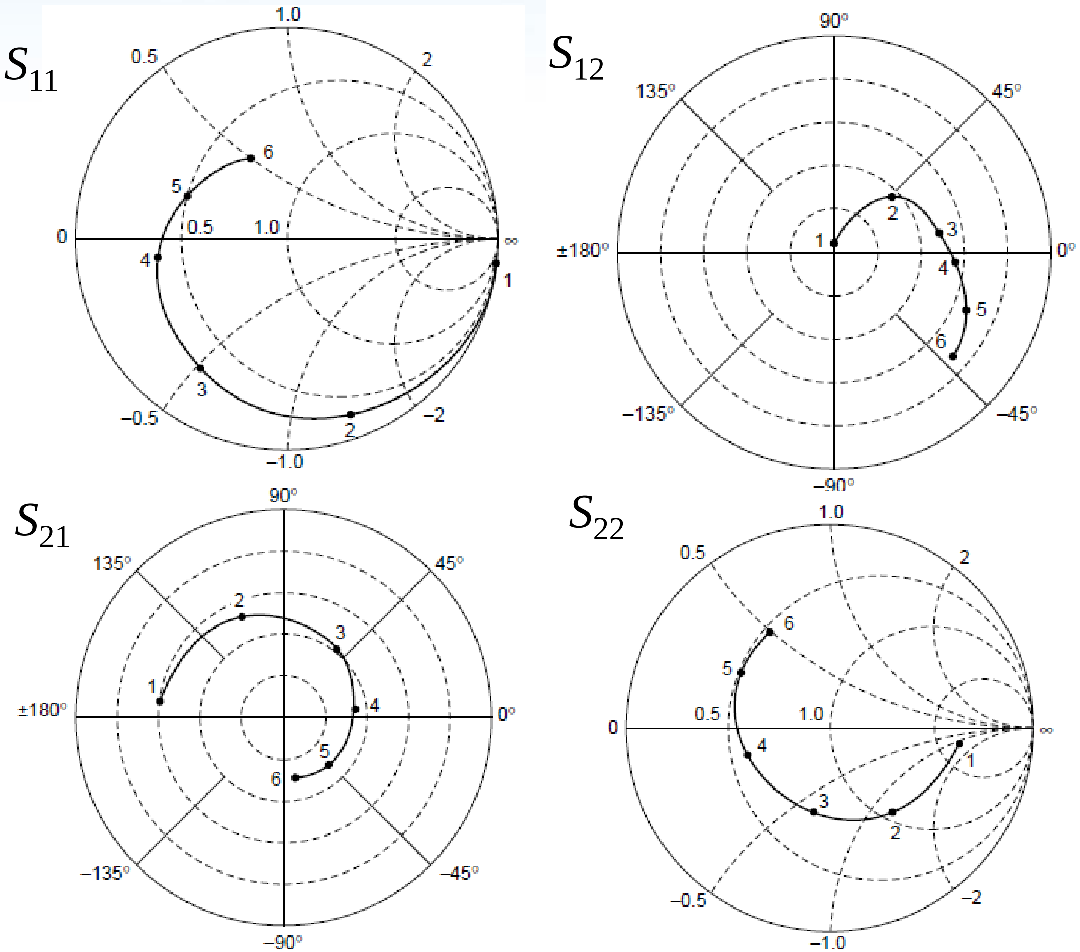
GaAs MES FET

NE76084

C to Ku BAND LOW NOISE AMPLIFIER
N-CHANNEL GaAs MES FET

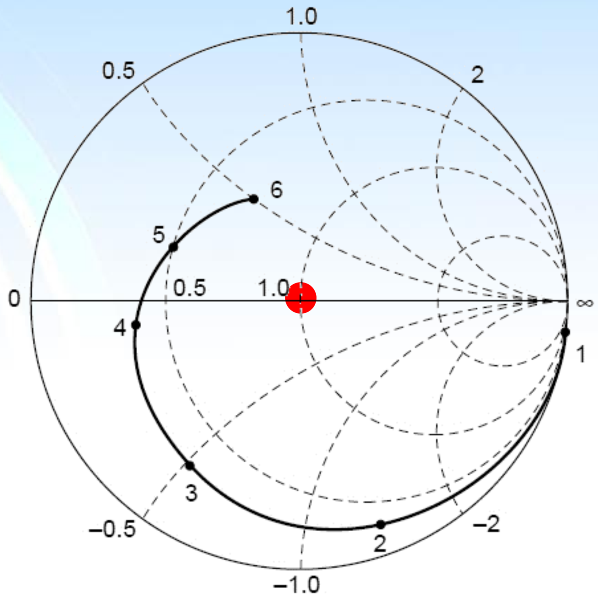
S-PARAMETERS

V_{DS} = 3 V, I_D = 10 mA
START 500 MHz, STOP 18 GHz, STEP 500 MHz



S-parameter representation of high-frequency devices

Ex) NE76084 MES FET 0.5～18GHz



S_{11}

The datasheet tells that we need impedance matching circuits with transmission lines.

If we know Z-parameters:

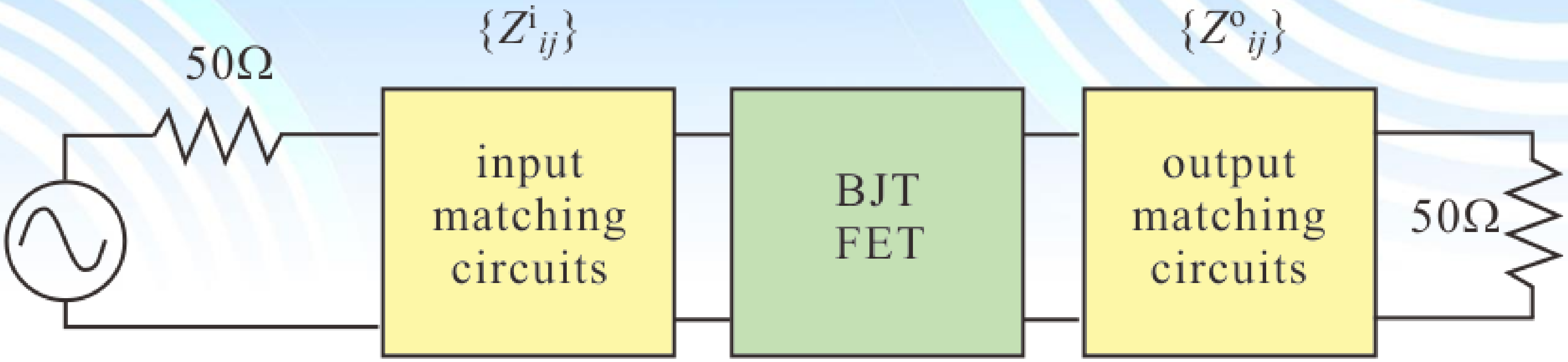
From Ho-Thevenin theorem

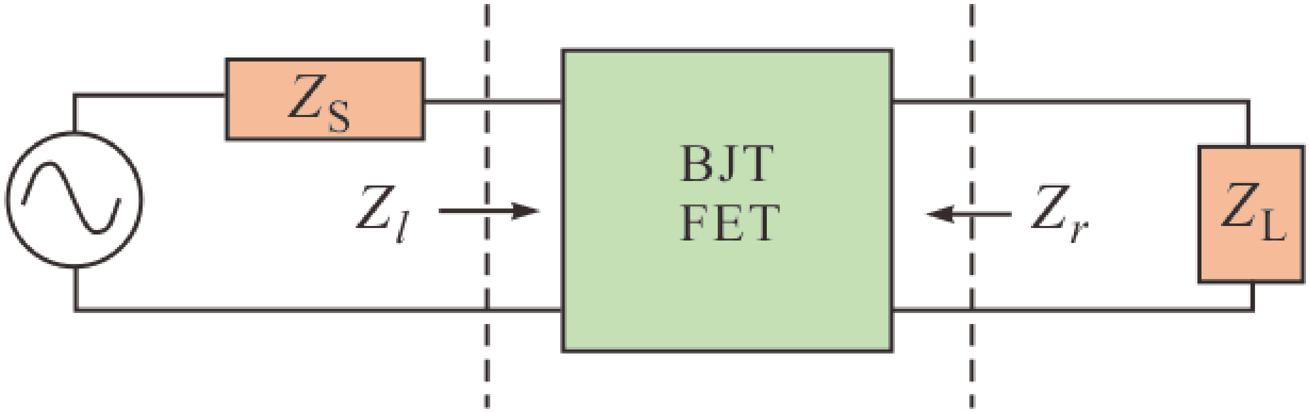
$$Z_{\mathrm{S}} = Z^{\mathrm{i}}_{22} - \frac{Z^{\mathrm{i}}_{12} Z^{\mathrm{i}}_{21}}{50 + Z^{\mathrm{i}}_{11}}, \quad Z_{\mathrm{L}} = Z^{\mathrm{o}}_{11} - \frac{Z^{\mathrm{o}}_{12} Z^{\mathrm{o}}_{21}}{50 + Z^{\mathrm{o}}_{22}}$$

$\{Z_{ij}\}$: BJT (FET) Z-parameters, again Ho-Thevenin says

$$Z_l = Z_{11} - \frac{Z_{12} Z_{21}}{Z_{\mathrm{L}} + Z_{22}}, \quad Z_r = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{\mathrm{S}} + Z_{11}}$$

S-parameter representation of high-frequency devices





Impedance matching with S-parameters

Generally the unitarity does not hold for amplification.

$$R_{\rm in} = S_{11} + \frac{S_{12}S_{21}R_{\rm L}}{1 - S_{22}R_{\rm L}} \qquad R_{\rm out} = S_{22} + \frac{S_{12}S_{21}R_{\rm S}}{1 - S_{11}R_{\rm S}}$$

Matching condition: $R_{\rm L} = R_{\rm out}^*, \quad R_{\rm S} = R_{\rm in}^*$

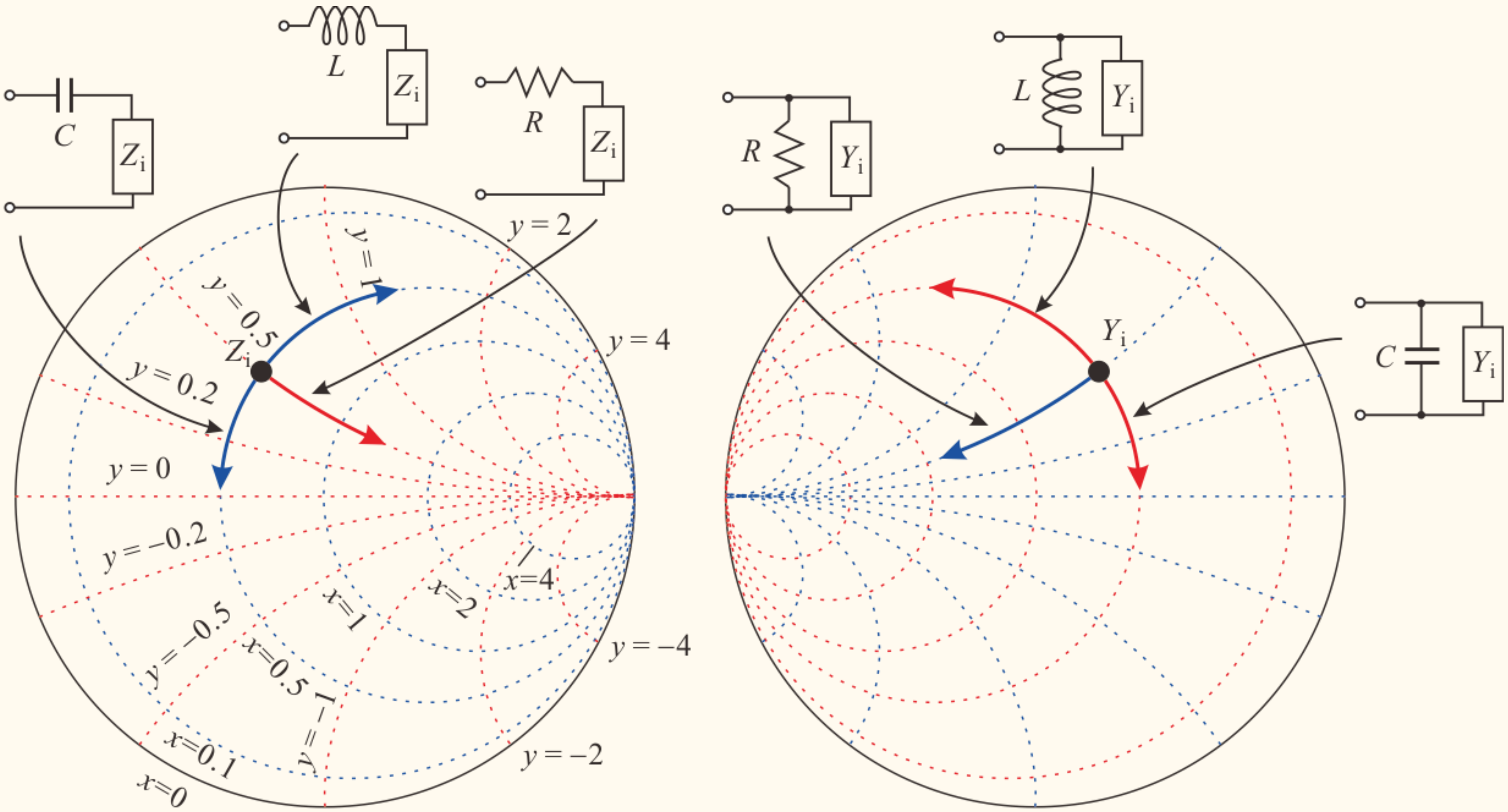
Solution $$R_{\rm S} = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad R_{\rm L} = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$ with

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |{\rm det}S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |{\rm det}S|^2,$$
$$N = S_{22} - S_{11}^*{\rm det}S, \quad M = S_{11} - S_{22}^*{\rm det}S$$

maximum available power gain $$G_{\rm max} = \left|\frac{S_{21}}{S_{12}}\right|(K - \sqrt{K^2 - 1})$$

$$K = \frac{1 + |{\rm det}S|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|}$$ stability factor

Practical impedance matching with Simth chart

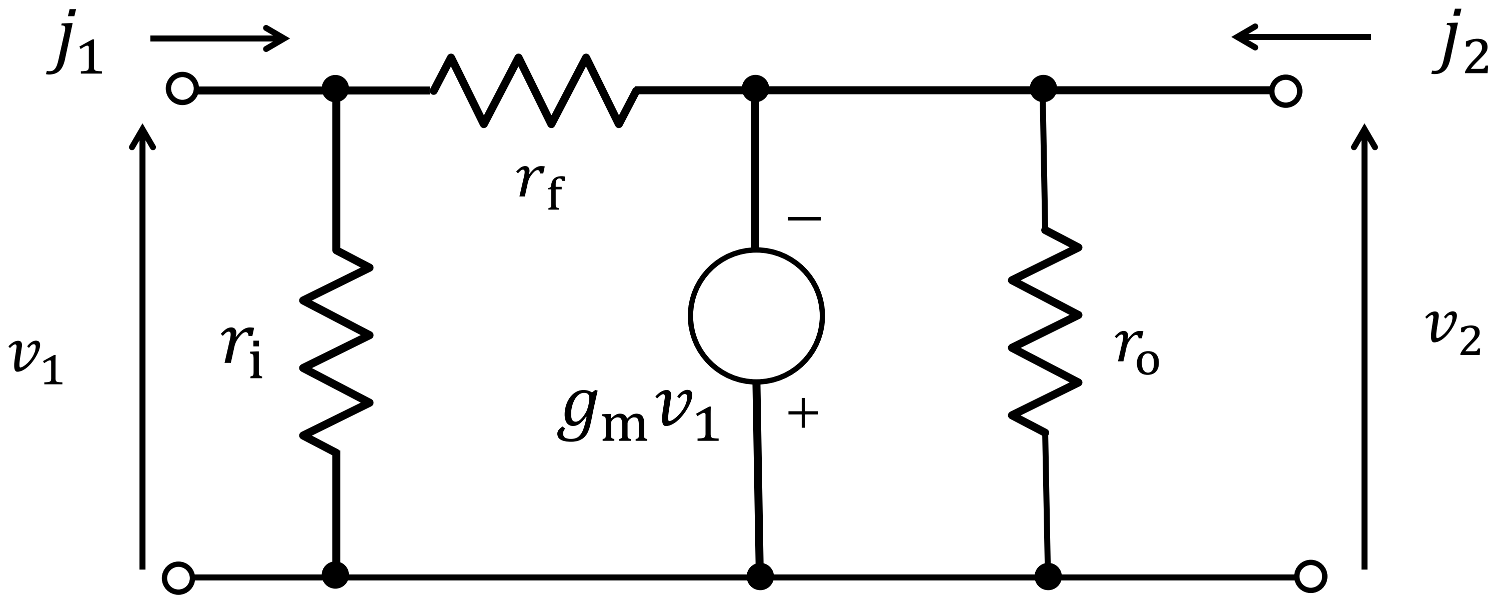


Impedance matching designer

http://home.sandiego.edu/~ekim/e194rfs01/jwmatcher/matcher2.html

http://leleivre.com/rf_lcmatch.html

Exercise D-1



Obtain the Y matrix for the above equivalent circuit (π-shape circuit).

Exercise D-2

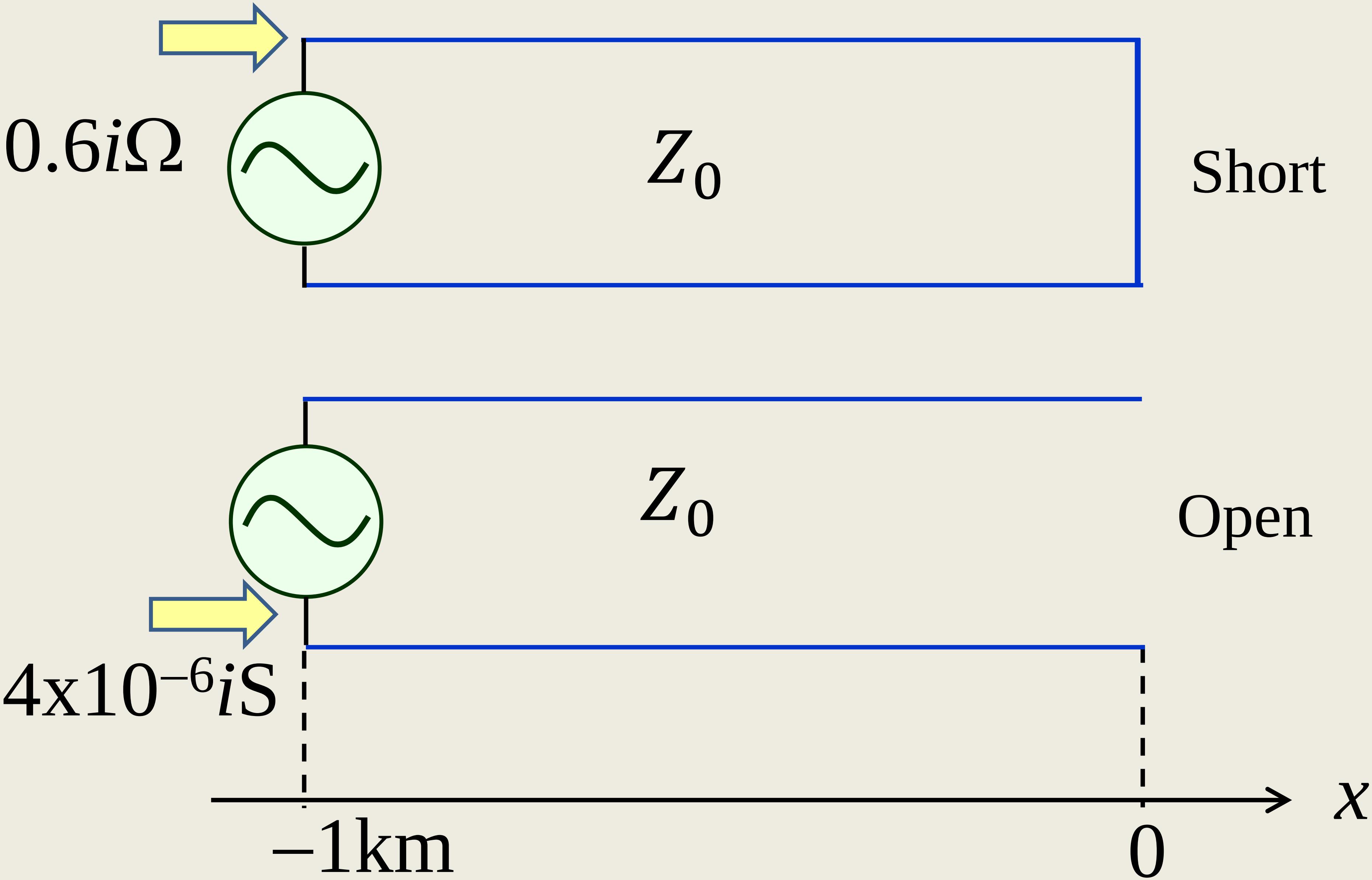
l =1km の伝送線路がある．終端側を短絡したところ，電源側から測定したインピダンスは0.6i Ωであった．一方，終端側を開放して電源側からアドミタンスを測定すると4x10^{-6}i Sであった．

この伝送線路の特性インピダンスを求めよ．

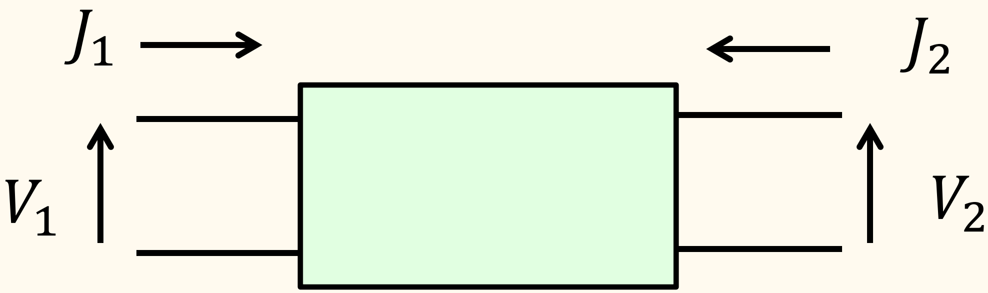
Consider a transmission line with the length l = 1km. First we short-circuited the end and measured the impedance from the signal source and obtained 0.6i Ω. Next we opened the end and measured the admittance from the signal source and obtained 4x10^{-6}i S.

What is the characteristic impedance of the transmission line?

Exercise D-2



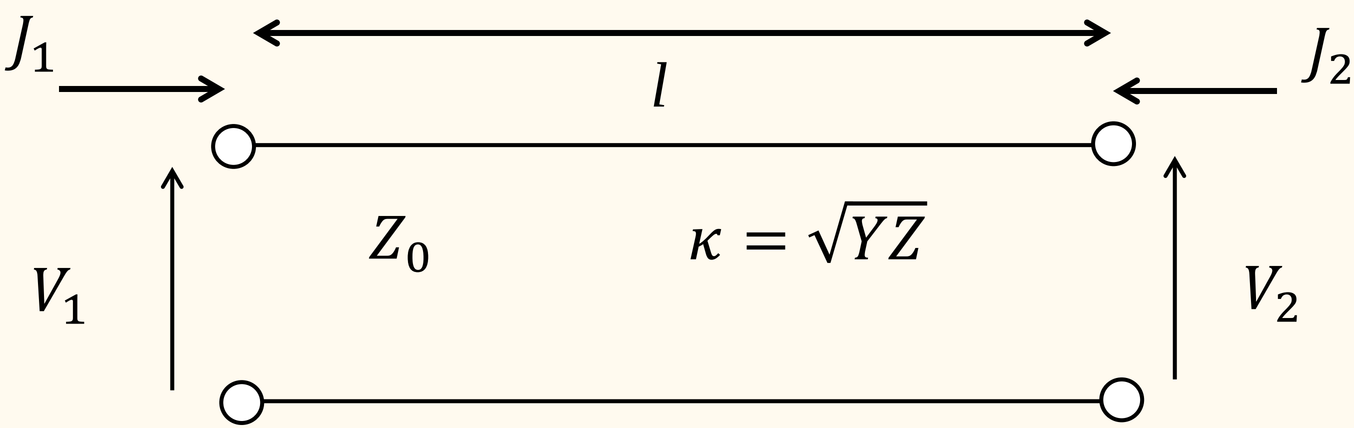
Exercise D-3



$$\begin{pmatrix} V_1 \\ J_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ J_2 \end{pmatrix}$$

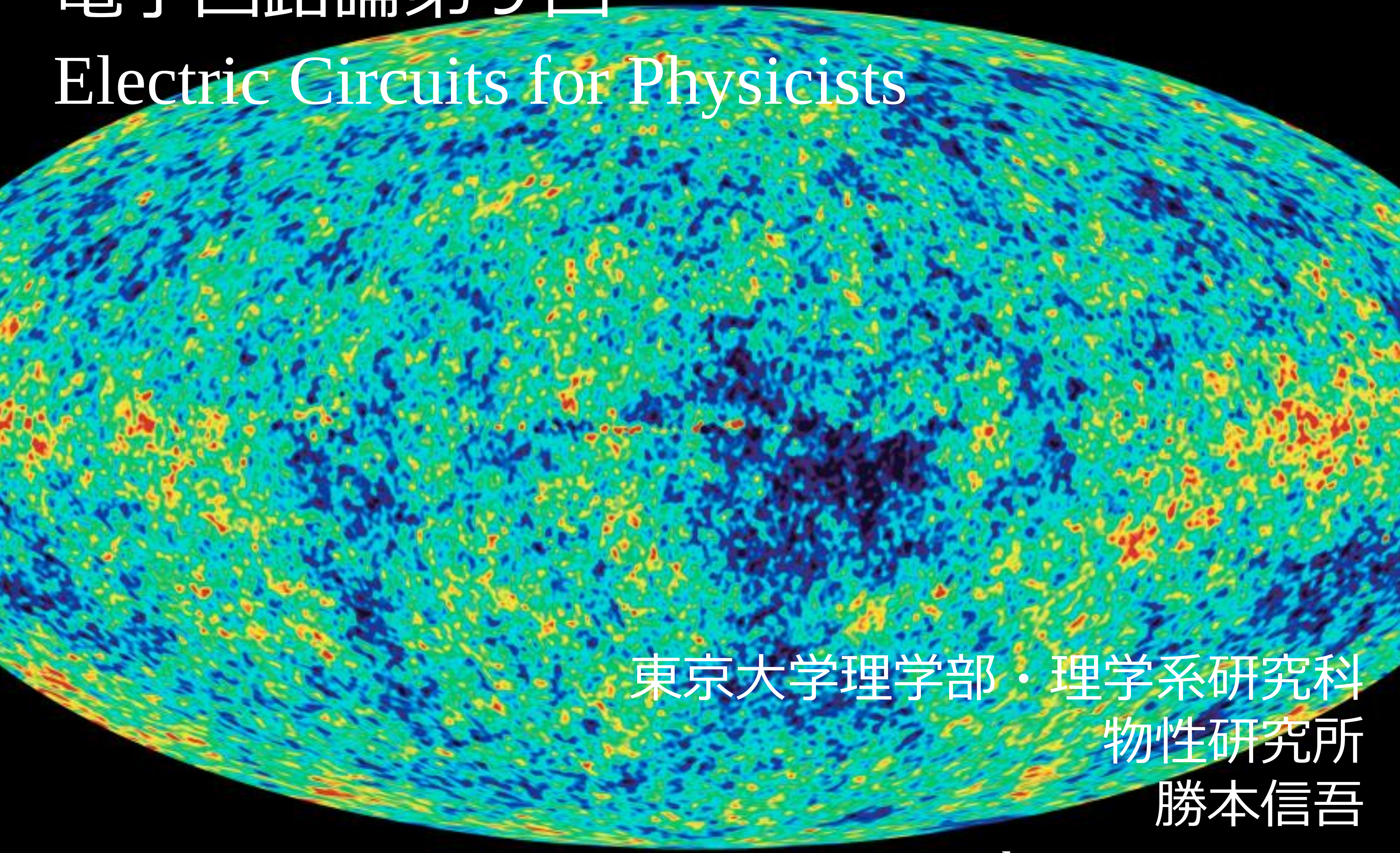
Remember F-matrix (cascade matrix) defined above.

Write down the F-matrix form of the transmission line shown below.



電子回路論第９回

Electric Circuits for Physicists



東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto


GSFC

Outline

5.3 S-parameter representation of devices

Impedance matching with Z, S parameters

Impedance matching with immittance chart

5.4 Non-TEM mode transmission lines

5.5 Non-linear elements and Toda lattice

Ch.6 Noises and Signals

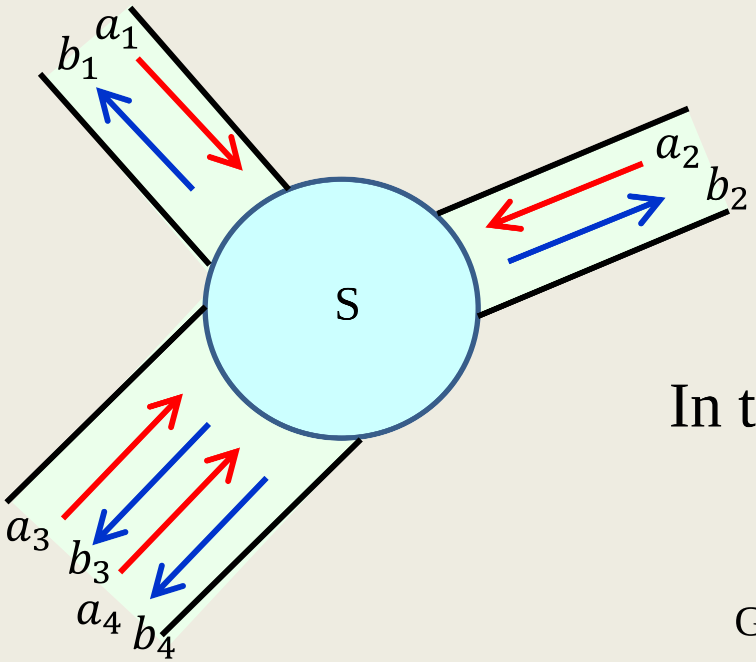
6.1 Fluctuations

6.2 Fluctuation-dissipation theorem

Review: Scattering (S) matrix (S parameters)

Transmission lines: wave propagating modes → Channels

Take $|a_i|^2$, $|b_i|^2$ to be powers (energy flow).

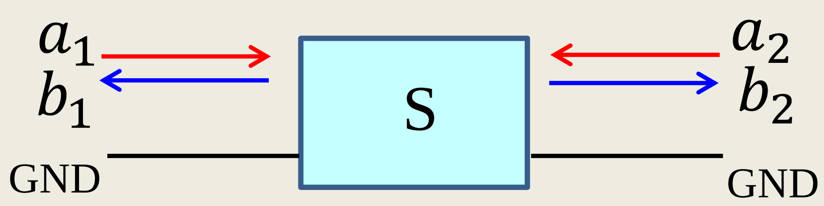


output S-matrix input

$$\begin{pmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} S_{11} & \cdots & S_{1i} & \cdots & S_{1n} \\ \vdots & \ddots & & & \vdots \\ S_{i1} & & S_{ii} & & S_{in} \\ \vdots & & & \ddots & \vdots \\ S_{n1} & \cdots & S_{ni} & \cdots & S_{nn} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix}$$

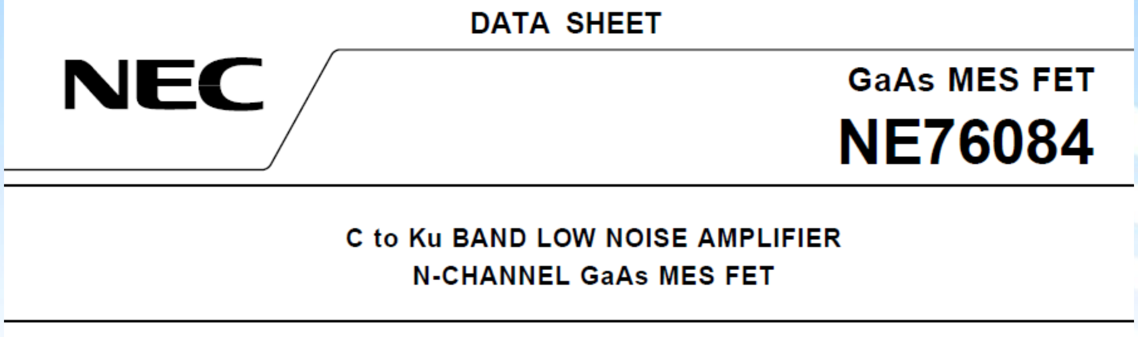
$$\boldsymbol{b} = \mathrm{S}\boldsymbol{a}$$

In the case of two-terminal pair circuit



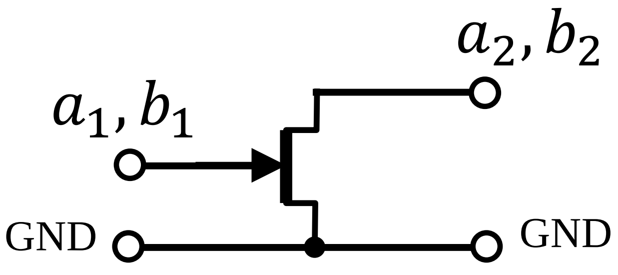
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} r_\mathrm{l} & t_\mathrm{r} \\ t_\mathrm{l} & r_\mathrm{r} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

S-parameter representation of high-frequency devices

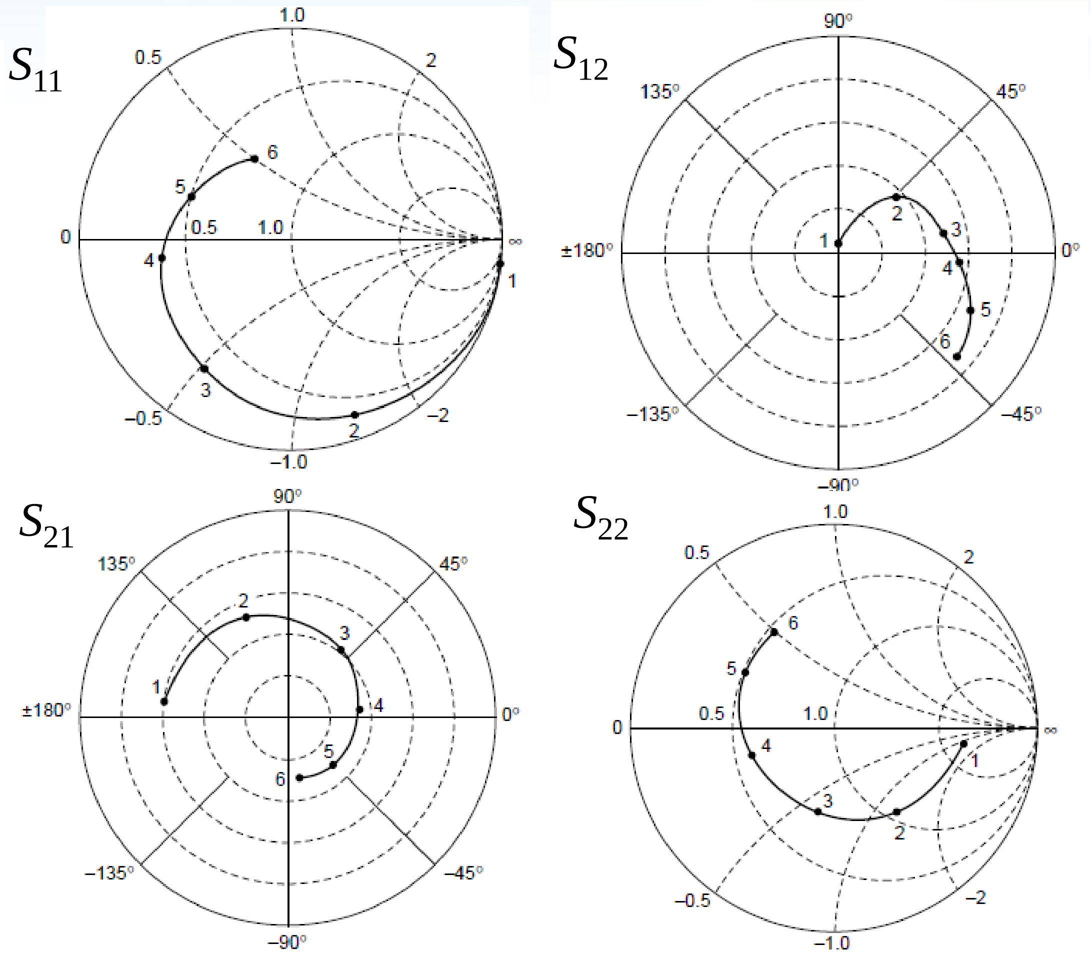


S-PARAMETERS

V_{DS} = 3 V, I_D = 10 mA
START 500 MHz, STOP 18 GHz, STEP 500 MHz

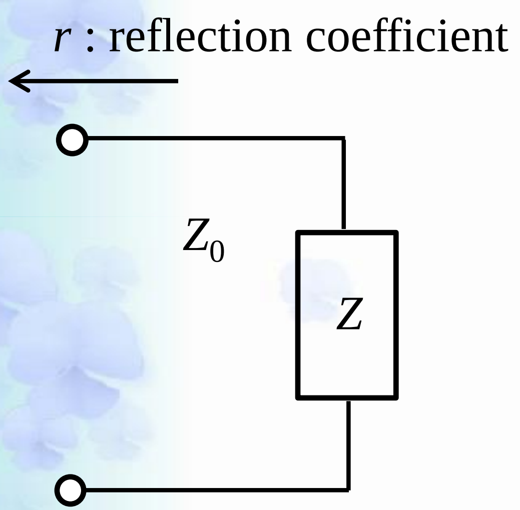


$$S_{11} = r_l, S_{22} = r_r$$



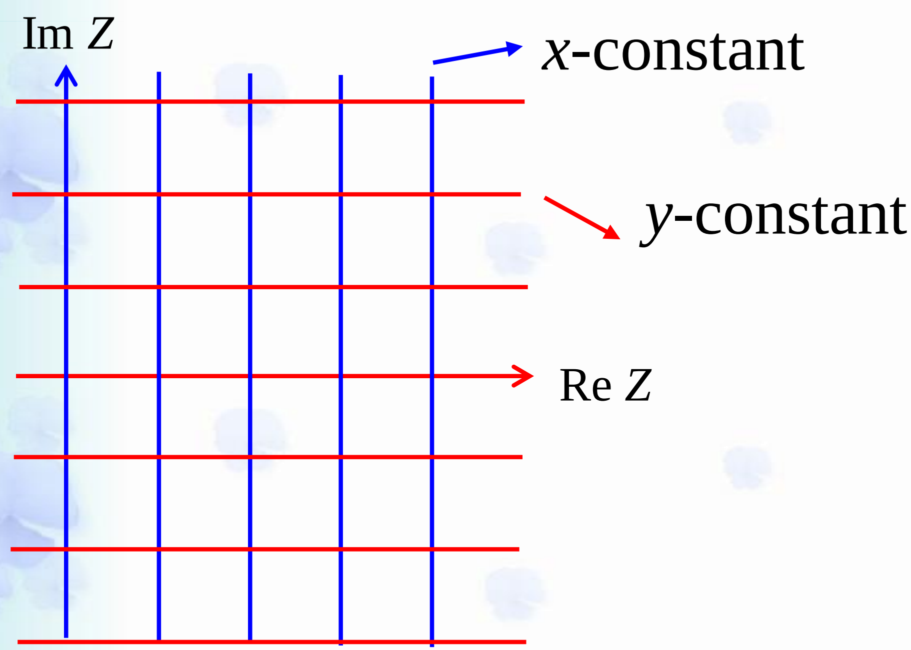
Review: Smith Chart

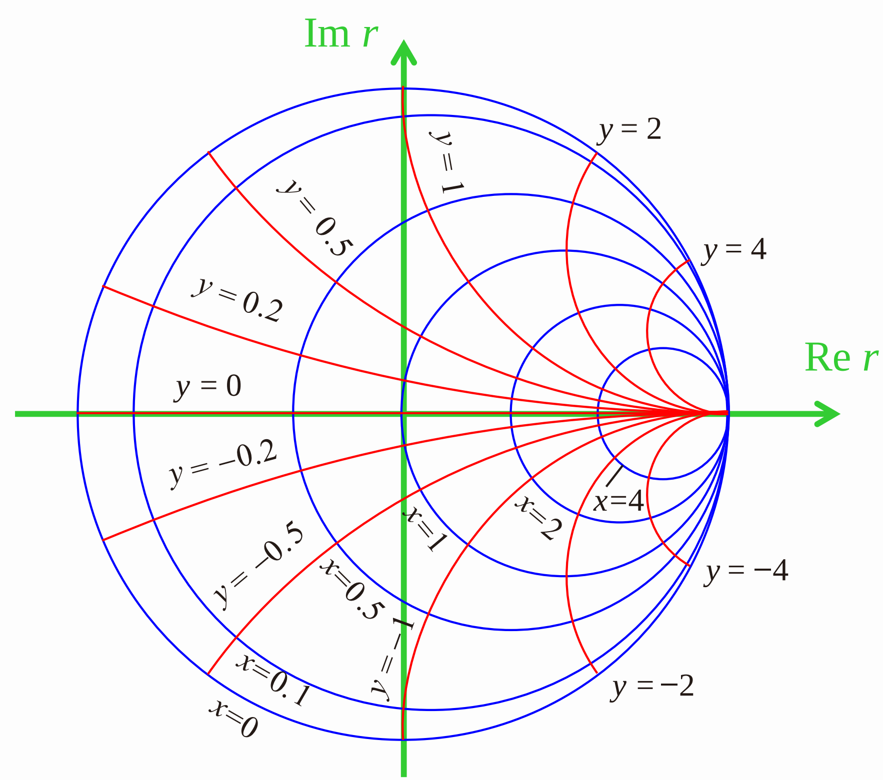
P.H. Smith
1905-1987



$$r = \frac{V_-}{V_+} = \frac{Z - Z_0}{Z + Z_0} = \frac{z-1}{z+1}$$

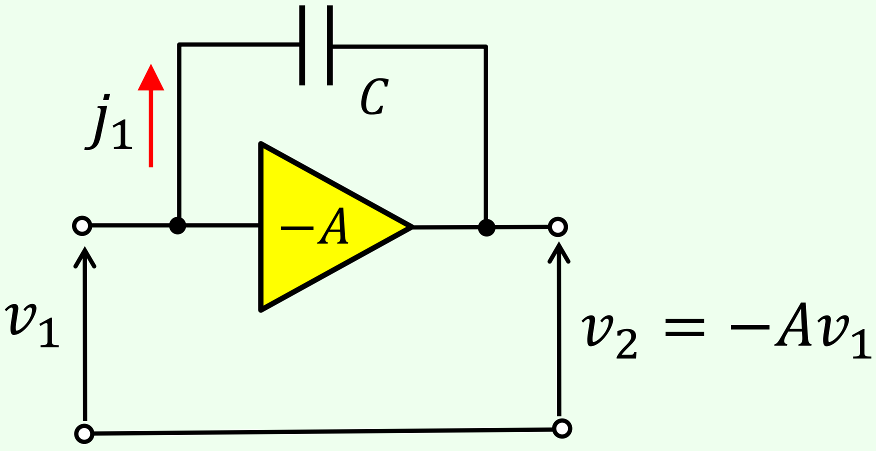
$$z = \frac{Z}{Z_0} = x + iy$$



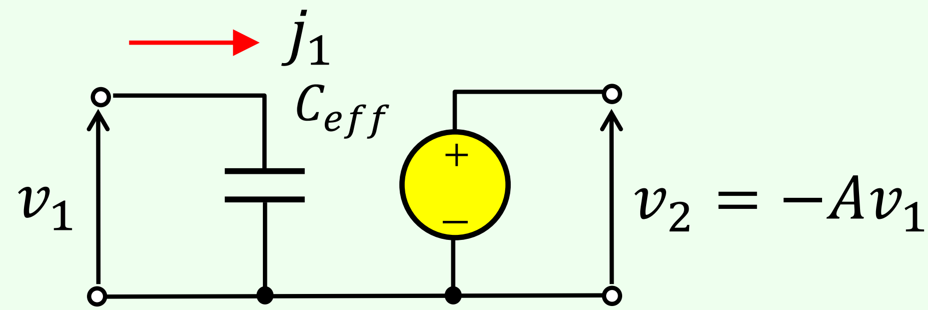


Comment: Mirror effect

An amplifier may change the effective impedance of passive elements.



equivalent circuit



$$j_1 = sC(v_1 - v_2) = sC(1 + A)v_1$$

$$C_{\text{eff}} \frac{dv_1}{dt} = sC_{\text{eff}} v_1 = j_1$$

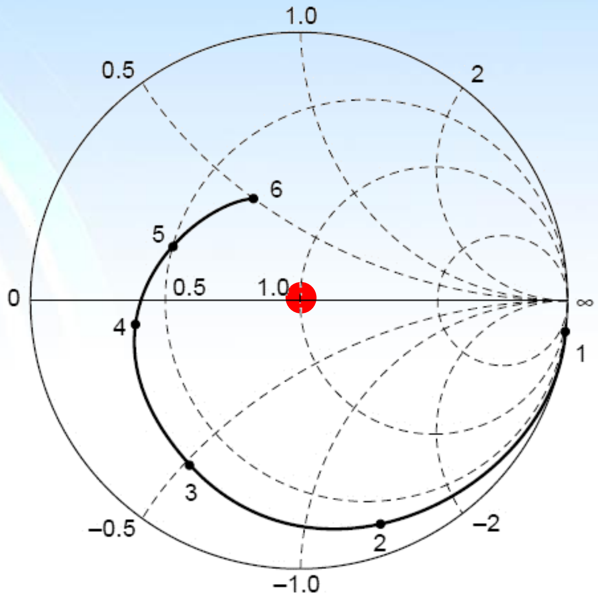
$$s = \frac{j_1}{(1 + A)Cv_1}$$

$$s = \frac{1}{C_{\text{eff}}} \frac{j_1}{v_1}$$

$$C_{\text{eff}} = (1 + A)C$$

: mirror effect

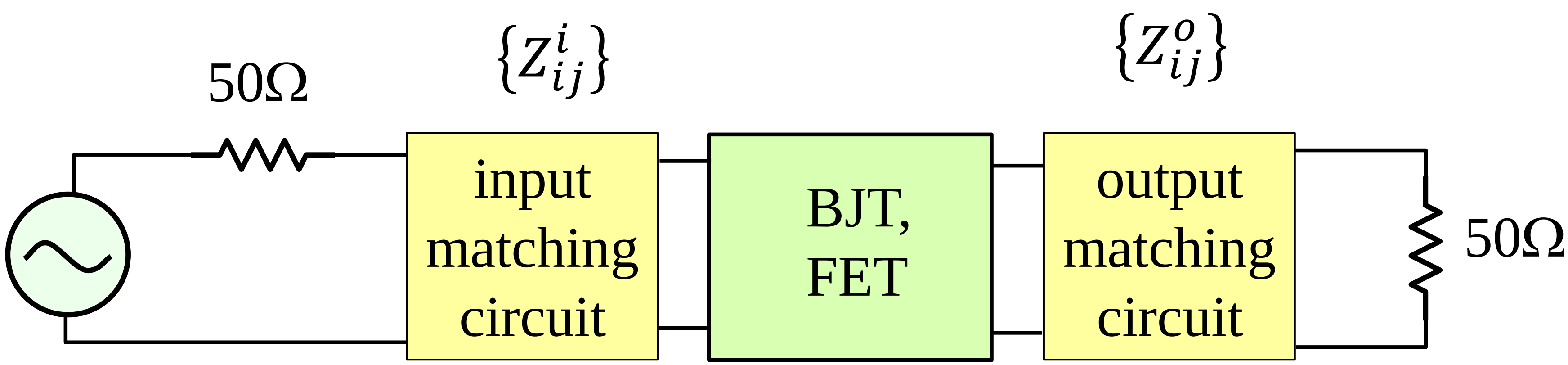
Ex) NE76084 MES FET 0.5～18GHz



S_{11}

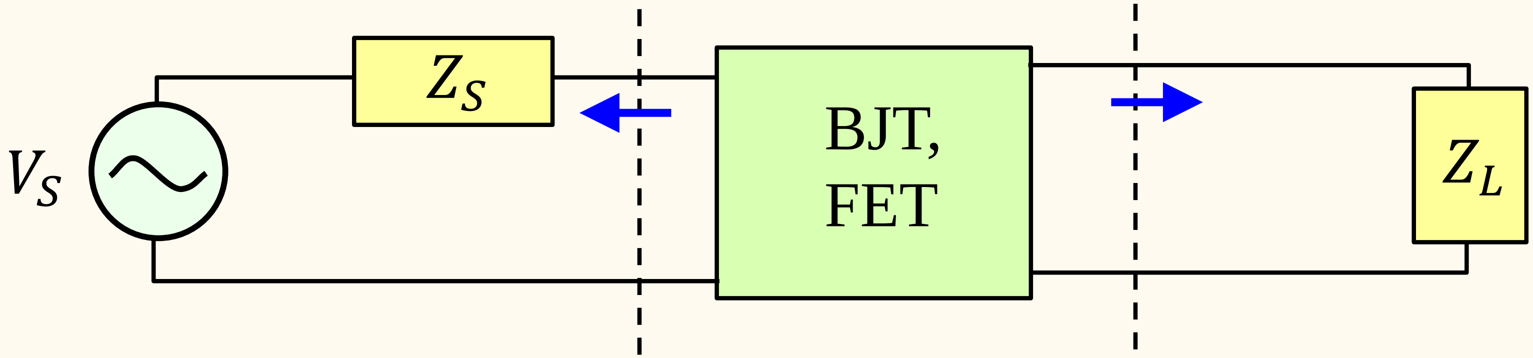
The datasheet tells that we need impedance matching circuits with transmission lines with $Z_0 = 50\ \Omega$.

Insert input and output matching circuits to kill reflections.



Impedance matching circuits

The circuit is summarized at the boundaries as

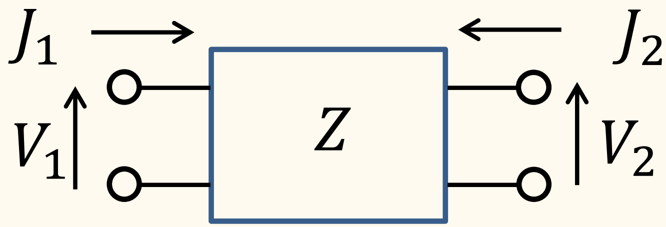


If we know Z-parameters of the input/output matching circuits, from Ho-Thevenin's theorem

$$Z_{\rm S} = Z_{22}^{\rm i} - \frac{Z_{12}^{\rm i} Z_{21}^{\rm i}}{50 + Z_{11}^{\rm i}}, \quad Z_{\rm L} = Z_{11}^{\rm o} - \frac{Z_{12}^{\rm o} Z_{21}^{\rm o}}{50 + Z_{22}^{\rm o}}.$$

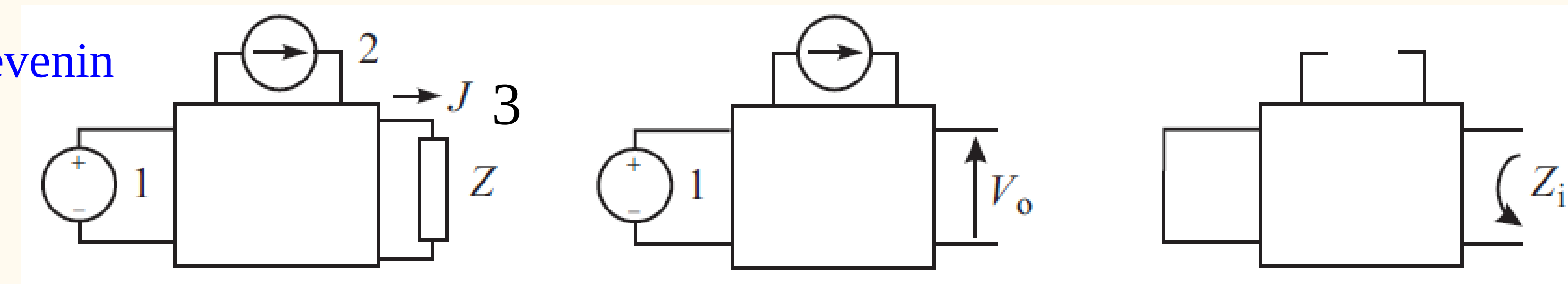
Z-matrix, Ho-Thevenin's theorem

Z-matrix



$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix}$$

Ho-Thevenin



1. Measure the open terminal voltage V_0.
2. Turn off all the power sources (voltage sources: short, current sources: open). Measure the open circuit impedance Z_i.

Then

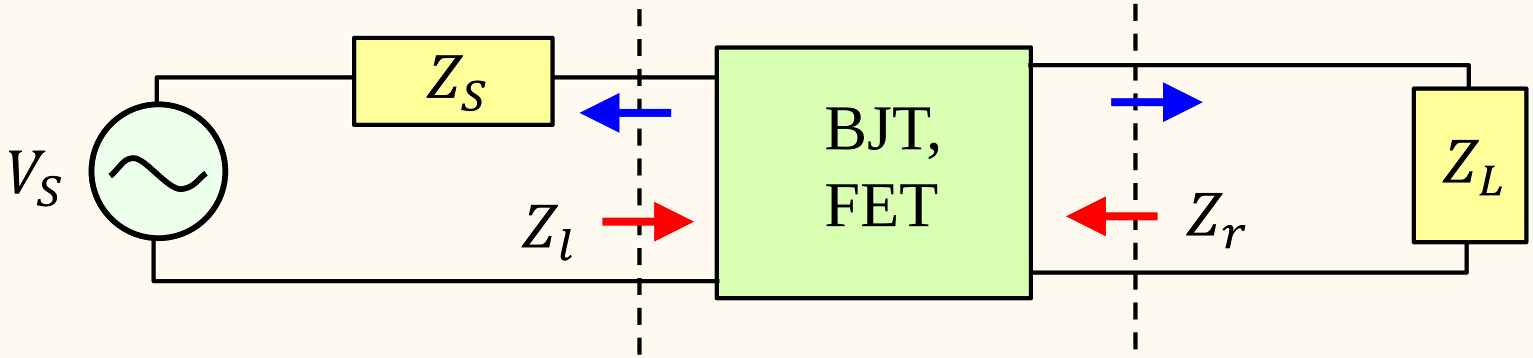
$$J = \frac{V_0}{Z + Z_i}$$

$$V_s = 0 \;\; \therefore V_1 = -50J_1 = Z_{11}J_1 + Z_{12}J_2 \quad \therefore J_1 = -\frac{Z_2}{50 + Z_{11}}J_2$$

$$V_2 = Z_{21}J_1 + Z_{22}J_2 = \left(Z_{22} - \frac{Z_{21}Z_{12}}{50 + Z_{11}} \right) J_2 \rightarrow Z_S$$

Impedance matching circuits

The circuit is summarized at the boundaries as



If we know Z-parameters of the input/output matching circuits, from Ho-Thevenin's theorem

$$Z_{\rm S} = Z^{\rm i}_{22} - \frac{Z^{\rm i}_{12}Z^{\rm i}_{21}}{50 + Z^{\rm i}_{11}}, \quad Z_{\rm L} = Z^{\rm o}_{11} - \frac{Z^{\rm o}_{12}Z^{\rm o}_{21}}{50 + Z^{\rm o}_{22}}.$$

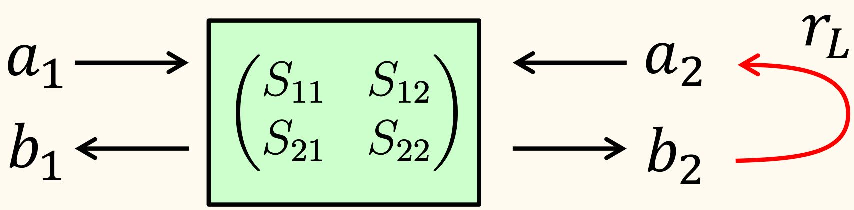
$\{Z_{ij}\}$: BJT (FET) Z-parameters, again Ho-Thevenin says

$$Z_l = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{\rm L} + Z_{22}}, \quad Z_r = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{\rm S} + Z_{11}}$$

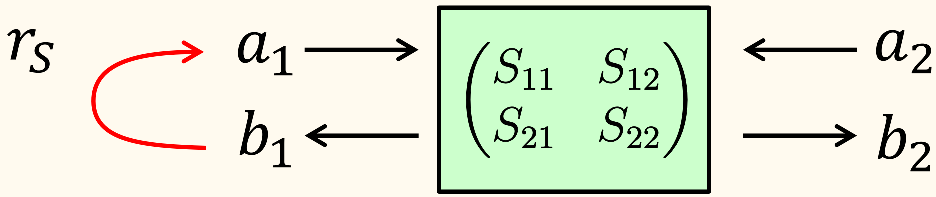
matching condition: $Z_l = Z_S^*,\ Z_r = Z_L^*$

Impedance matching with S-parameters

In S-parameter treatment, we use complex reflection coefficients to express load, source etc.



$$r_{\rm in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}r_{\rm L}}{1 - S_{22}r_{\rm L}}$$



$$r_{\rm out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}r_{\rm S}}{1 - S_{11}r_{\rm S}}$$

Matching condition: $r_{\rm L} = r_{\rm out}^*, \quad r_{\rm S} = r_{\rm in}^*$

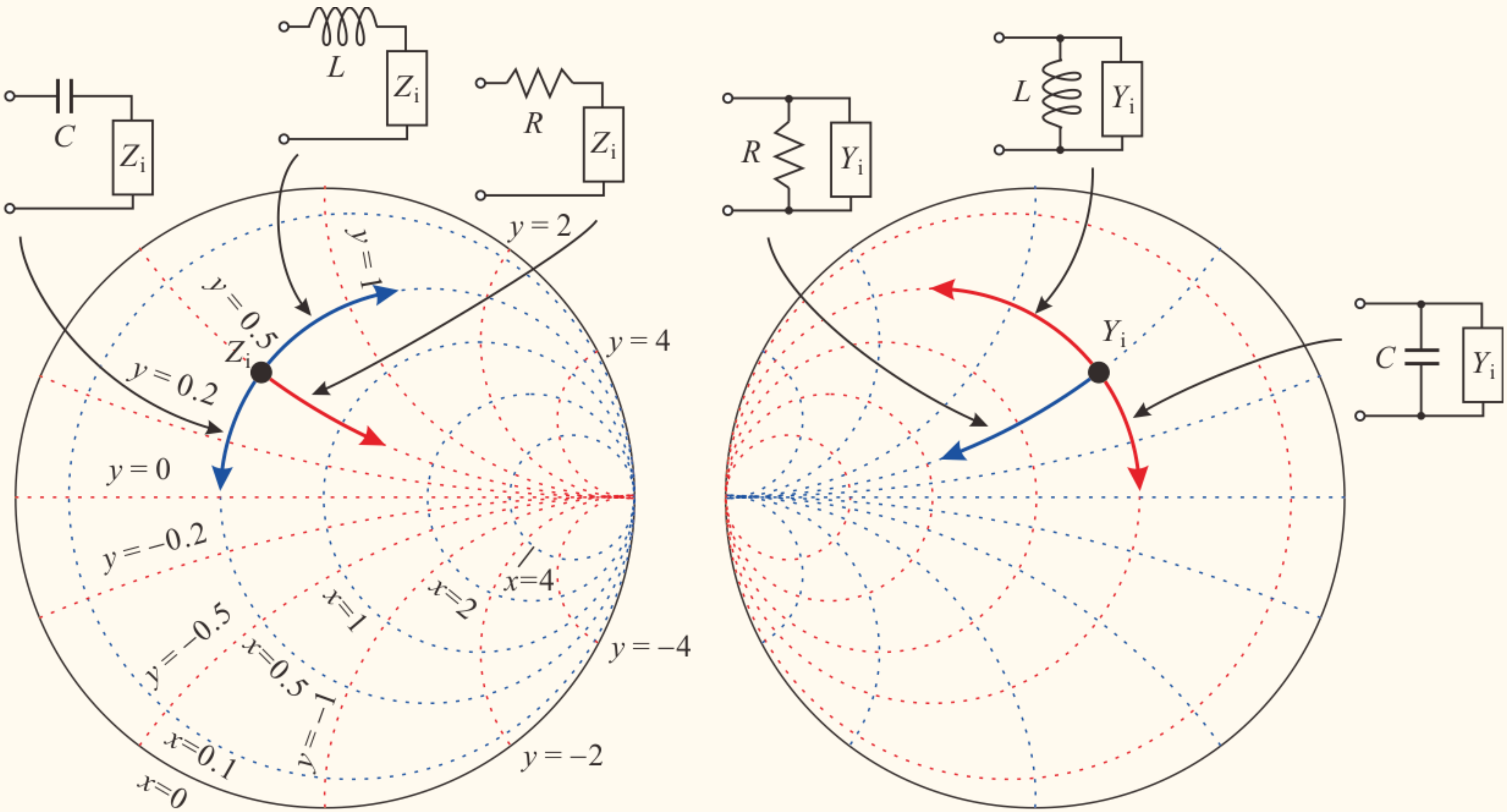
Solution $$r_{\rm S} = \frac{B_1 \pm \sqrt{B_1^2 - 4|M|^2}}{2M}, \quad r_{\rm L} = \frac{B_2 \pm \sqrt{B_2^2 - 4|N|^2}}{2N}$$

with

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\det S|^2, \quad B_2 = 1 - |S_{11}|^2 + |S_{22}|^2 - |\det S|^2,$$
$$N = S_{22} - S_{11}^* \det S, \quad M = S_{11} - S_{22}^* \det S$$

Practical impedance matching with Smith chart

Series and parallel connection of passive elements and traces on charts



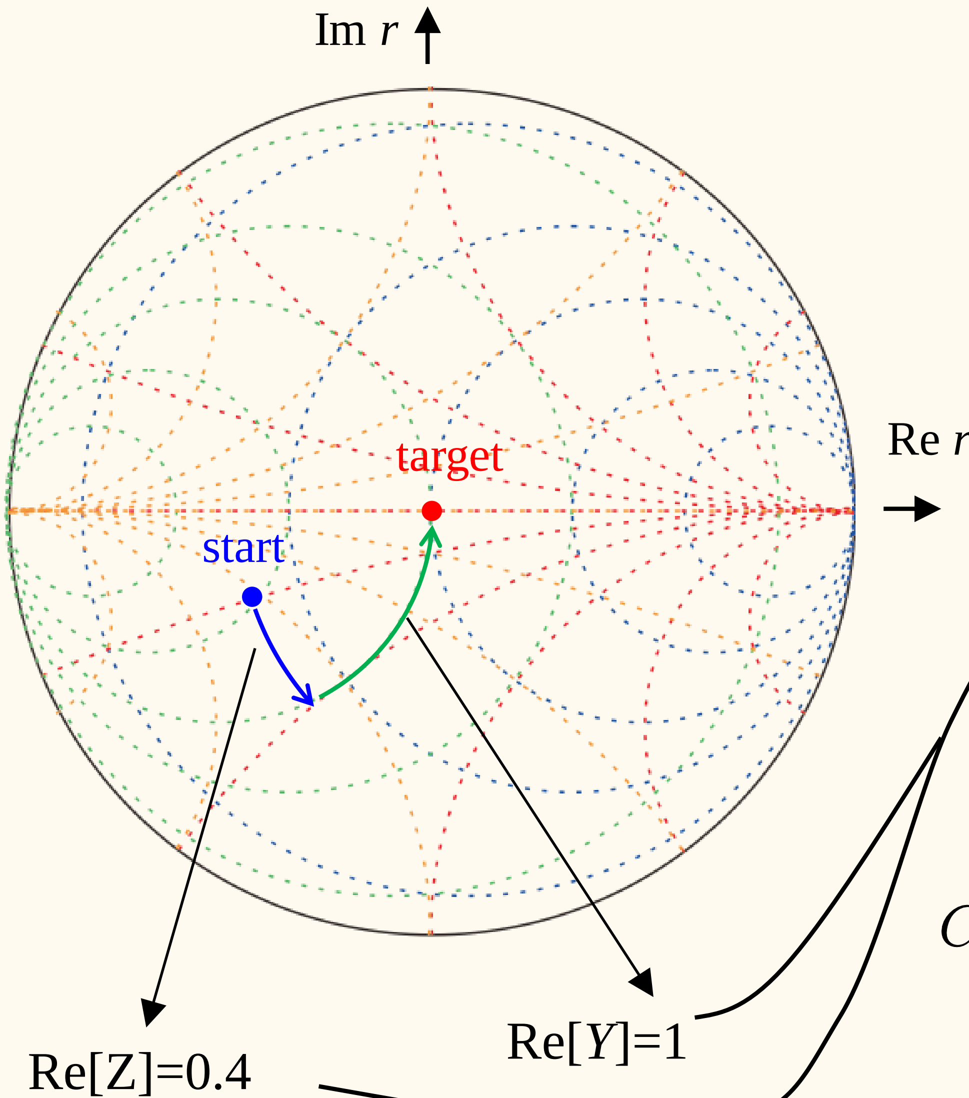
Smith chart

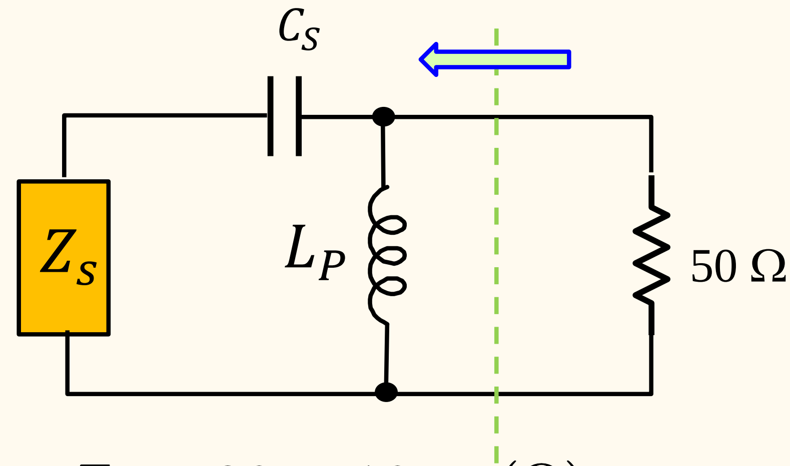
Admittance chart

An example of impedance matching

frequency 100 MHz ≈ 628 Mrad/s

immittance chart





$$Z_s = 20 - 10i \quad (\Omega)$$
$$= 0.4 - 0.2i \quad (Z_0)$$

equalize: $0.4 + iy = \dfrac{1}{1 + iq}$

$$y = -\sqrt{0.24} \approx -0.49$$

$$-0.49 = -0.2 - \frac{1}{\omega C_S Z_0}$$

$$C_S \approx \frac{1}{2\pi \times 10^6 \times 50 \times 0.29} \approx 110 \text{ pF}$$

similarly $L_P \approx 65$ nH

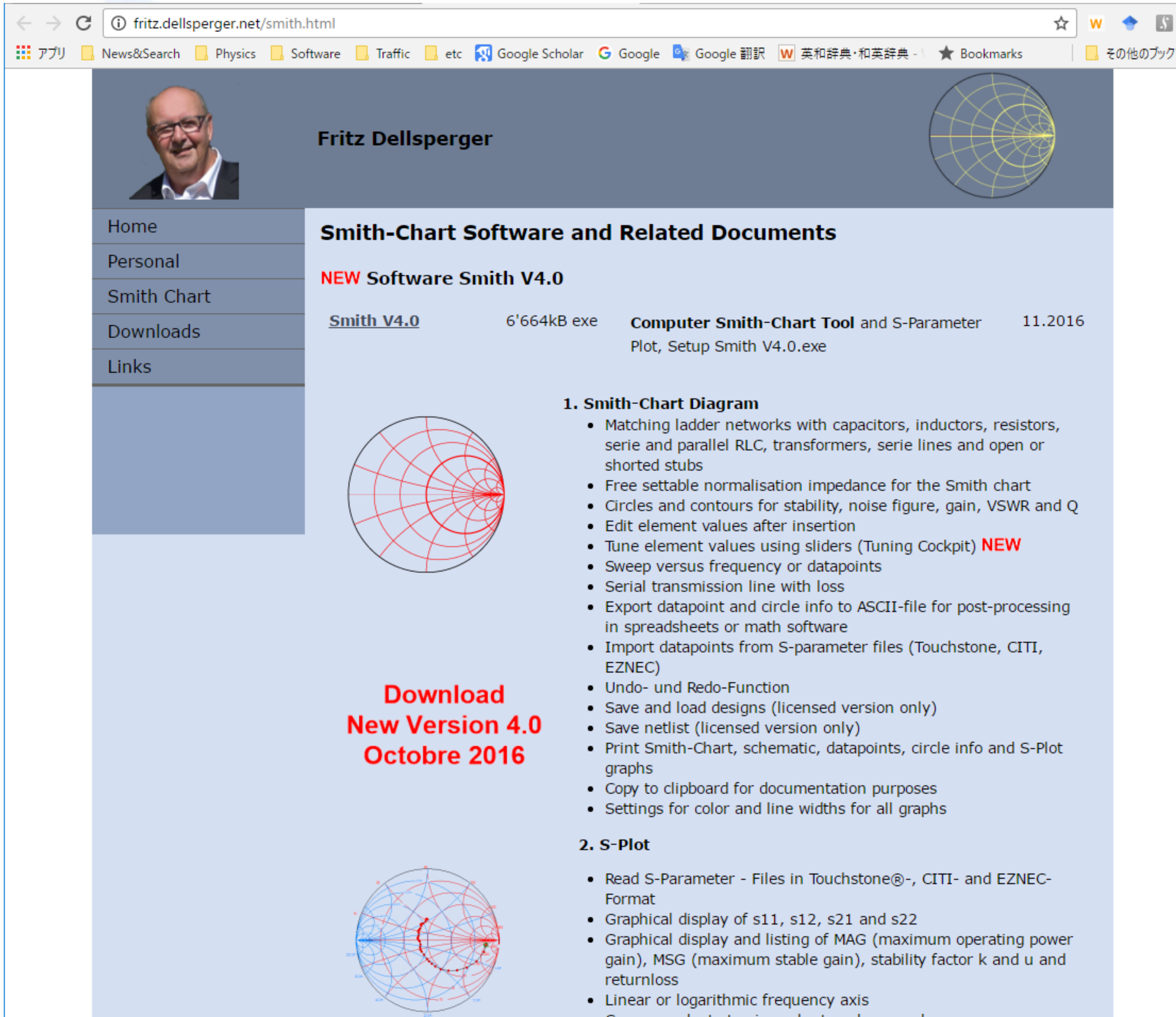
Impedance matching designer

http://home.sandiego.edu/~ekim/e194rfs01/jwmatcher/matcher2.html

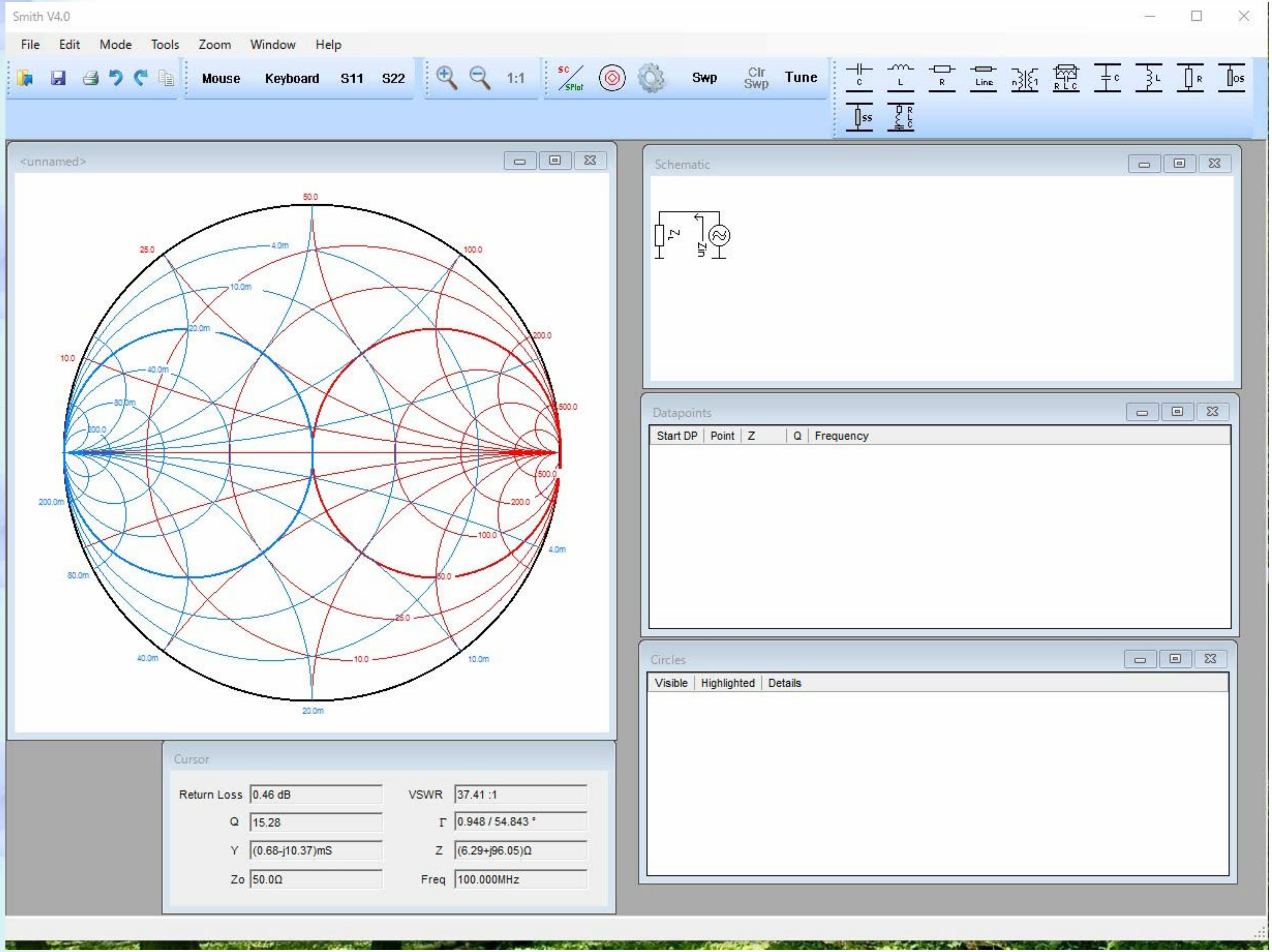
http://leleivre.com/rf_lcmatch.html

Useful freeware: Smith v4.0

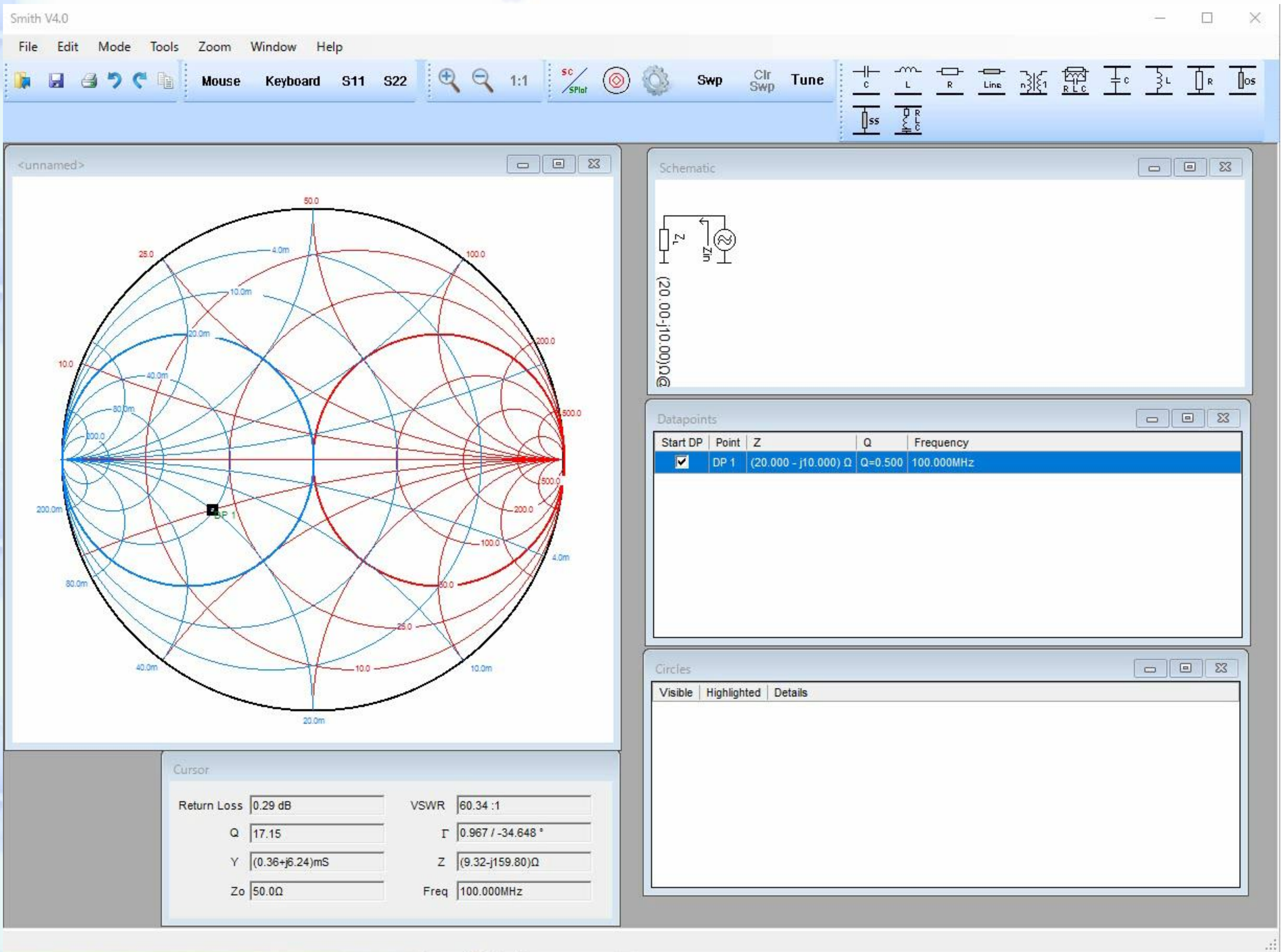
http://fritz.dellsperger.net/smith.html



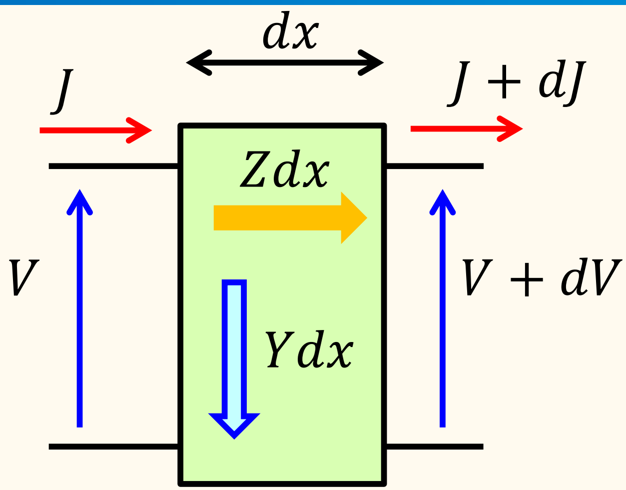
Impedance matching with Smith V4 (1)



Impedance matching with Smith V4 (2)



5.4 Non-TEM mode transmission line



$$\frac{V}{J} = \mp\frac{Z}{\kappa} = \mp\sqrt{\frac{Z}{Y}}$$

Characteristic impedance

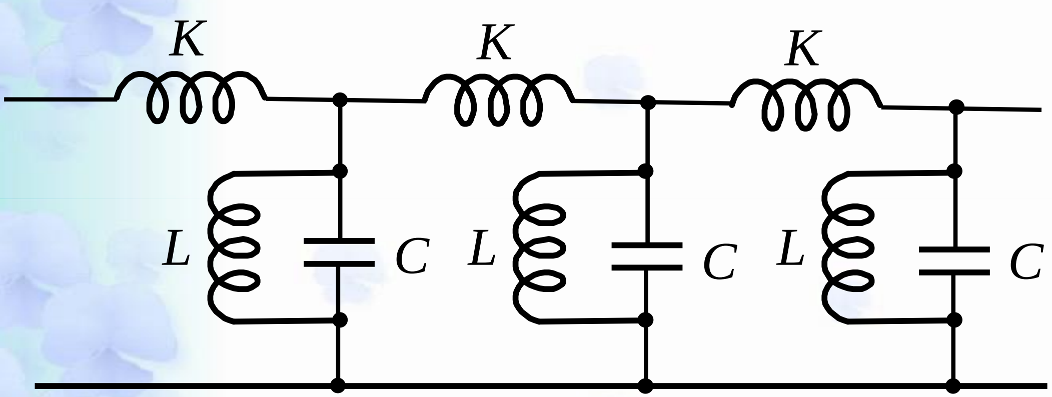
LC model: $Z = i\omega L,\ Y = i\omega C$

The inductance represents magnetic fields circulating the core and the capacitance electric fields directing from the core to the shield.

$$Z_0 = \sqrt{\frac{L}{C}}$$: real, dispersionless (no ω-k relation)

Non-linear ω-term in Z or Y → dispersion (longitudinal components)

5.4 Non-TEM mode gives mass in transmission line



C: capacitance per unit length
L: inductance per inverse unit length
K: inductance per unit length

$$Y = i\omega C + \frac{1}{i\omega L}$$

$$-k^2 = YZ = \left(i\omega C + \frac{1}{i\omega L}\right) i\omega K = -CK\omega^2 + \frac{K}{L}$$

Constant finite mass: $E = \hbar\omega \propto k^2$

(Schrodinger eq.: Parabolic partial differential equation)

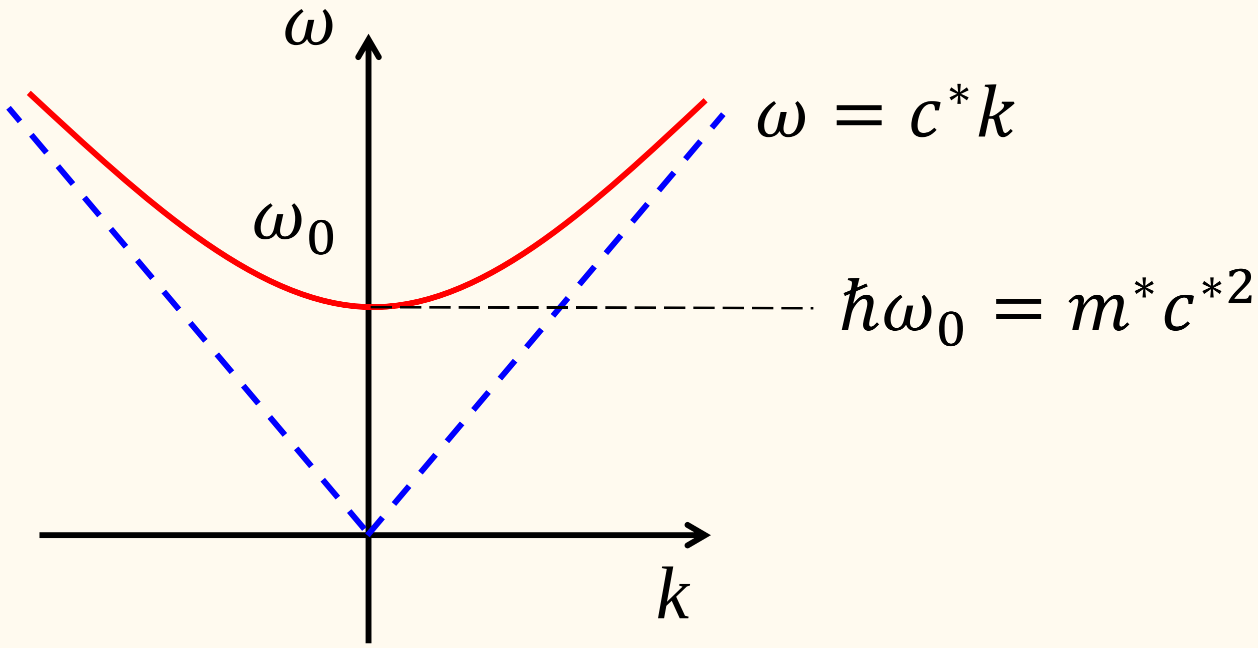
Coupling between linear dispersions: mass mechanism *cf.* Higgs

5.4 Non-TEM mode gives mass in transmission line

$$\frac{1}{\sqrt{LC}} = \omega_0 \text{ unchanged with } dx \to 0$$

$$Z = i\omega K, \quad Y = \frac{1-(\omega/\omega_0)^2}{i\omega L}$$

$$ik = \kappa = \sqrt{YZ} = i\sqrt{\frac{K}{L}\left[\left(\frac{\omega}{\omega_0}\right)^2 - 1\right]} \qquad \eta^2 \equiv \frac{K}{L}$$



5.4 Giving mass to LC transmission line

$$\omega \gg \omega_0 \rightarrow k \sim \eta \frac{\omega}{\omega_0}$$ No dispersion

Velocity: $$c^* = \frac{\omega}{k} = \frac{\omega_0}{\eta} = \frac{1}{\sqrt{KC}}$$

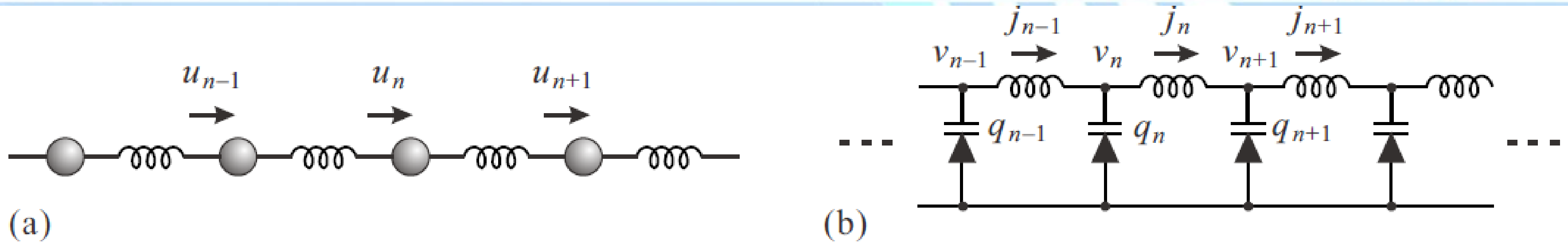
$$\omega \sim \omega_0 \quad \omega = \omega_0 + \delta\omega$$

$$k^2 \approx 2\eta^2 \frac{\delta\omega}{\omega_0} \qquad \therefore \epsilon \equiv \hbar\delta\omega = \frac{\hbar k^2}{2(\eta^2/\omega_0)} = \frac{\hbar^2 k^2}{2m^*}$$

$$m^* \equiv \frac{\hbar\eta^2}{\omega_0}$$

$$E_0 = \hbar\omega_0 = \frac{\hbar\eta^2}{\omega_0} \cdot \left(\frac{\omega_0}{\eta}\right)^2 = m^* c^{*2}$$

5.5 Non-linear LC transmission line and Toda lattice



Toda lattice is a typical non-linear system with exact (soliton) solutions. It is defined as follows:

The springs in (a) have Toda-potential: $\phi(r) = \frac{a}{b} e^{-br} + ar \quad (ab > 0)$

Equation of motion:

$$m\frac{d^2 u_n}{dt^2} = -a\exp[-b(u_{n+1} - u_n)] + a\exp[-b(u_n - u_{n-1})]$$

For relative shift

$r_n = u_{n+1} - u_n$

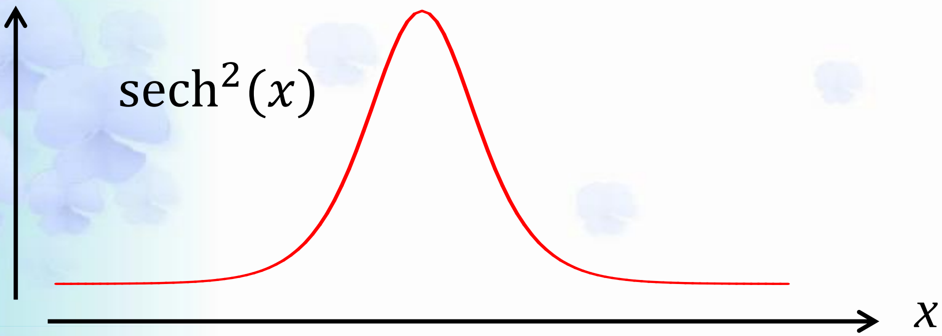
$$m\frac{d^2 r_n}{dt^2} = a(2e^{-br_n} - e^{-br_{n+1}} - e^{-br_{n-1}})$$

Force of a spring: $f = -\phi'(r) = a(e^{-br} - 1)$

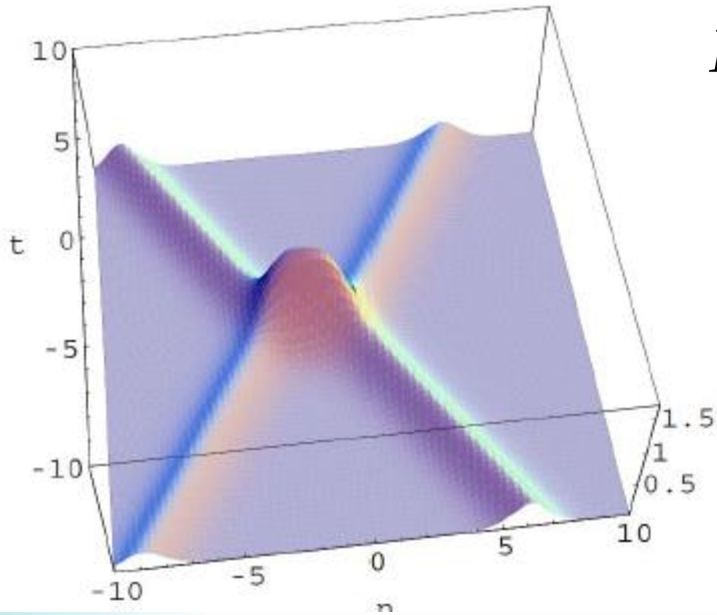
Solitons in Toda lattice

$$\frac{d^2}{dt^2}\log\left(1+\frac{f_n}{a}\right)=\frac{b}{m}(f_{n+1}+f_{n-1}-2f_n)$$

Soliton solution:



$$u_n=\omega^2\mathrm{sech}^2(\kappa n+\sigma\omega t+\delta),$$
$$\sigma=\pm1,\quad \omega=\sinh\kappa,$$
$$\kappa,\quad \delta:\text{constants}$$

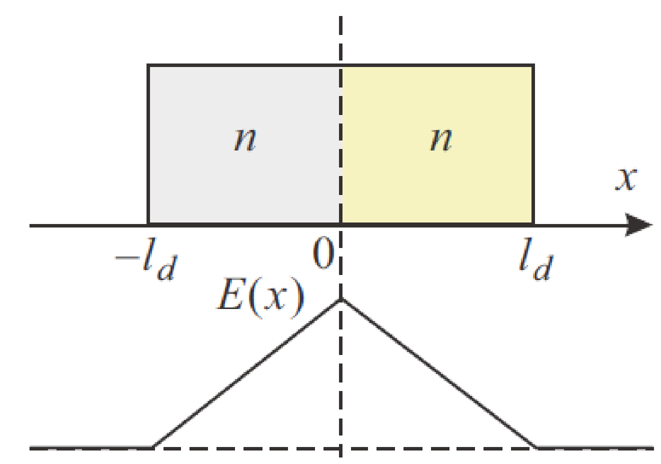


$N=2$ soliton solution:

$$u_n=\frac{\tau_{n+1}\tau_{n-1}}{\tau_n^2}-1,$$
$$\tau_n=1+e^{2\eta_1}+e^{2\eta_2}+A_{12}e^{2(\eta_1+\eta_2)},$$
$$\eta_i=\kappa_i n+\sigma_i\omega_i t+\delta_i,\quad \sigma_i=\pm1,\ \omega_i=\sinh\kappa_i,$$
$$A_{12}=\frac{ab\sinh^2(\kappa_1-\kappa_2)-m(\sigma_1\omega_1-\sigma_2\omega_2)^2}{m(\sigma_1\omega_1+\sigma_2\omega_2)^2-ab\sinh^2(\kappa_1+\kappa_2)}$$

Non-linear capacitance: Vari-cap

Varicap BB505

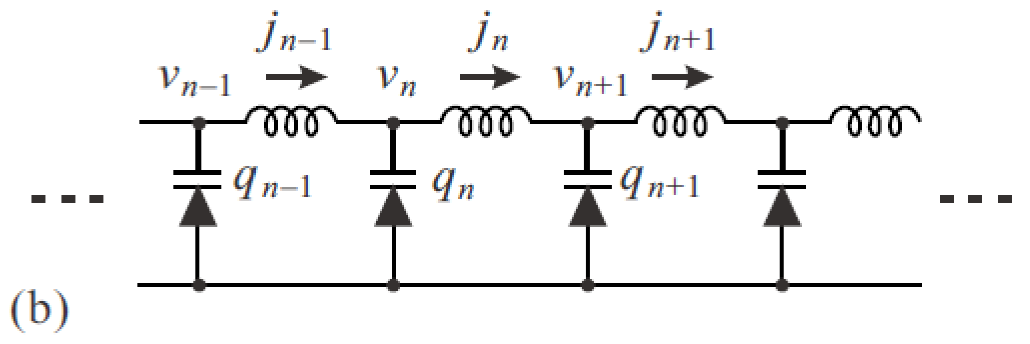


$$V_{\rm b} = \frac{en}{\epsilon}\int_{-l_d}^{0} 2(x+l_d)dx + \frac{en}{\epsilon}\int_{0}^{l_d} 2(l_d-x)dx = \frac{2enl_d^2}{\epsilon}$$

$$V + V_{\rm b} = \frac{2en}{\epsilon}\left(l_d + \frac{Q}{nS}\right)^2 \quad \therefore C = \frac{dQ}{dV} = \sqrt{\frac{\epsilon}{2en}}\frac{nS}{\sqrt{V+V_{\rm b}}}$$

$$V + V_{\rm b} = V_0 + \delta V \qquad \delta V \to V$$

L-Varicap transmission line



$$L\frac{dJ_n}{dt} = v_n - v_{n-1},$$

$$\frac{dq_n}{dt} = J_{n-1} - J_n,$$

$$q_n = \int_0^{v_n} C(V)dV, \quad C(V) = \frac{Q(V_0)}{F(V_0) + V - V_0}$$

$$q_n = Q(V_0)\log\left[1 + \frac{V_n}{F(V_0)}\right] + \text{const.}$$

$$\frac{d^2}{dt^2}\log\left[1 + \frac{V_n}{F(V_0)}\right] = \frac{1}{LQ(V_0)}(V_{n-1} + V_{n+1} - 2V_n)$$

Solitons in non-linear circuit

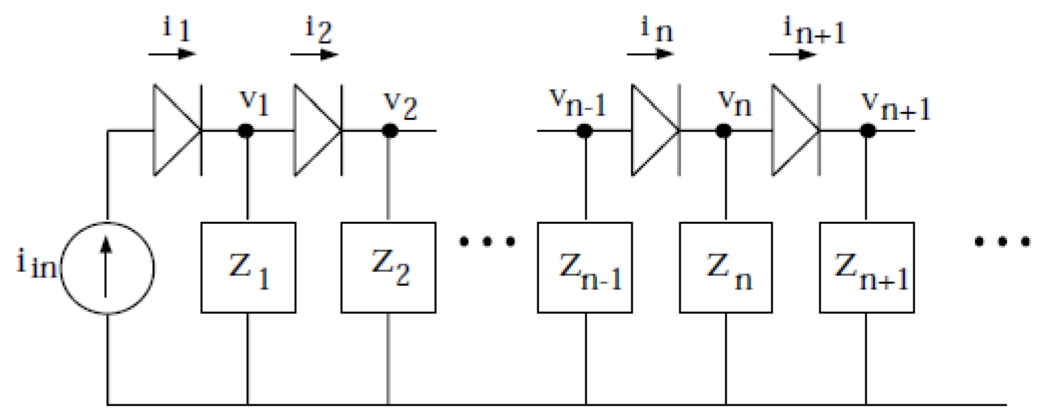
International Journal of Bifurcation and Chaos, Vol. 9, No. 4 (1999) 571–590

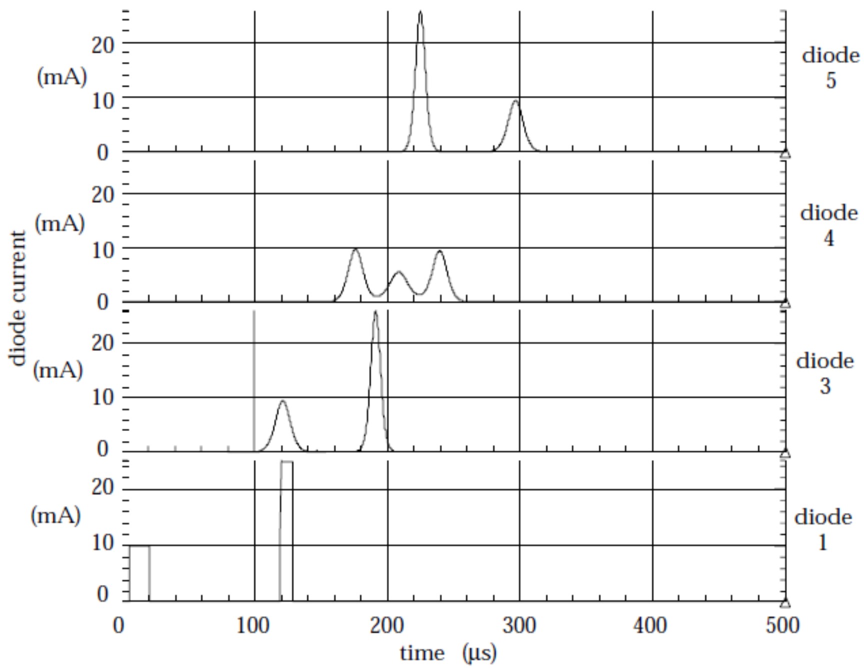
CIRCUIT IMPLEMENTATIONS OF SOLITON SYSTEMS*

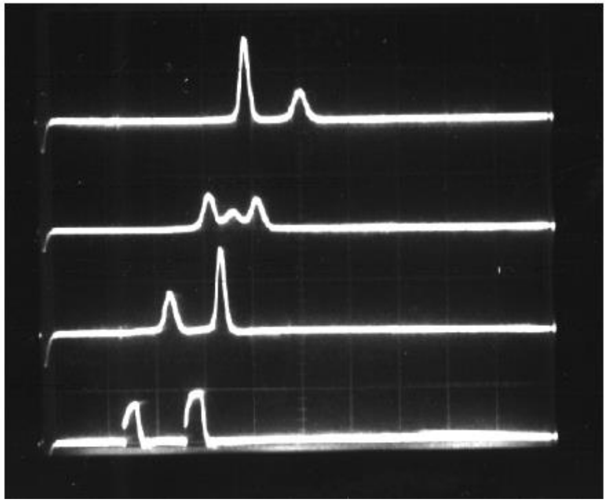
ANDREW C. SINGER
Department of Electrical and Computer Engineering, University of Illinois, Urbana, IL 61801, USA

ALAN V. OPPENHEIM
Department of Electrical Engineering, MIT, Cambridge, MA 02139, USA

Received May 27, 1998; Revised October 6, 1998







Toda lattice circuit, Soliton circuit

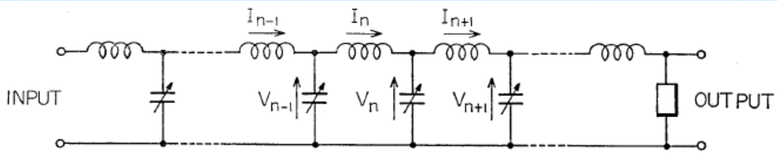
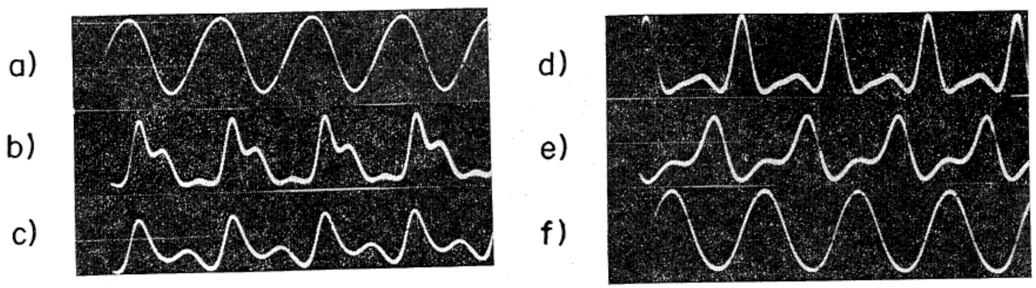


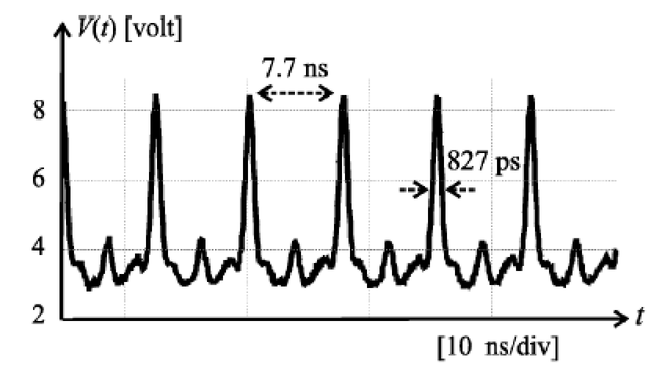
Fig. 1. A nonlinear network equivalent to a one-dimensional anharmonic lattice. The circuit element have an inductance $L=22\ \mu$H or capacitance $C(V)=27\ V^{-0.48}$ pF.



J. Phys. Soc. Japan **28** (1970) 1366~1367

Studies on Lattice Solitons by Using Electrical Networks

Ryogo Hirota and Kimio Suzuki



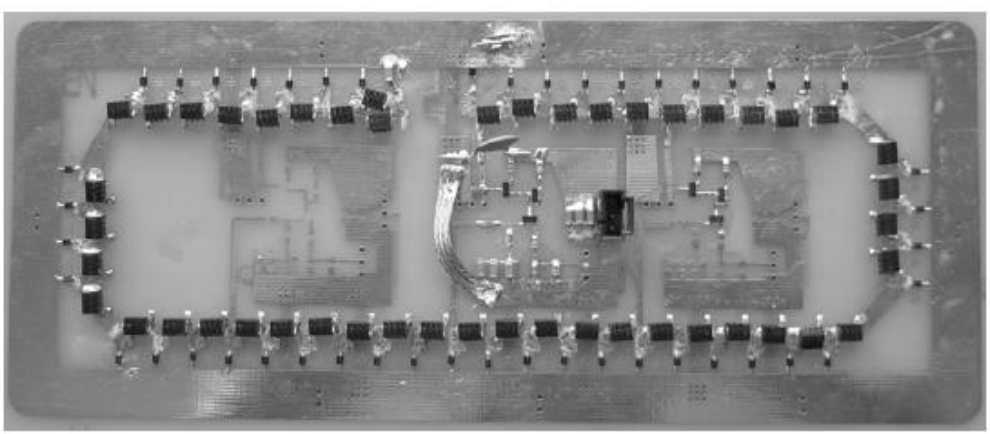
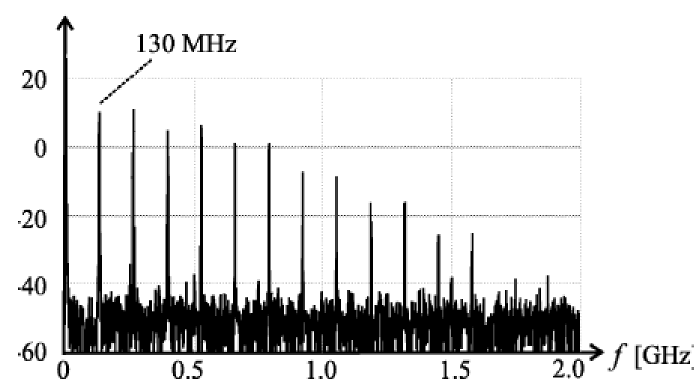


Fig. 16. Microwave soliton oscillator prototype.



電子回路論第10回
Electric Circuits for Physicists

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Comment: Impedance match/mismatch

Propagation of a wave:

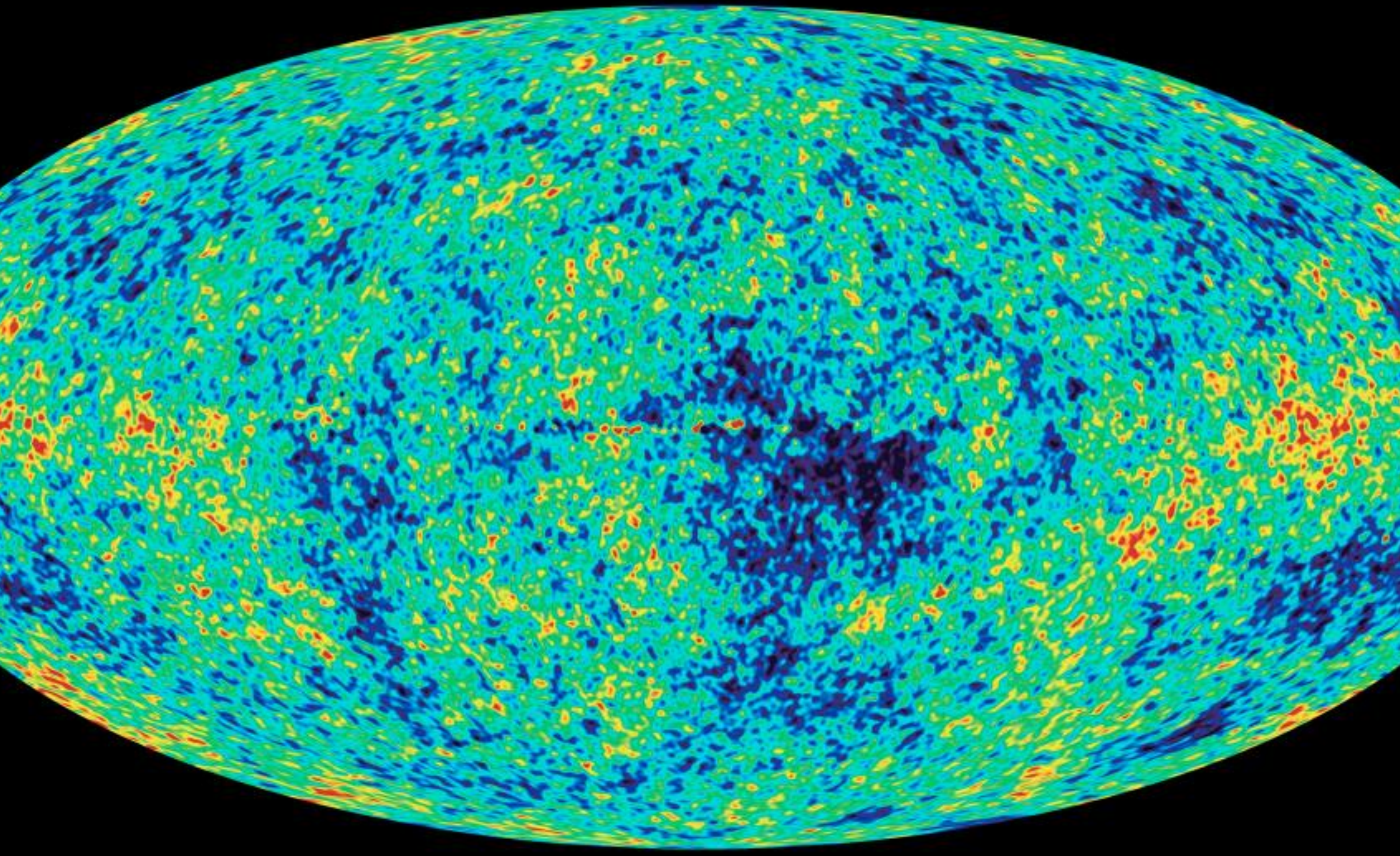
Impedance match: complete absorption
(propagation without reflection)

Mismatch: wave reflection

Impedance match/mismatch is an important concept applicable to a broad area of physics.

- Antenna: should be matched to the vacuum.
  EM wave propagation simulation: boundary is shunted with the characteristic impedance of vacuum.
- Optics: impedance mismatch → disagreement in refractive index
- Plasma: should be matched to electrodes for excitation.
- Phonon impedance mismatch at low temperatures: Kapitza resistance
- Sound insulated booth: should have sound impedance mismatch.

Ch6. Noises and Signals



Outline

Noises

Electric circuits transform: 1) Information

2) Electromagnetic power

on some physical quantities like voltages, current, …

Noises: stochastic (uncontrollable, unpredictable by human) variation in other words, fluctuation in such a quantity.

Internal noise

Intrinsic noise: Thermal noise (Johnson-Nyquist noise), Shot noise

Noise related to a specific physical phenomenon

Avalanche, Popcorn, Barkhausen, etc.

1/f noise: Name for a group of noises with spectra 1/f.

External noise

EMI, microphone noise, etc.

6.1 Fluctuation

Quantity x, fluctuation $\delta x = x - \bar{x}$

$$\overline{(\delta x)^2} = \overline{(x-\bar{x})^2} = \overline{x^2} - \bar{x}^2 \quad (\overline{\delta x} = 0)$$

$g(x)$: distribution function of x

Fourier transform: $$u(q) = \mathscr{F}\{g(x)\} = \int_{-\infty}^{\infty} g(x) e^{ixq} \frac{dx}{\sqrt{2\pi}}$$

$u(q)$: **characteristic function** of the distribution

From Taylor expansion, any moment can be obtained as

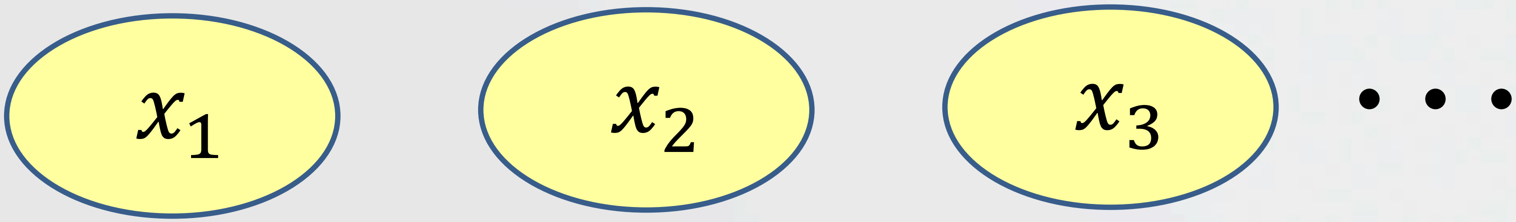
$$\overline{x^n} = \frac{\sqrt{2\pi}}{i^n} \left[\frac{d^n}{dq^n} u(q) \right]_{q=0}$$

Moments to high orders → reconstruction of $g(x)$

6.1 Fluctuation

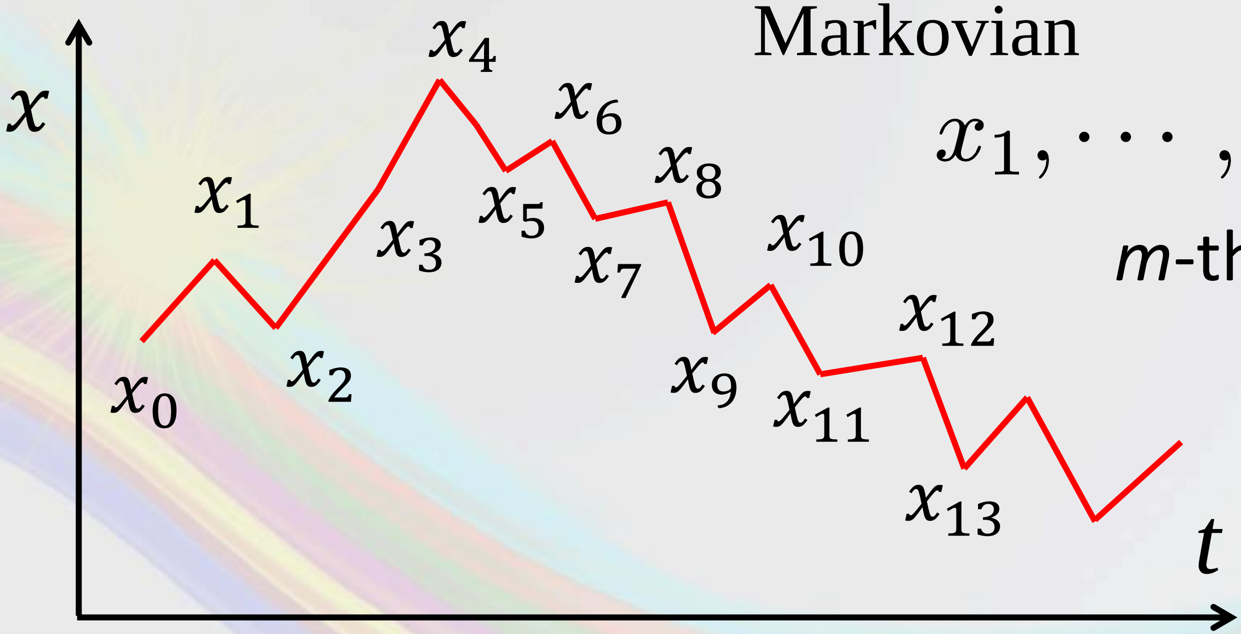
In electric circuits we need to consider two kinds of averages:

substance 1 substance 2 substance 3



x_j: independent

Ensemble average: $\bar{x}$



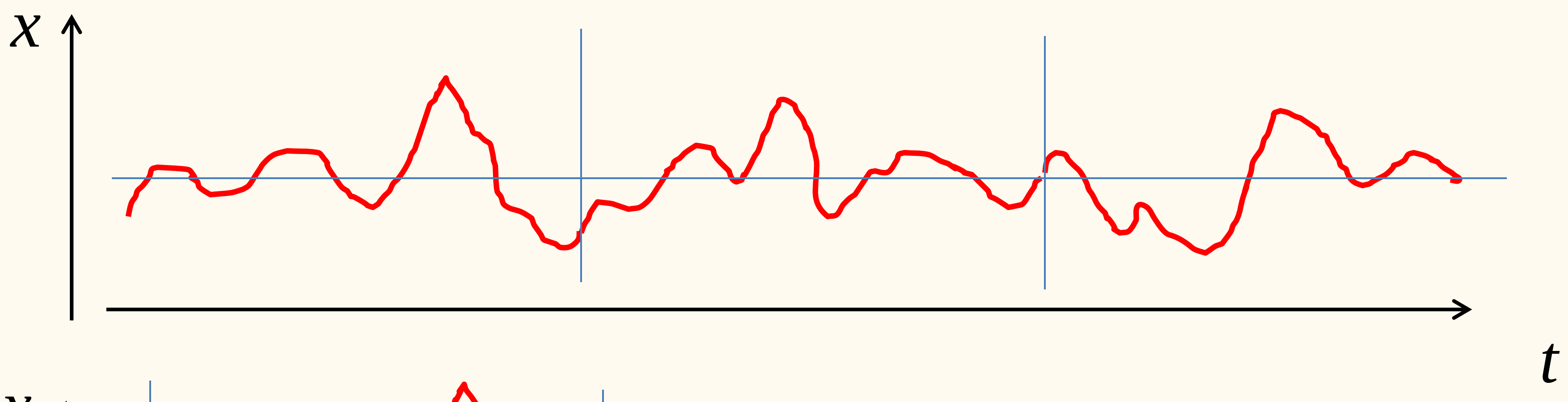
Markovian

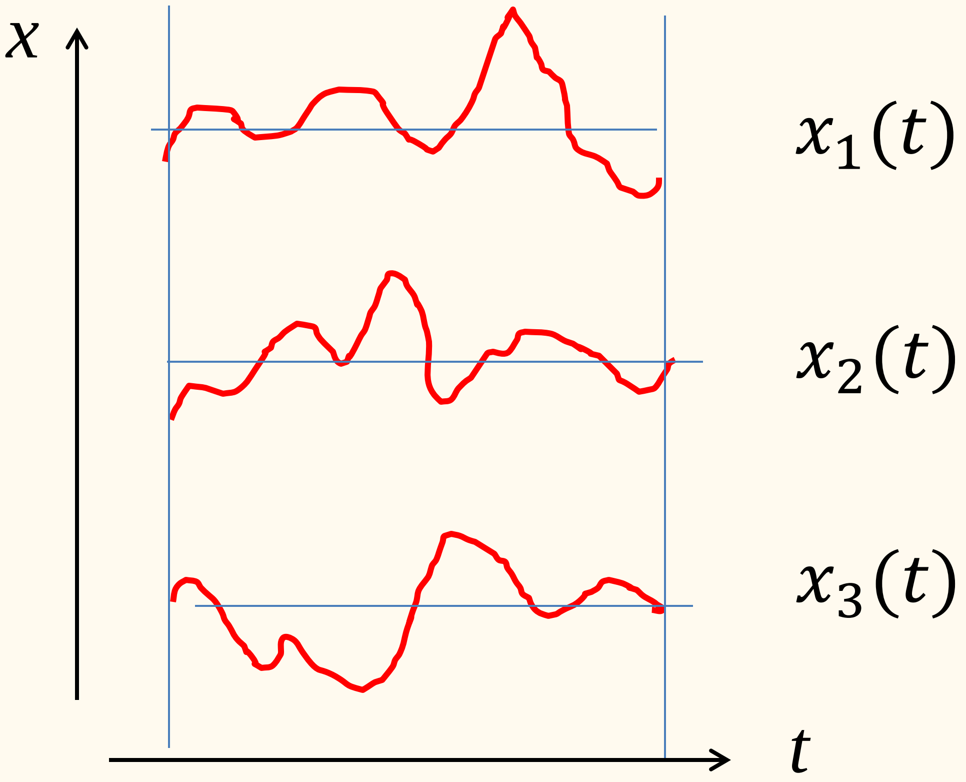
$$x_1, \cdots, x_m \xrightarrow{\text{affect}} x_{m+1}$$

m-th order Markovian

Time average of fluctuating variable: $\langle x \rangle$

Random process to distribution





The averaging interval should be longer than *m* in *m*-th order Markovian.

6.1.1 Fluctuation-Dissipation Theorem

久保亮五
Ryogo Kubo 1920-1995

Harry Nyquist
1889-1976

Nobert Wiener
1894-1964

Aleksandr Khinchin
1894-1959

Power Spectrum

Consider probability sets in the interval [0,T).

set index: j $$x_j(t) = \sum_{n=1}^{\infty}(a_{jn}\cos\omega_n t + b_{jn}\sin\omega_n t), \quad \omega_n = \frac{2n\pi}{T}$$

$$\mathscr{P}_{jn} = (a_{nj}\cos\omega_n t + b_{nj}\sin\omega_n t)^2 \qquad \text{(Power)}$$

$$\langle\mathscr{P}_n\rangle = \frac{1}{2}\langle a_n^2 + b_n^2\rangle$$ $\because$ cross product terms are averaged out

---

Random process:
Gaussian distribution in time

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{x^2}{2\sigma^2}\right]$$

$$\overline{(\delta x)^2} = \sigma^2, \quad \overline{\left(\sum_{j=1}^{m}\delta x_j\right)^2} = m\sigma^2$$

Then $\overline{\langle\mathscr{P}_n\rangle} = \sigma_n^2$ (non-Markovian)

Power Spectrum

Power spectrum $G(\omega)$

Frequency band width $\delta\omega$: separation between two adjacent frequencies

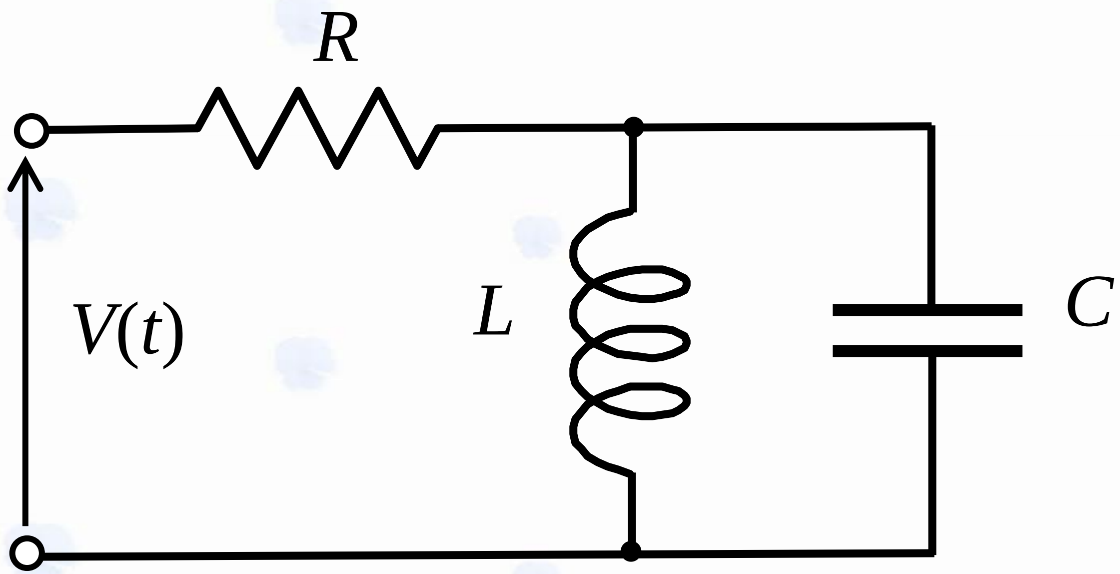
$$\delta\omega = \omega_{n+1} - \omega_n = \frac{2(n+1)\pi}{T} - \frac{2n\pi}{T} = \frac{2\pi}{T}$$

$$G(\omega_n)\frac{\delta\omega}{2\pi} = \overline{\langle \mathscr{P}_n \rangle} \;(= \sigma_n^2)$$

$$\overline{\langle x^2(t) \rangle} = \sum_{n=1}^{\infty} \overline{\langle \mathscr{P}_n \rangle} \quad (\overline{x(t)} = 0)$$

$$= \sum_n G(\omega_n)\frac{\delta\omega}{2\pi} \to \int_0^{\infty} G(\omega)\frac{d\omega}{2\pi}$$

6.1.1 Fluctuation-Dissipation Theorem



$$\omega_0 \equiv 1/\sqrt{LC}$$

$$Z(i\omega) = \frac{R(\omega_0^2 - \omega^2) + i\omega_0^2 \omega L}{\omega_0^2 - \omega^2},$$

$$Y(i\omega) = \frac{\omega_0^2 - \omega^2}{R(\omega_0^2 - \omega^2) + i\omega_0^2 \omega L}$$

Johnson-Nyquist noise

$V(t)$ noise power spectrum $\rightarrow G_v(\omega)$

$$G_v(\omega) = 4k_{\mathrm{B}}T\mathrm{Re}[Z(i\omega)]$$

$$G_v(\omega) = 4k_{\mathrm{B}}TR$$

Johnson-Nyquist noise
Thermal noise

White noise

One representation of the fluctuation-dissipation theorem

6.1.2 Wiener-Khintchine Theorem

Autocorrelation function $C(\tau) = \overline{\langle x(t)x(t+\tau)\rangle}$

$$= \overline{\sum_{n,m} \langle [a_n \cos\omega_n t + b_n \sin\omega_n t][a_m \cos\omega_m (t+\tau) + b_m \sin\omega_m (t+\tau)]\rangle}$$

$$= \frac{1}{2}\sum_n \overline{\langle a_n^2 + b_n^2\rangle} \cos\omega_n\tau = \sum_n \overline{\langle \mathscr{P}_n\rangle} \cos\omega_n\tau$$

$$= \int_0^\infty G(\omega)\cos\omega\tau \frac{d\omega}{2\pi}$$

$$C(\tau) = \int_0^\infty G(\omega)\cos\omega\tau \frac{d\omega}{2\pi}$$

$$G(\omega) = 4\int_0^\infty C(\tau)\cos\omega\tau\, d\tau$$

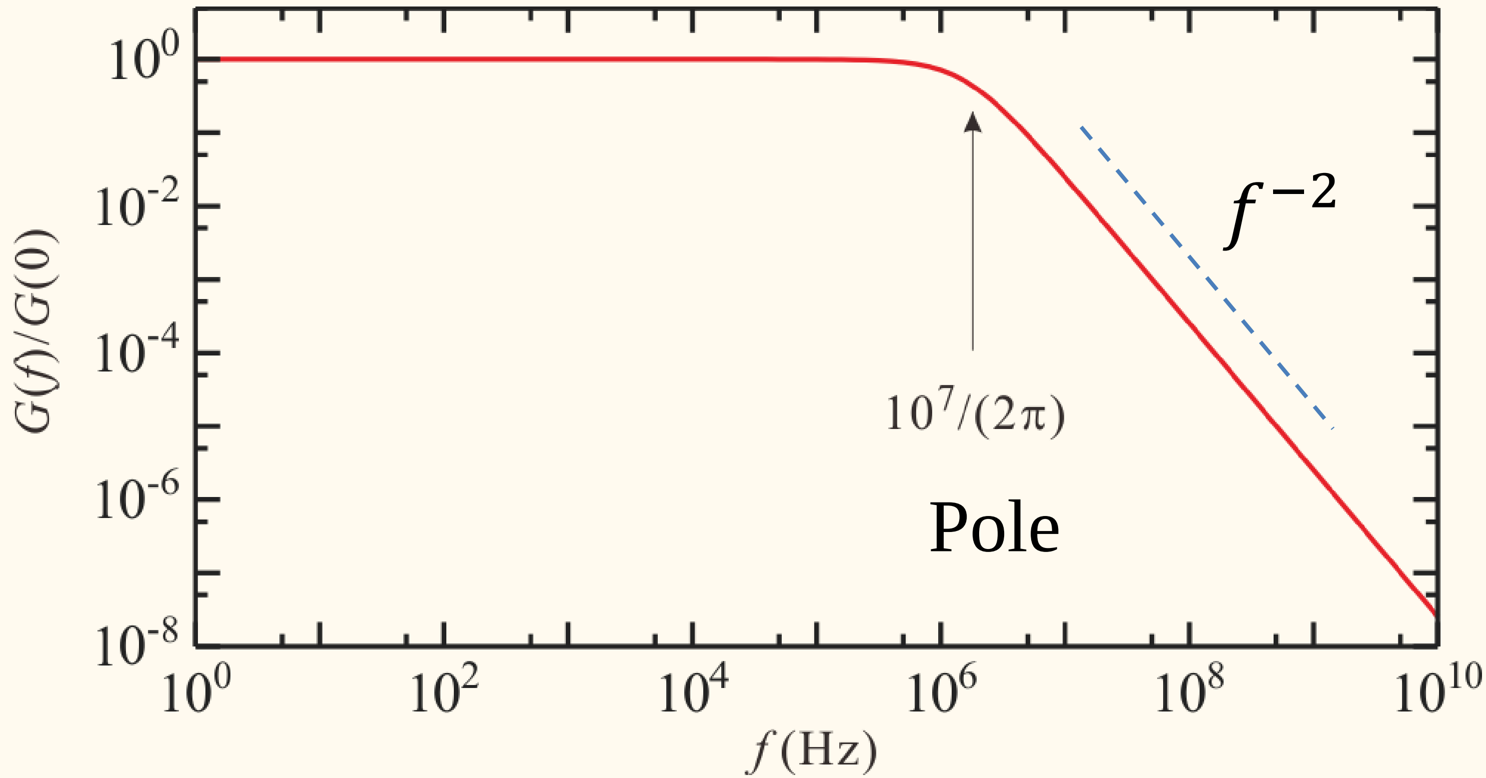
Wiener-Khintchine theorem

6.1.2 Wiener-Khintchine Theorem

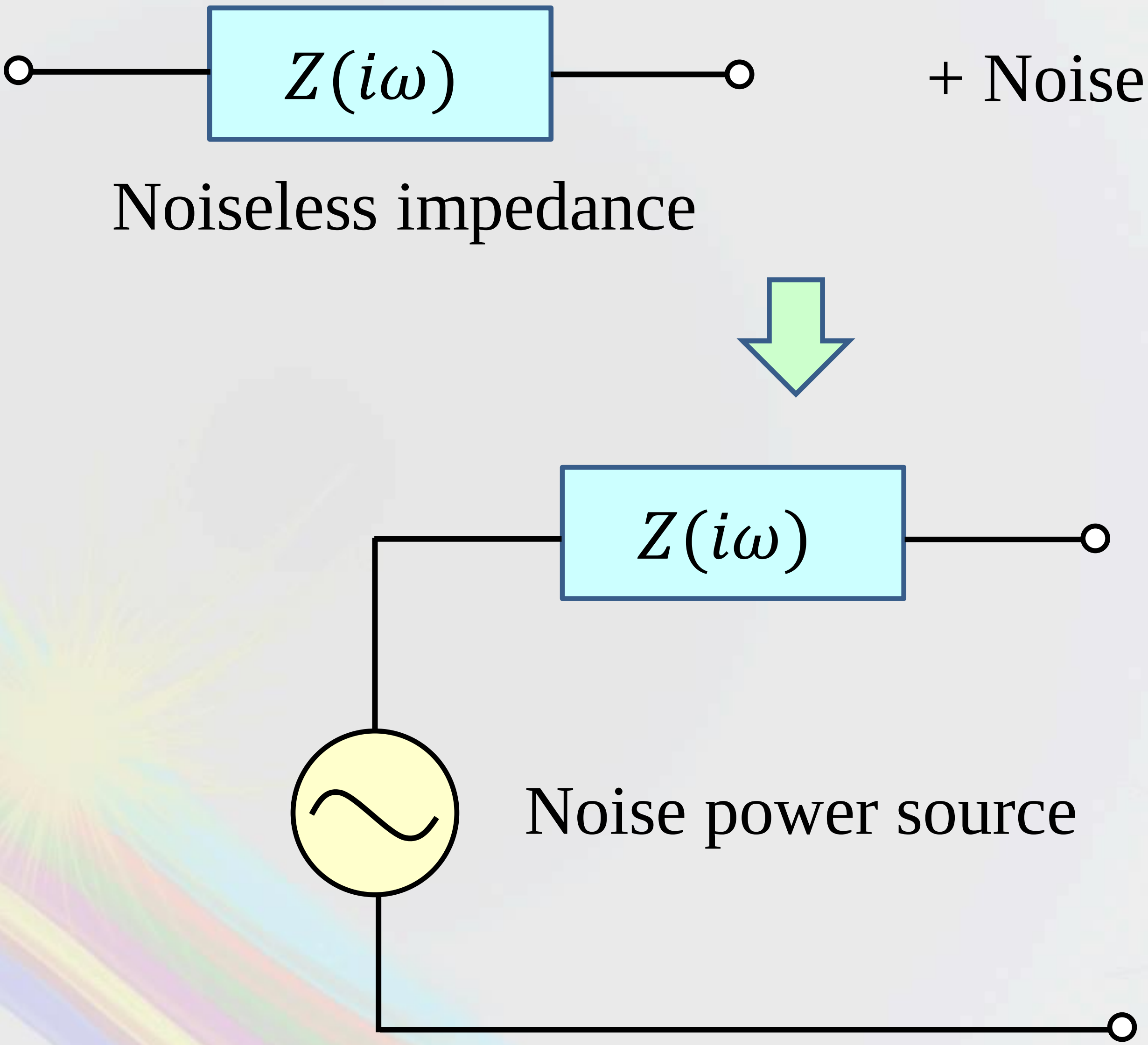
Example) $C(\tau) = \exp\left(-\frac{\tau}{\tau_0}\right)$

$$G(f) = 4\int_0^\infty e^{-\tau/\tau_0}\cos(2\pi f\tau)d\tau = \frac{4\tau_0}{1+(2\pi f\tau_0)^2}$$

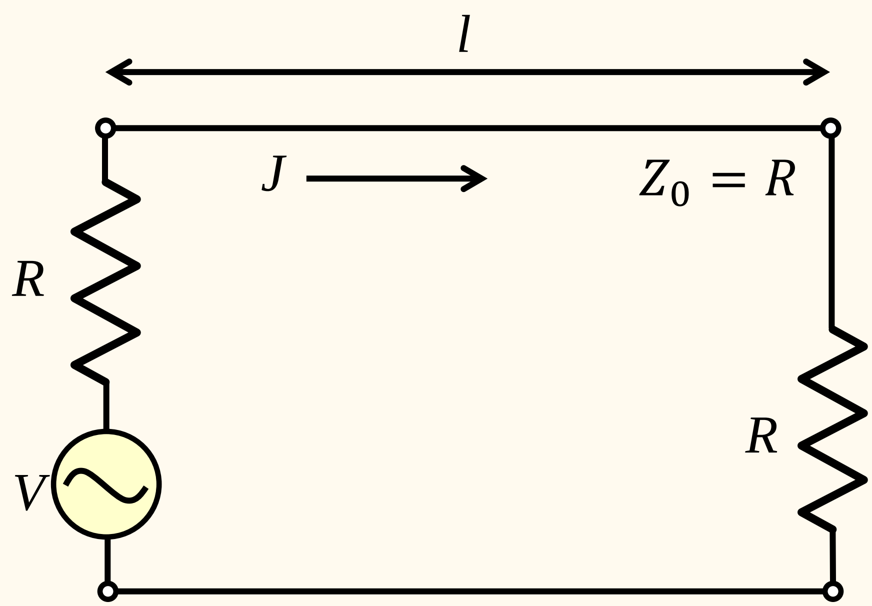
$\tau_0 = 10^{-7}$s (10MHz)



6.1.3 Electric circuit treatment of noise



6.1.4 Nyquist Theorem



Mode density on a transmission line with length *l*

$$\omega_n = c^* k_n = \frac{2\pi n c^*}{l} \qquad \therefore \quad \delta\omega = \frac{2\pi c^*}{l}$$

Bidirectional → Freedom × 2

Bose distribution $$f(\hbar\omega, T) = \frac{1}{\exp(\hbar\omega/k_{\mathrm{B}}T) - 1}$$

6.1.4 Nyquist Theorem

Thermal energy density per freedom

$$\frac{\hbar\omega}{\exp(\hbar\omega/k_\mathrm{B}T)-1} \sim \frac{\hbar\omega}{1+(\hbar\omega/k_\mathrm{B}T)-1} = k_\mathrm{B}T \quad (k_\mathrm{B}T \gg \hbar\omega)$$

Thermal energy density in band $\Delta\omega$

$$2\frac{\Delta\omega}{\delta\omega}k_\mathrm{B}T = \frac{2k_\mathrm{B}Tl}{2\pi c^*}\Delta\omega$$, a half of which flows in one-direction

Energy flowing out from the end:

$$\frac{k_\mathrm{B}Tl}{2\pi c^*}\Delta\omega \times \frac{1}{l} \times c^* = k_\mathrm{B}T\Delta f \quad (2\pi f = \omega)$$

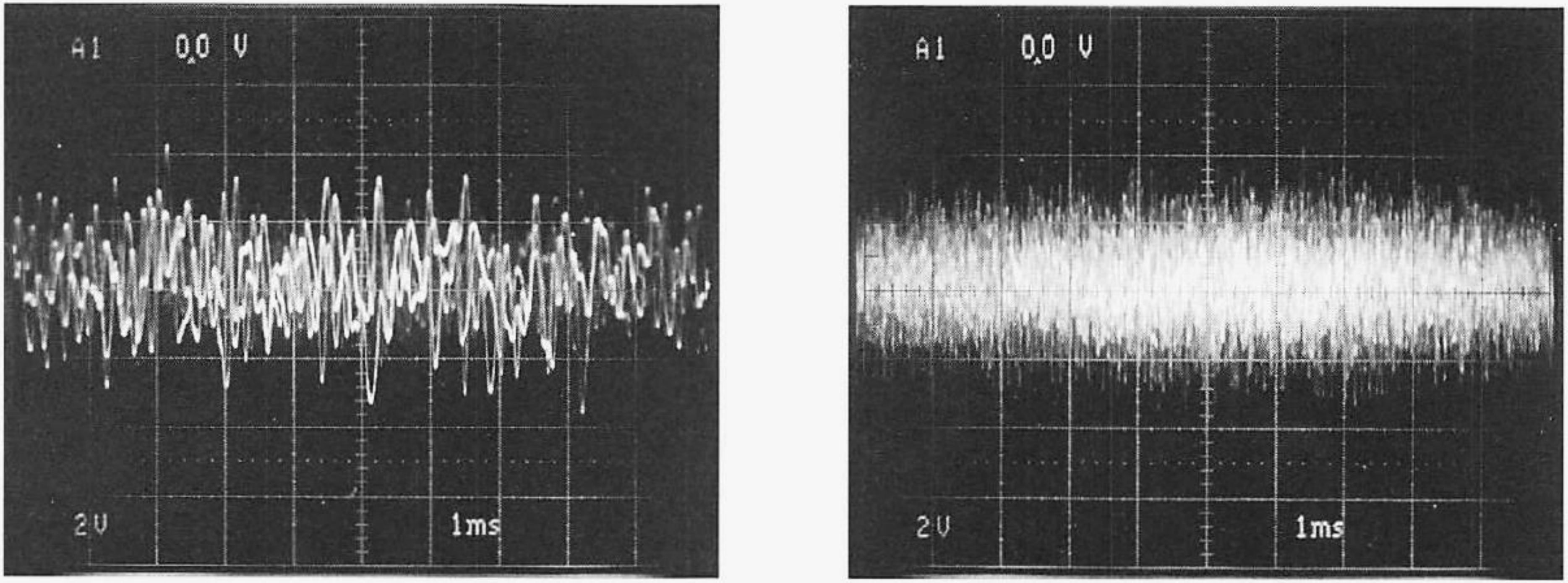
equals the energy supplied from the noise source.

$$\overline{J^2}R = k_\mathrm{B}T\Delta f, \quad \overline{V^2} = 4Rk_\mathrm{B}T\Delta f \quad (V = 2RJ)$$

$$\sqrt{\overline{J^2V^2}} = 2k_\mathrm{B}T\Delta f$$

→ Noise Temperature

Thermal noise



(a) 上限周波数5 kHz (−3dB) 1 V_{rms}の熱雑音を1 ms/divで観測

(b) 上限周波数100 kHz (−3dB)1 V_{rms}の熱雑音を1 ms/divで観測

〈写真 1-1〉 熱雑音の測定

6.1.5 Shot Noise

Single Electron

Time domain: δ-function approximation

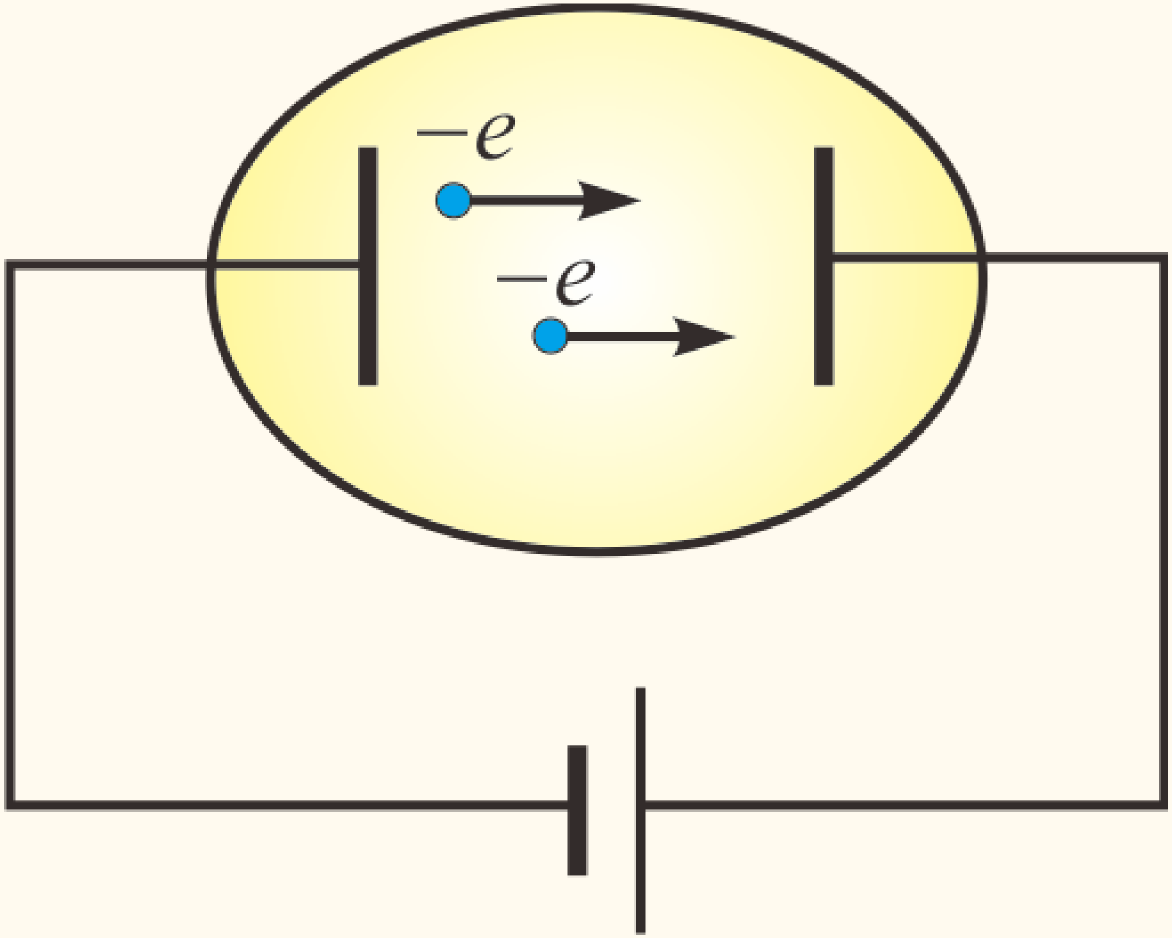
$$J_e(t) = e\delta(t - t_0)$$
$$= e\int_{-\infty}^{\infty} e^{2\pi i f(t-t_0)}df = 2e\int_0^{\infty} \cos\left[2\pi f(t - t_0)\right]df$$

Uniform 2e in frequency domain: fluctuation at each frequency
Coherent only at $t = t_0$

Current fluctuation density for infinitesimal band df

$$\delta J = d\sqrt{\langle J_e^2\rangle} = \frac{2e}{\sqrt{2}}df = \sqrt{2}edf$$

6.1.5 Shot Noise



6.1.5 Shot Noise

Double Electron

$$\overline{\langle \delta j^2 \rangle} = (j_p + j_q e^{i\phi})(j_p + j_q e^{-i\phi}) = j_p^2 + j_q^2 + 2 j_p j_q \cos\phi$$

ϕ: coherent phase shift → averaged out

$$\overline{\langle \delta j^2 \rangle} = j_p^2 + j_q^2 = 2 \times (\sqrt{2} e)^2 df$$

N-Electron

$$\overline{\langle \delta J^2 \rangle} = N \times 2e^2 df = 2e\overline{J} df \quad (\overline{J} = eN)$$

Quantum mechanical correlation → Modification from random

6.1.5 Shot Noise

Example: pn junction

Current-Voltage characteristics: $J(V) = J_0 \left[\exp\left(\frac{eV}{k_B T} \right) - 1 \right]$

Differential resistance $r_d = \left(\frac{dJ}{dV} \right)^{-1} = \left[\frac{eJ_0}{k_B T} \exp\left(\frac{eV}{k_B T} \right) \right]^{-1} = \frac{k_B T}{e} \frac{1}{J + J_0}$

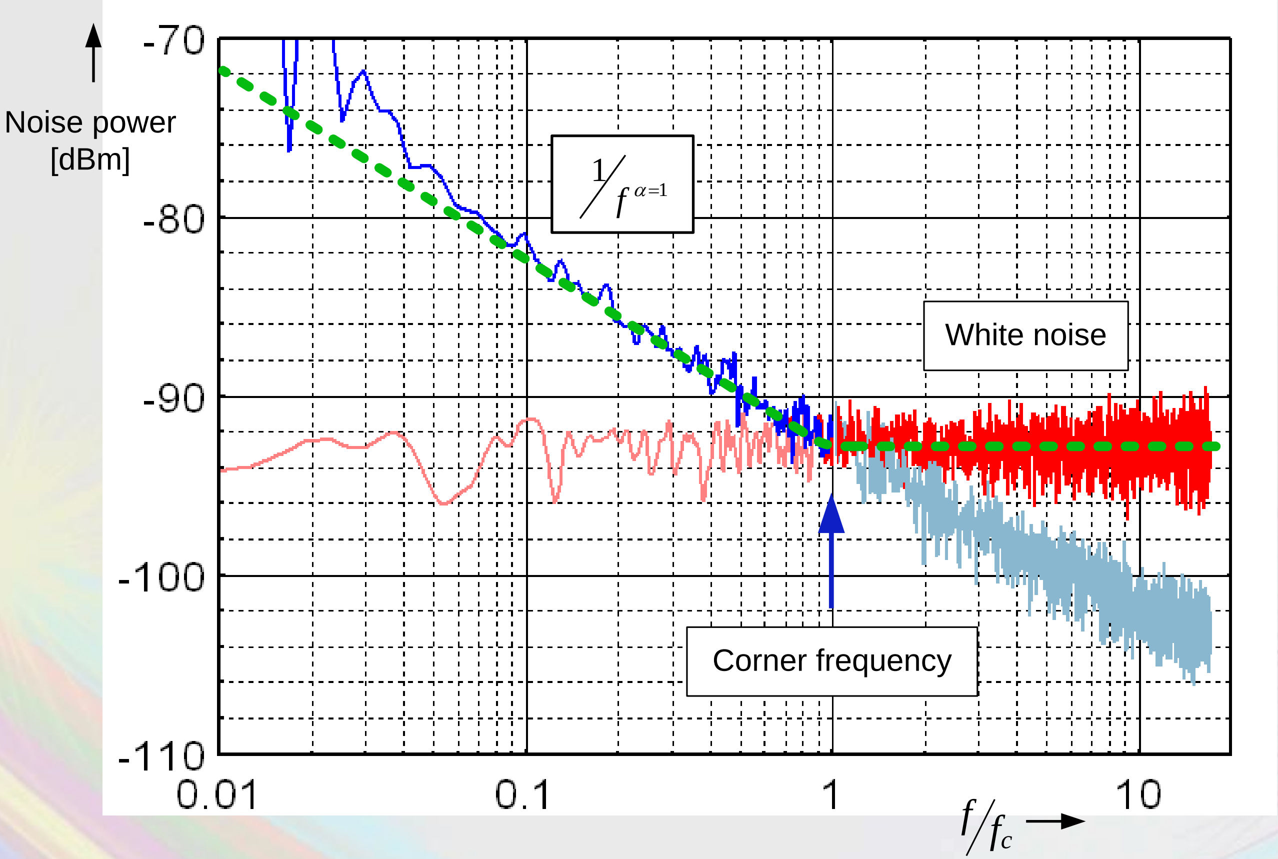
$$J \gg J_0 \;\rightarrow\; r_d \sim k_B T / eJ$$

$$\overline{\langle (\delta J)^2 \rangle} = 2e \frac{k_B T}{e r_d} df = 4 k_B T \frac{1}{2 r_d} df$$

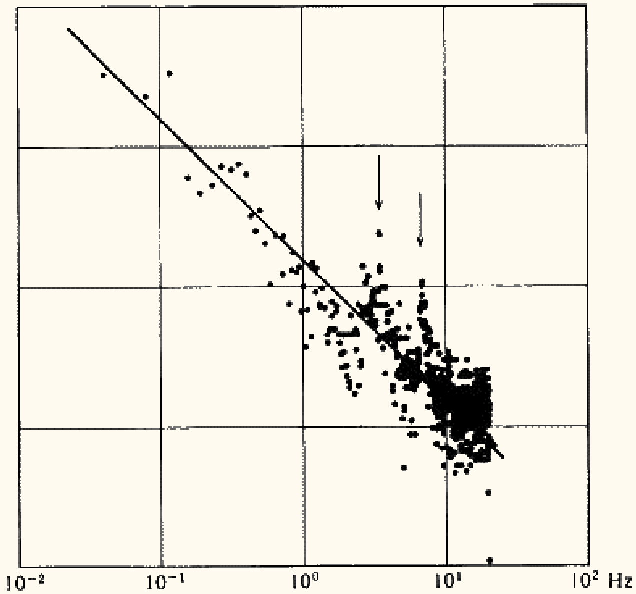
$$(\delta V)^2 = 4 \frac{r_d}{2} k_B T \Delta f$$

6.1.6 1/f noise

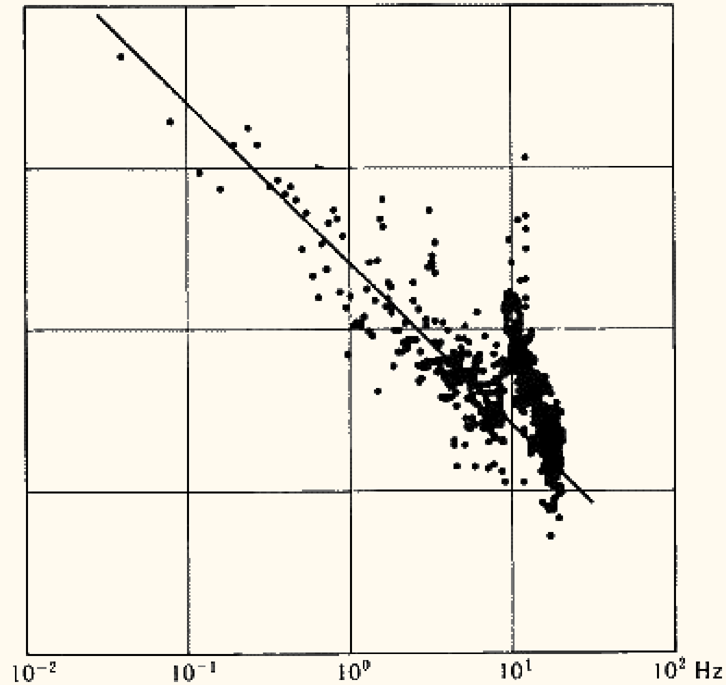
$$(\delta V)^2 = K J^a R^2 \frac{\Delta f}{f^{\alpha}}$$



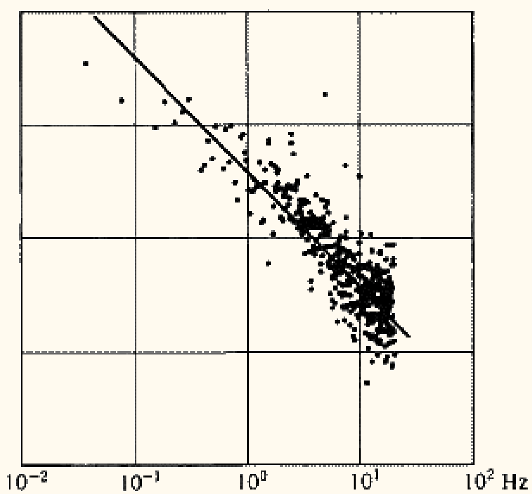
6.1.6 1/f noise



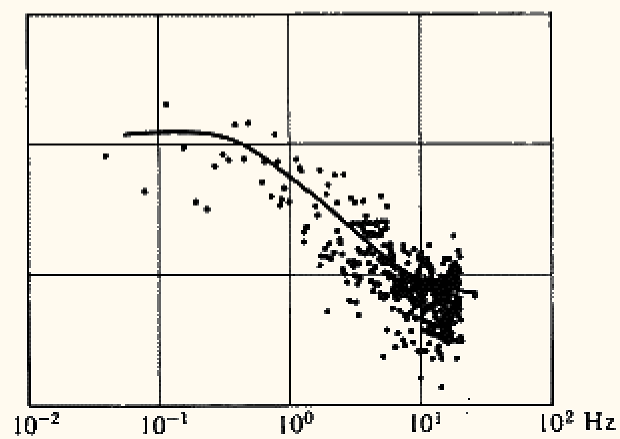
J. S. Bach, Brandenburg Concerto No.1



A. Vivaldi, Four Seasons, Spring



Kawai Naoko, Smile for me



S. Sato, Keshin (incarnation) II

“Unit” of Noise

Noise: Power spectrum per frequency

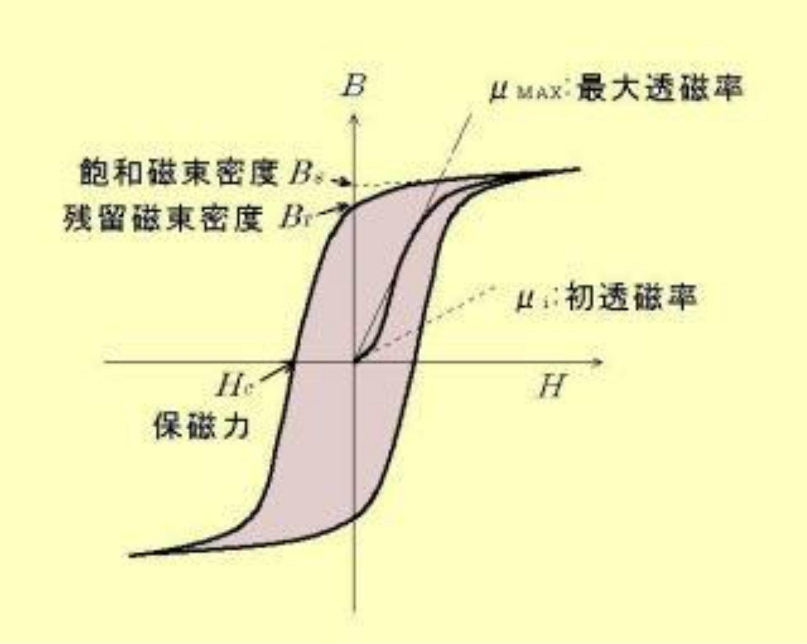
$$\overline{j_n^2} = \overline{\delta J^2}/\Delta f, \quad \overline{e_n^2} = \overline{\delta V^2}/\Delta f$$

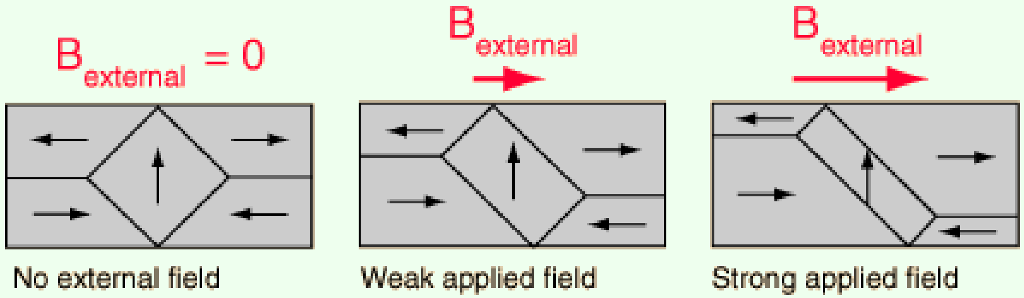
↓

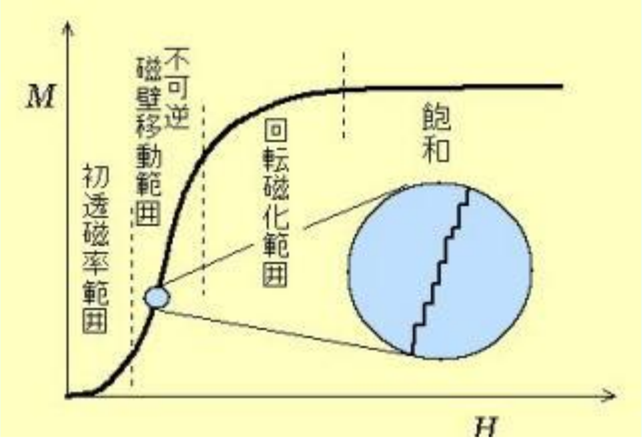
unit of $\sqrt{\overline{j_n^2}}, \quad \sqrt{\overline{e_n^2}}$

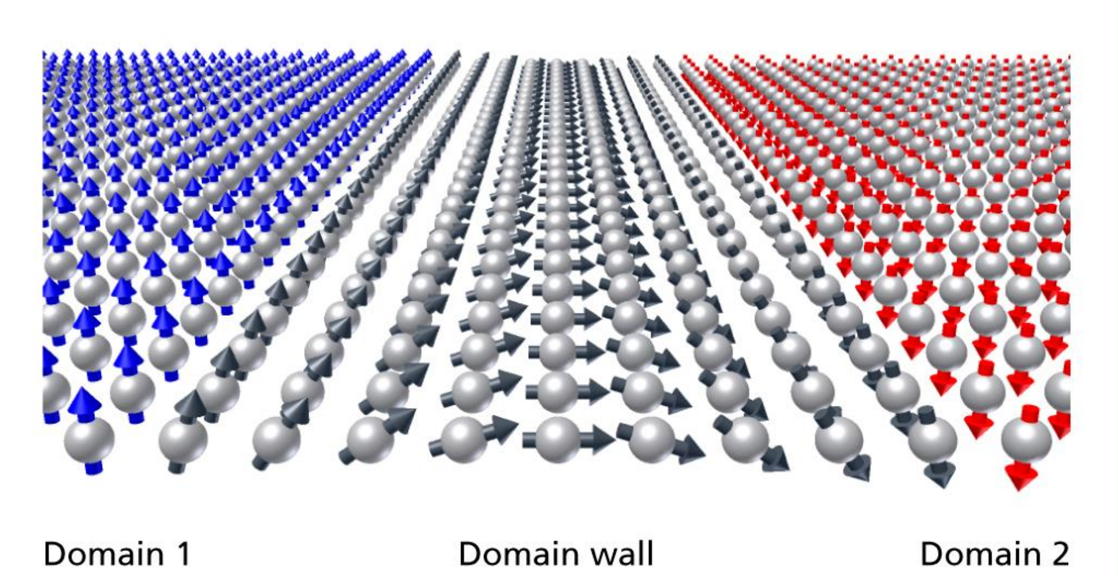
$$\mathrm{A}/\sqrt{\mathrm{Hz}}, \quad \mathrm{V}/\sqrt{\mathrm{Hz}}$$

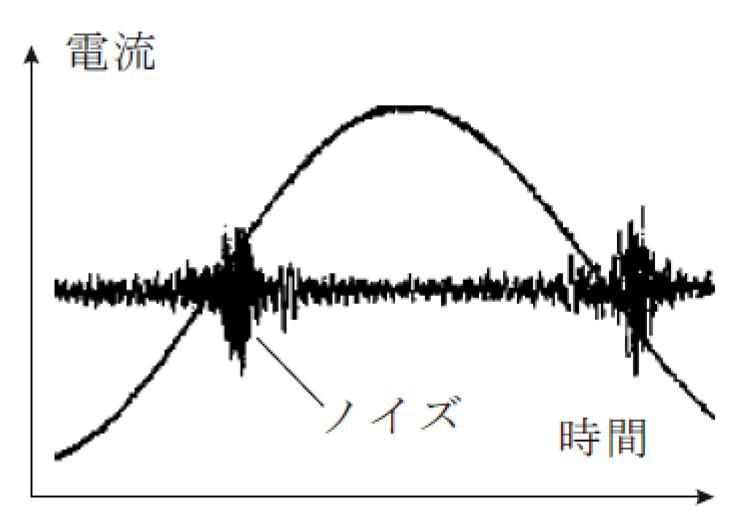
Other noises: Barkhausen noise





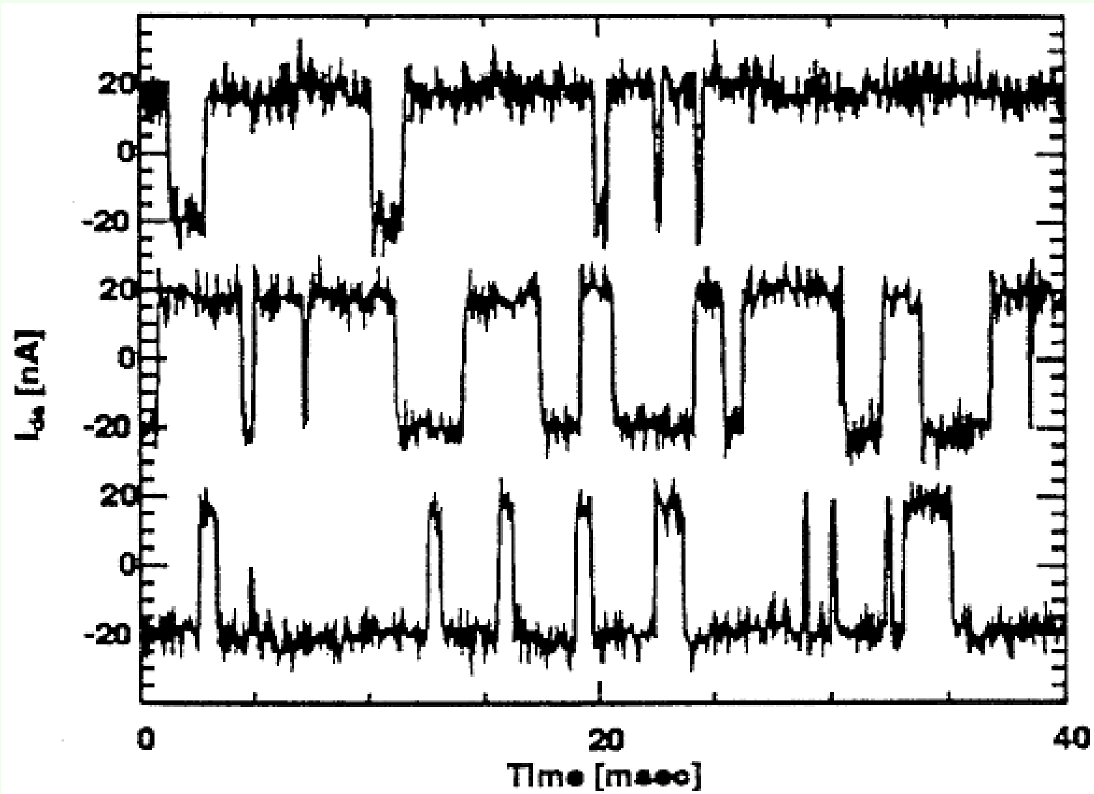






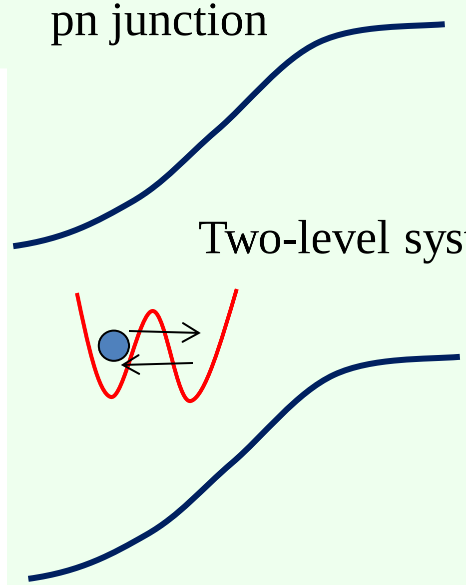
Popcorn noise

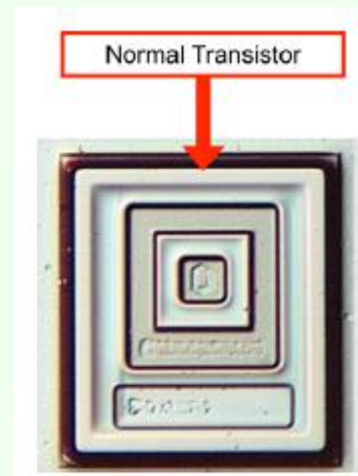
Popcorn noise, Burst noise

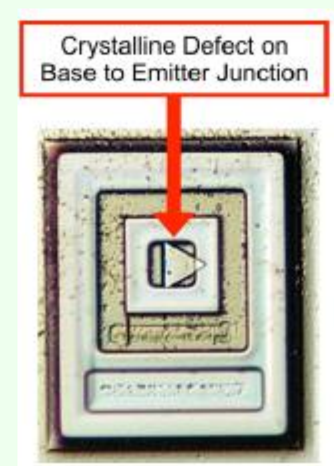


pn junction

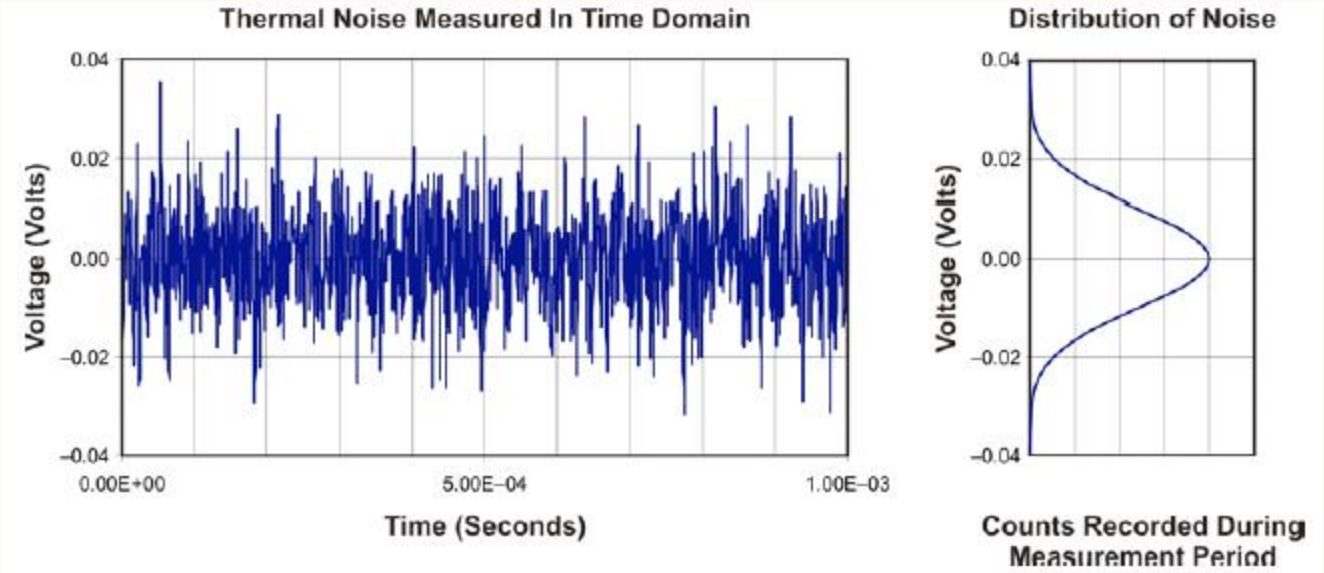
Two-level system

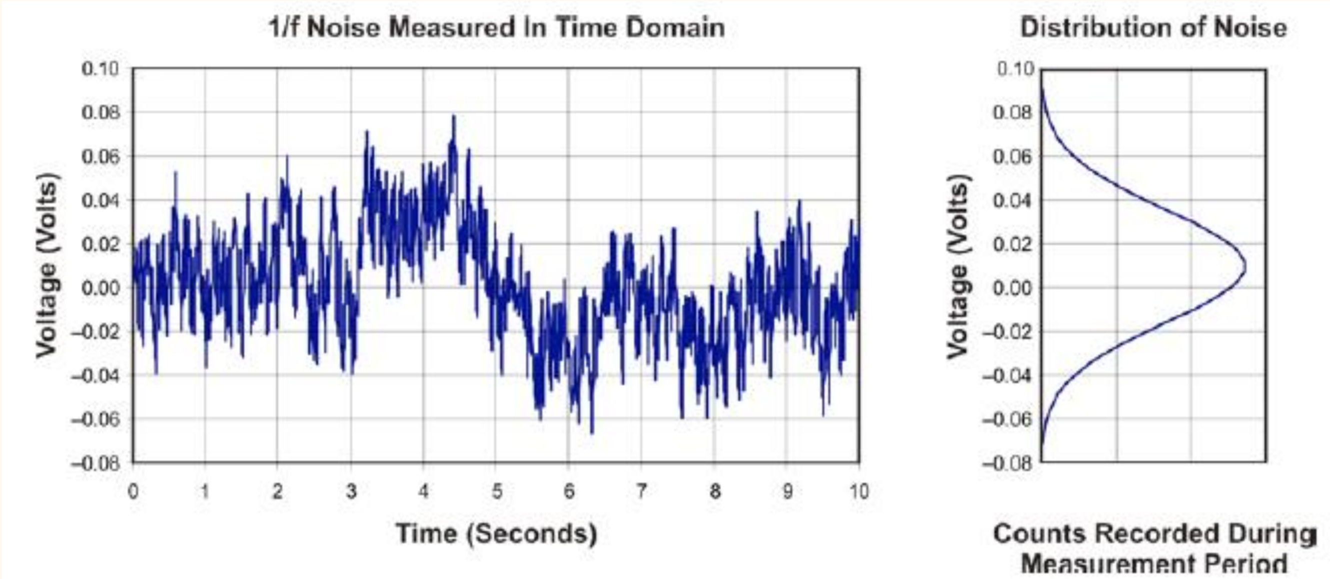




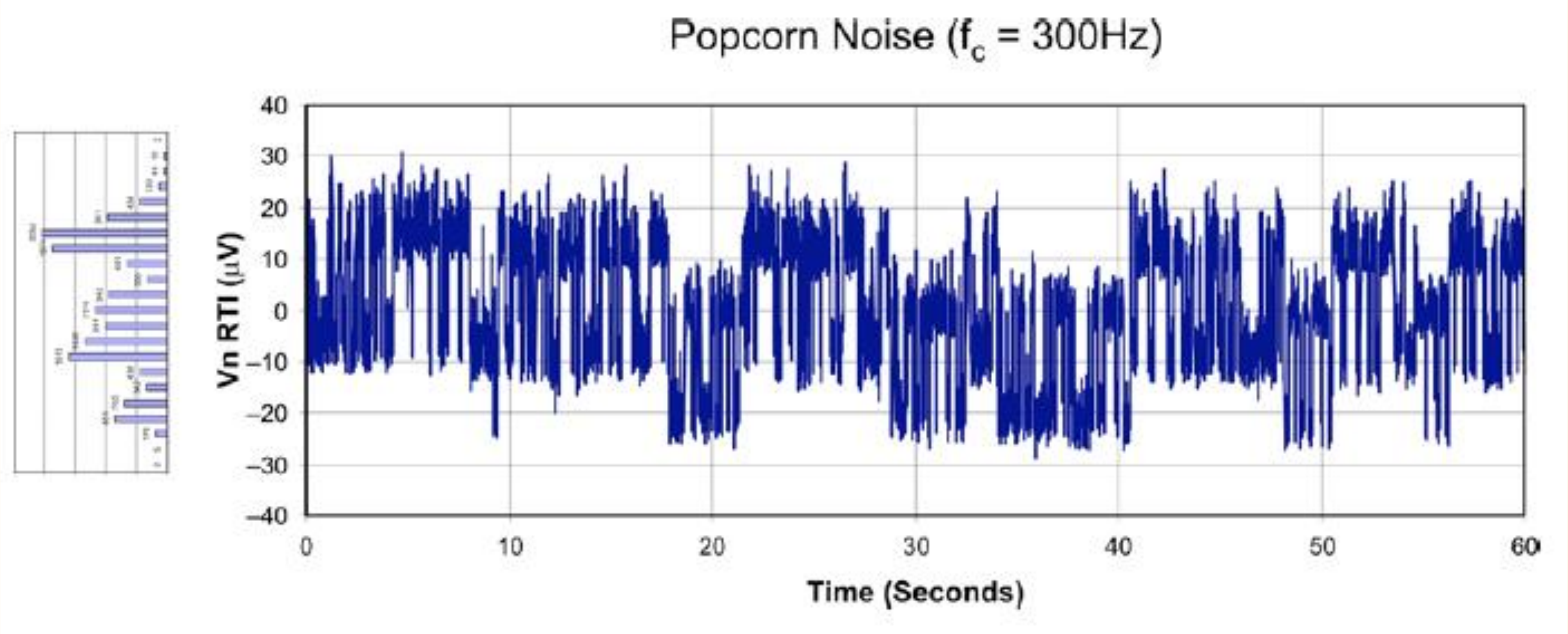


Amplitude distributions of random-type noises

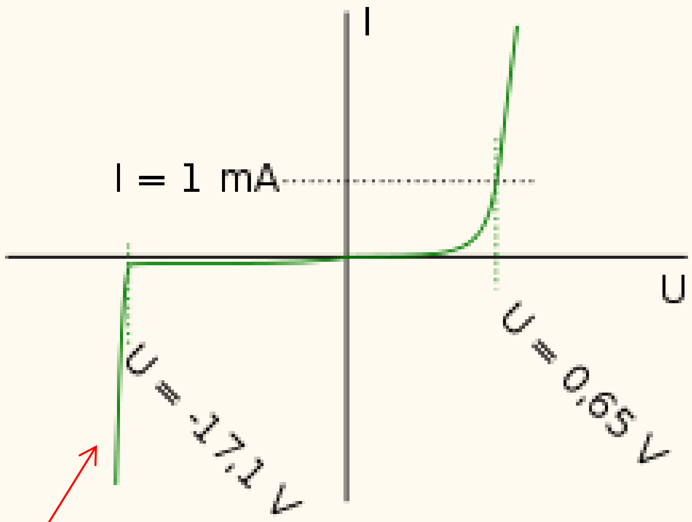




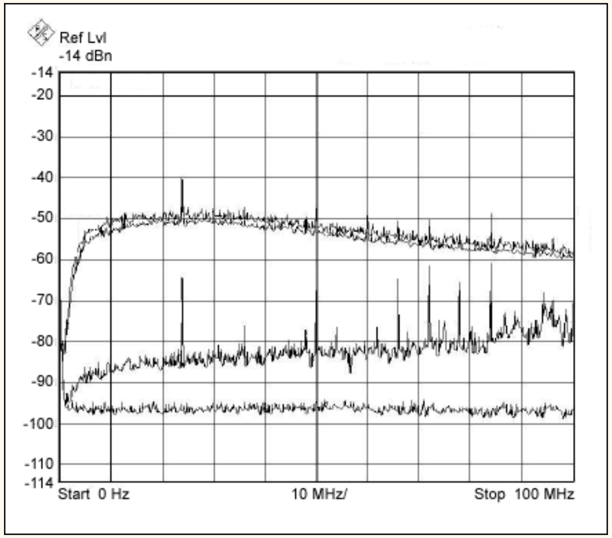
Amplitude distribution of popcorn noise



Avalanche noise



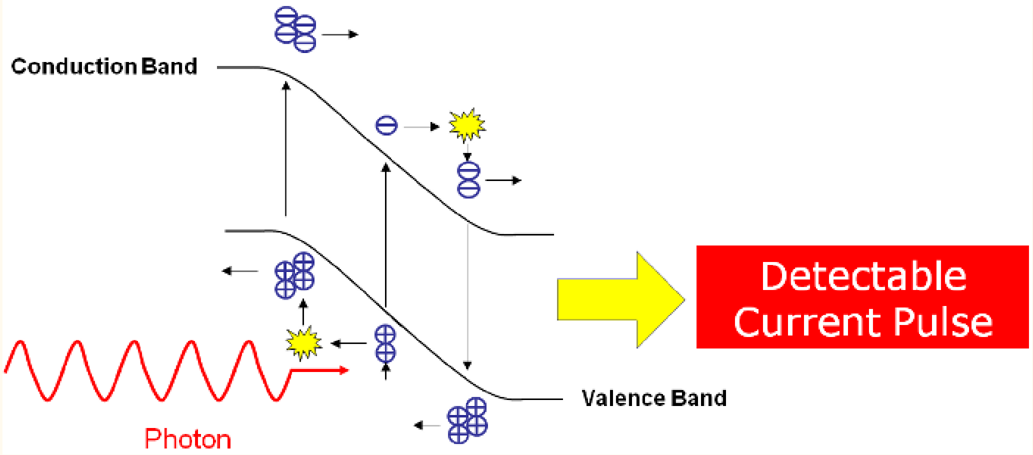
avalanche or Zener breakdown



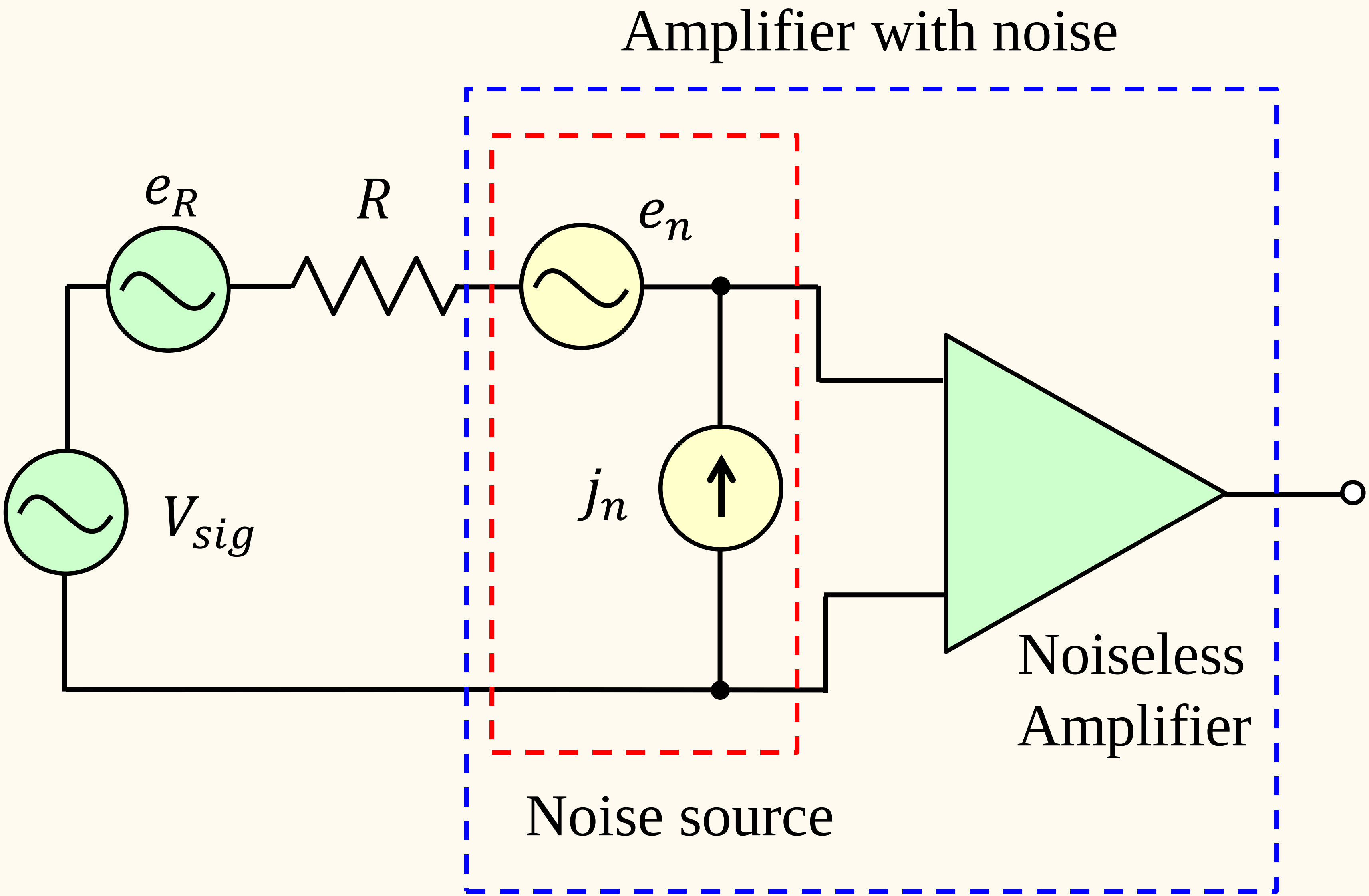
white noise

Avalanche Photo-Diode (APD)

Zener voltage standard diode



6.2 Noises from Amplifiers



6.2 Noises from Amplifiers

Amplifiers: the elements have characteristic noises,
power sources work as noise sources

→ Noiseless amplifier + Noise source = Amplifier with noise

Power gain G_p

$$e_{\mathrm{intotal}}^2 = j_n^2 R^2 + e_{\mathrm{R}}^2 + e_n^2 = e_{\mathrm{out}}^2 / G_{\mathrm{p}}$$

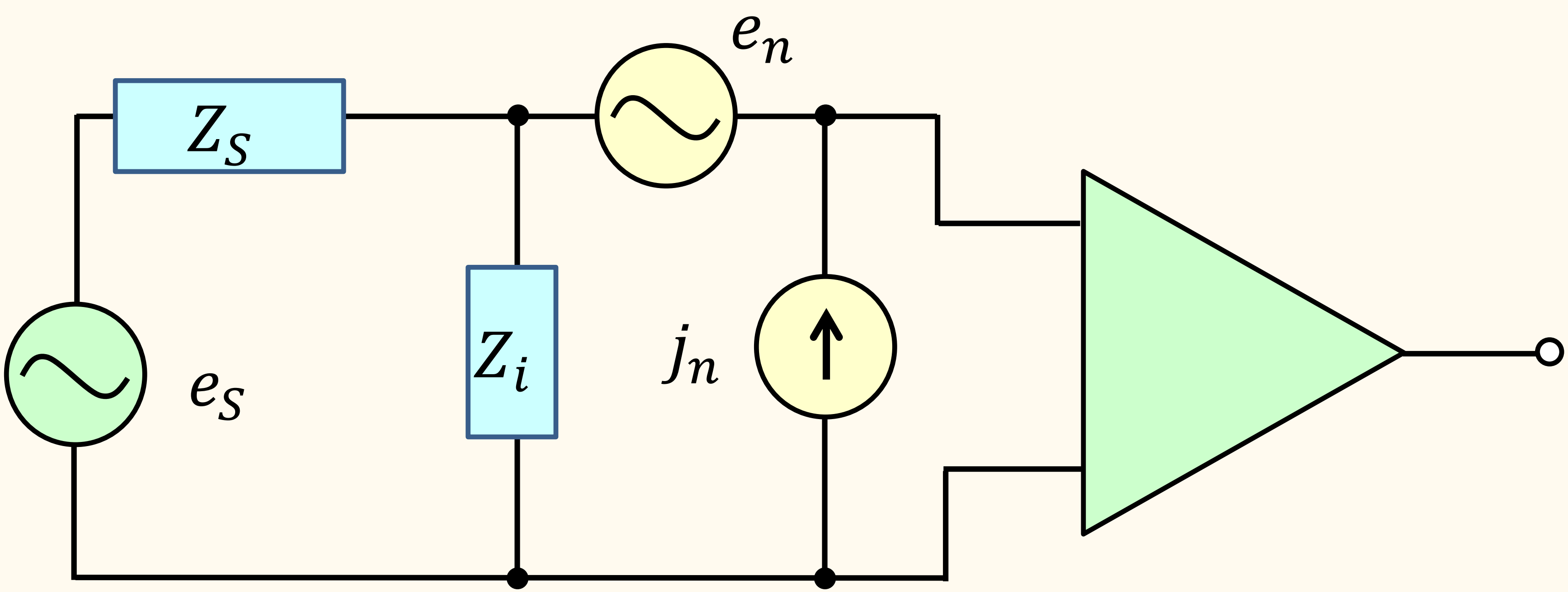
Signal to noise ratio: S/N ratio

Noise Figure: $$\mathrm{NF} = 10\log_{10}\frac{(S/N)_{\mathrm{in}}}{(S/N)_{\mathrm{out}}} = 10\log_{10}\frac{S_{\mathrm{in}}N_{\mathrm{out}}}{S_{\mathrm{out}}N_{\mathrm{in}}}$$

$$N_{\mathrm{out}} = G_{\mathrm{p}}\overline{e_{\mathrm{N}}^2}$$

$$\mathrm{NF} = 10\log_{10}\frac{S_{\mathrm{in}}G_{\mathrm{p}}\overline{e_{\mathrm{N}}^2}}{S_{\mathrm{in}}G_{\mathrm{p}}\overline{e_{\mathrm{R}}^2}} = 10\log_{10}\frac{\overline{e_{\mathrm{N}}^2}}{\overline{e_{\mathrm{R}}^2}} = 10\log_{10}\frac{\overline{e_n^2} + \overline{e_{\mathrm{R}}^2} + \overline{j_n^2}R^2}{\overline{e_{\mathrm{R}}^2}}$$

6.2.2 Noise impedance matching



6.2.2 Noise impedance matching

Noise temperature and matched source impedance

$$T_{\rm a} = \frac{\sqrt{\overline{e_n^2}\ \overline{j_n^2}}}{2k_{\rm B}}, \qquad R_{\rm bs} = \sqrt{\frac{\overline{e_n^2}}{\overline{j_n^2}}}$$

Output noise temperature:

$$T_{\rm n} = \left(1 + \frac{{\rm Re}(1/Z_{\rm i})}{{\rm Re}(1/Z_{\rm s})}\right) \frac{T_{\rm a}}{2{\rm Re}Z} \left(\frac{|Z|^2}{R_{\rm bs}} + R_{\rm bs}\right), \qquad \frac{1}{Z} \equiv \frac{1}{Z_{\rm i}} + \frac{1}{Z_{\rm s}}$$

Minimize T_n: $\quad Z_{\rm i} = \dfrac{1}{R_{\rm bs}^{-1} - Z_{\rm s}^{-1}}$ Noise matching condition

$$T_{\rm n} = \left(1 + \frac{{\rm Re}(1/Z_{\rm i})}{{\rm Re}(1/Z_{\rm s})}\right) T_{\rm a}$$

References

C. Kittel, "Elementary Statistical Physics", (Dover, 2004).

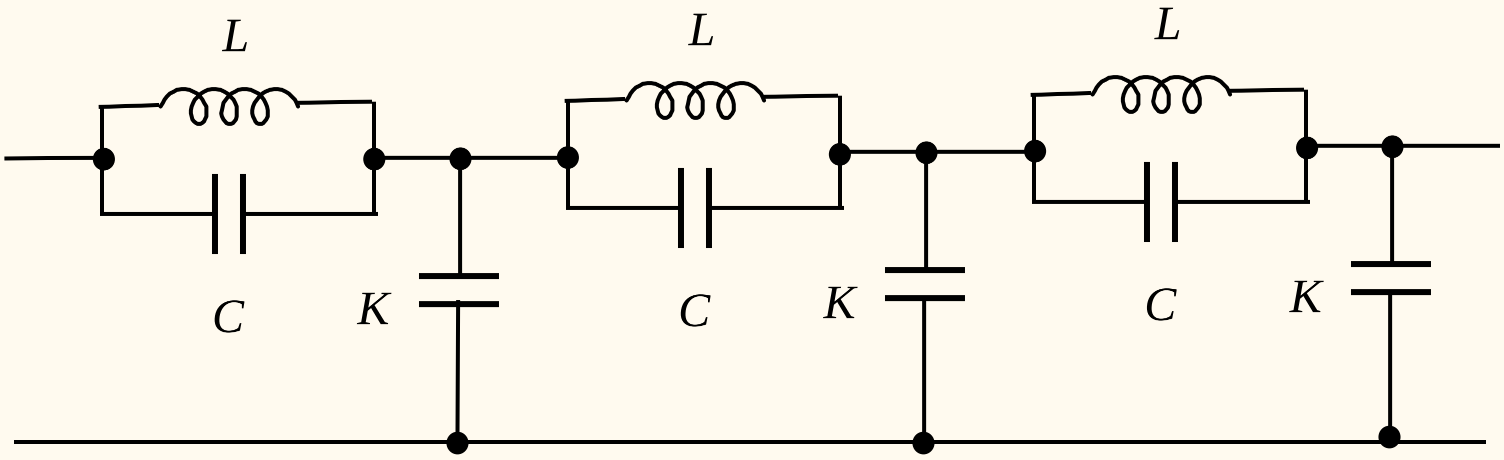
遠坂俊昭「計測のためのアナログ回路設計」(CQ出版社，1997).

Anton F. P. van Putten, "Electronic Measurement Systems", (IOP pub., 1996).

寺本英，広田良吾，武者利光，山口昌哉 「無限・カオス・ゆらぎ」(培風館，1985).

Exercise E-1

Obtain the dispersion relation in the following transmission line.



Exercise E-2

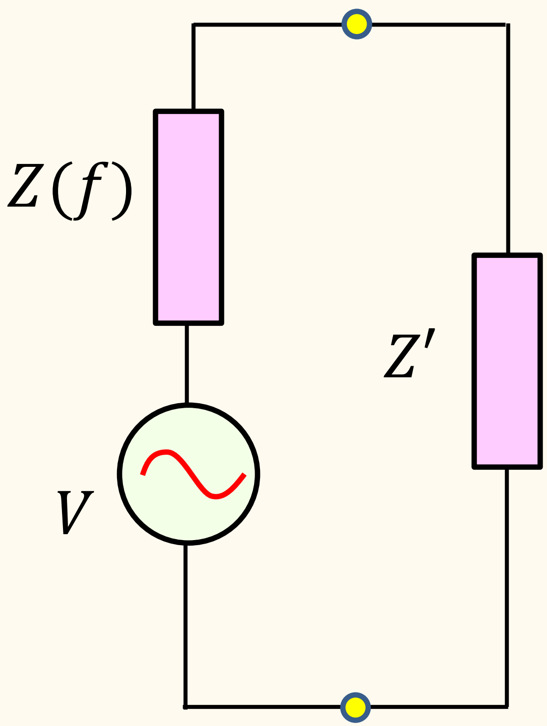
Show that the power spectrum $G(f)$ of voltage noise across the impedance

$$Z(f) = R(f) + iY(f)$$

is given as

$$G(f) = 4R(f)k_{\mathrm{B}}T.$$

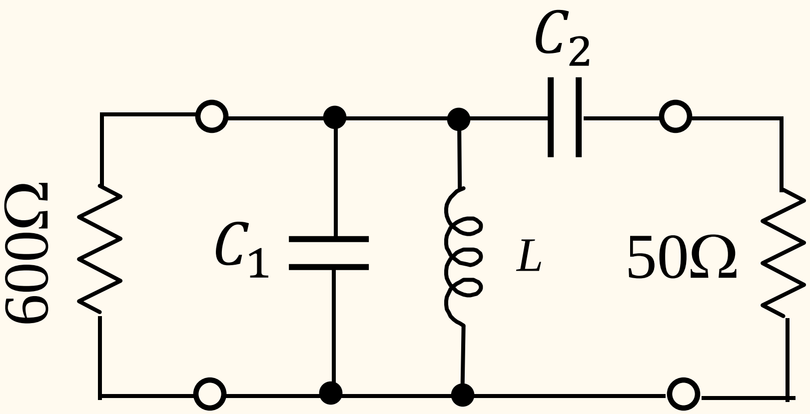
Assume that thermal noise energy per unit time is $k_B T\Delta f$.



(hint) From the above assumption we can skip the discussion on the mode energy in transmission line. Instead consider the case in the left figure, in which Z' is matched to Z as

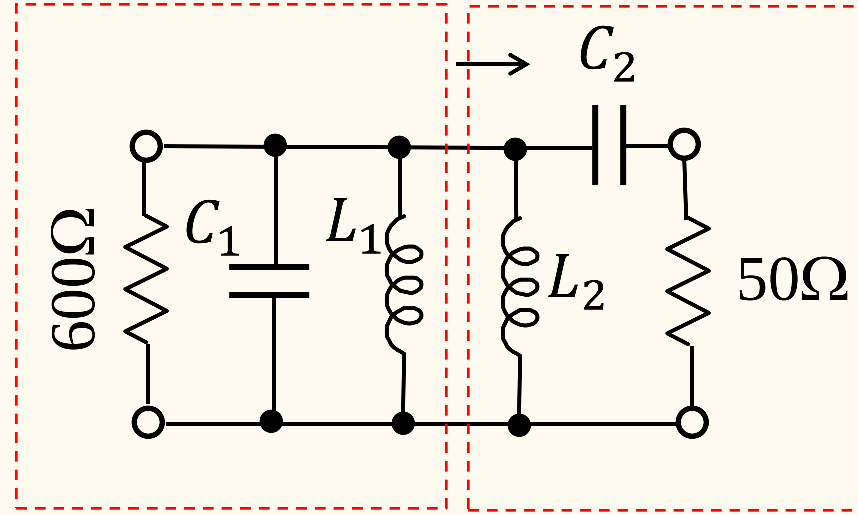
$$Z'(f) = Z^*(f) = R(f) - iY(f)$$

Exercise E-3



(a)

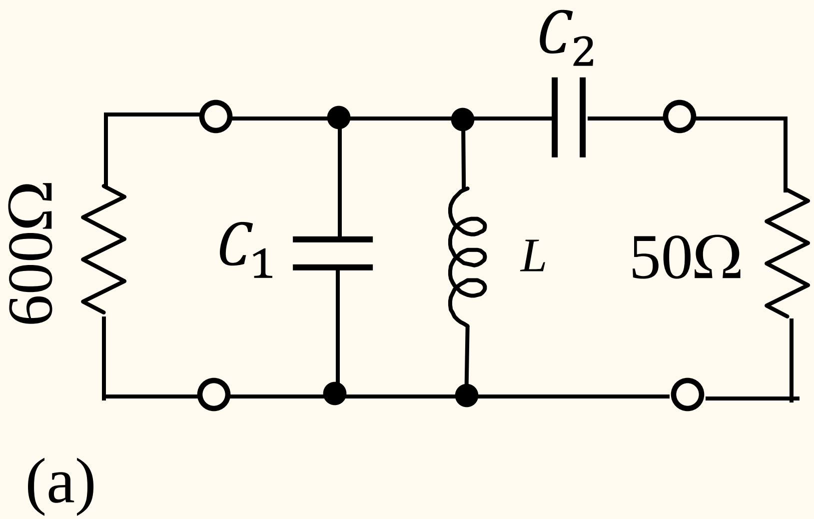
(b)

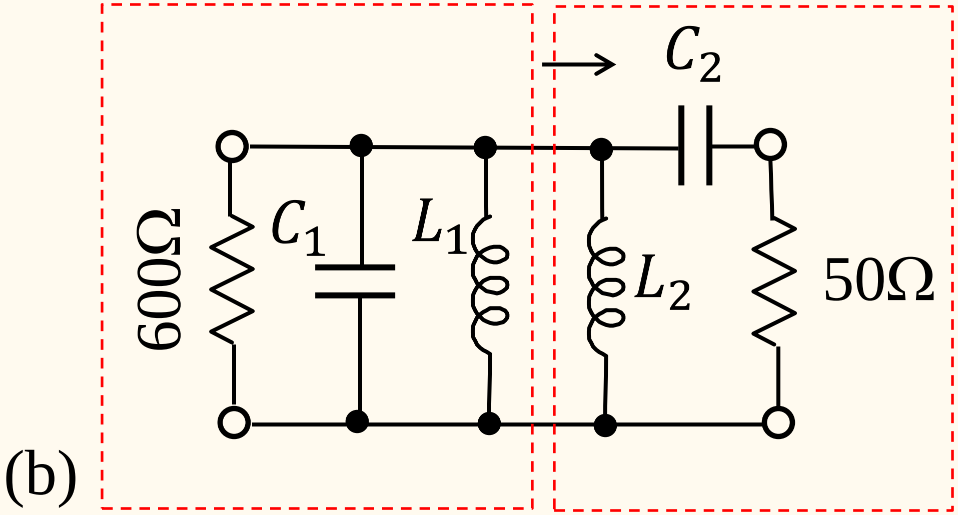


A preamplifier with FETs for an FM receiver has the output impedance of 600Ω. The FM receiver has the input impedance of 50Ω and we need to make impedance matching. The central frequency is 85MHz, the effective with of amplification is 10MHz. Obtain C_1, C_2, L in the matching circuit with 3 digits significant figures.

(hint) Express L with a parallel of L_1 and L_2 as shown in (b). The left resonance circuit should be tuned to 85MHz, width10MHz. Then the left and the right circuit should be impedance matched.

Exercise E-3





FM受信機のプリアンプをFETで作ったところ，出力インピーダンスが600Ωになった．受信機の入力インピーダンスは50Ωなので，インピーダンスマッチを取る必要がある．中心周波数を85MHz，有効周波数幅を10MHz，として(a)のような回路でマッチを取ると，回路定数C_1, C_2, Lはどうなるか．有効数字3桁で答えよ．

(ヒント) (b)のようにインダクタンスを２つに分割し，左の共鳴回路で85MHz，10MHz幅に同調させる．この後，左右のインピーダンスが一致するように定数を求める．

電子回路論第11回
Electric Circuits for Physicists

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Outline

6.2 Noise from amplifiers

6.2.1 Noise figure

6.2.2 Noise impedance matching

6.3 Modulation and signal transfer

6.3.1 Modulation/demodulation

6.3.2 Amplitude modulation

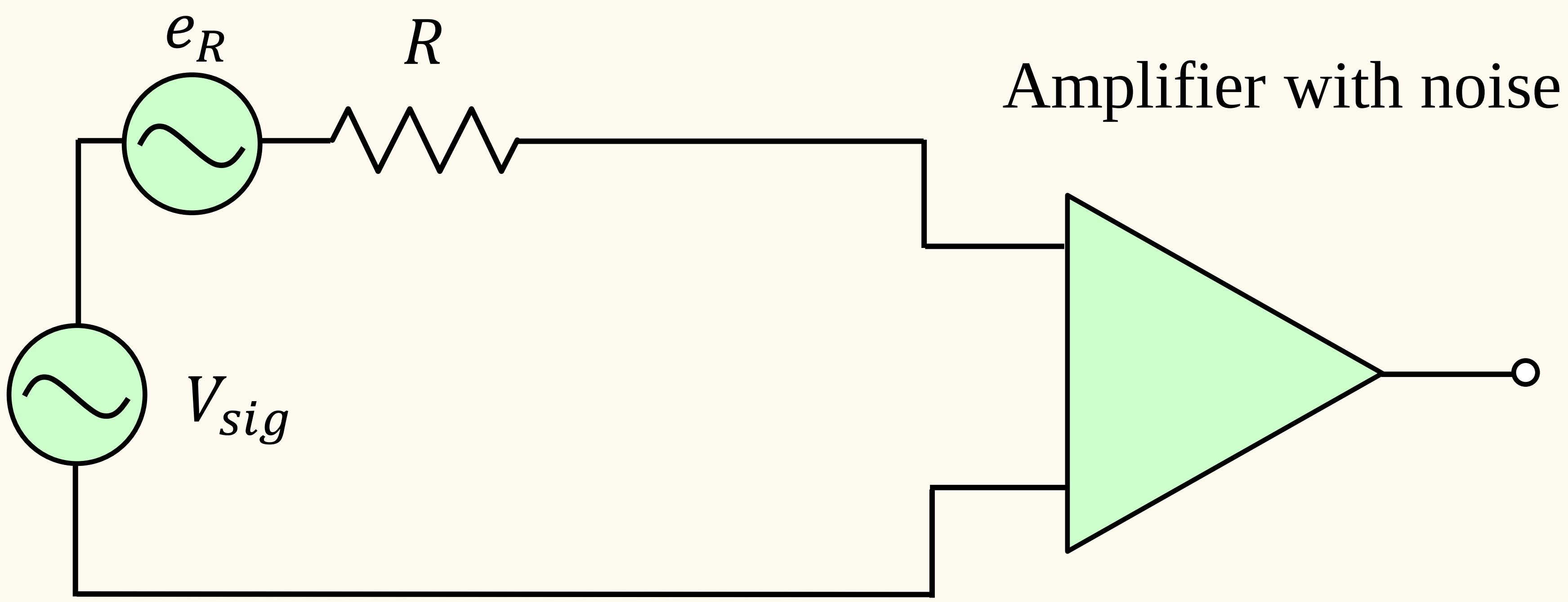
6.3.3 Angle modulation

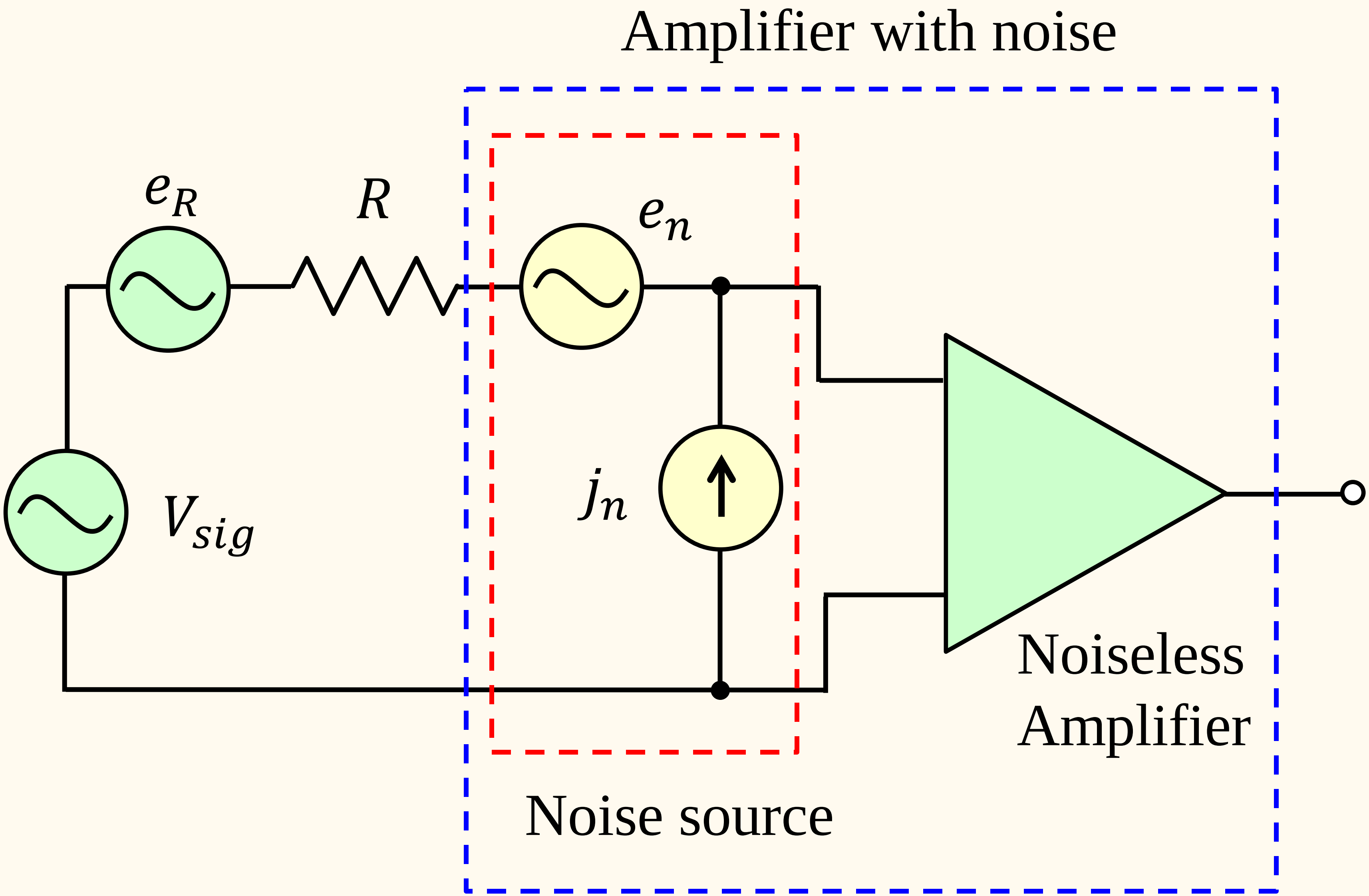
6.3.4 Demodulation of frequency modulated signal

6.3.5 Modulation and noise

FM broadcast test

6.2 Noise from Amplifiers




Amplifier with noise
e_R
R
e_n
j_n
V_{sig}
Noiseless
Amplifier
Noise source

6.2 Noise from Amplifiers

Amplifiers: the elements have characteristic noises,
power sources work as noise sources

→ Noiseless amplifier + Noise source = Amplifier with noise

Power gain G_p $$e^2_{\mathrm{intotal}} = j_n^2 R^2 + e_{\mathrm{R}}^2 + e_n^2 = e^2_{\mathrm{out}}/G_{\mathrm{p}}$$

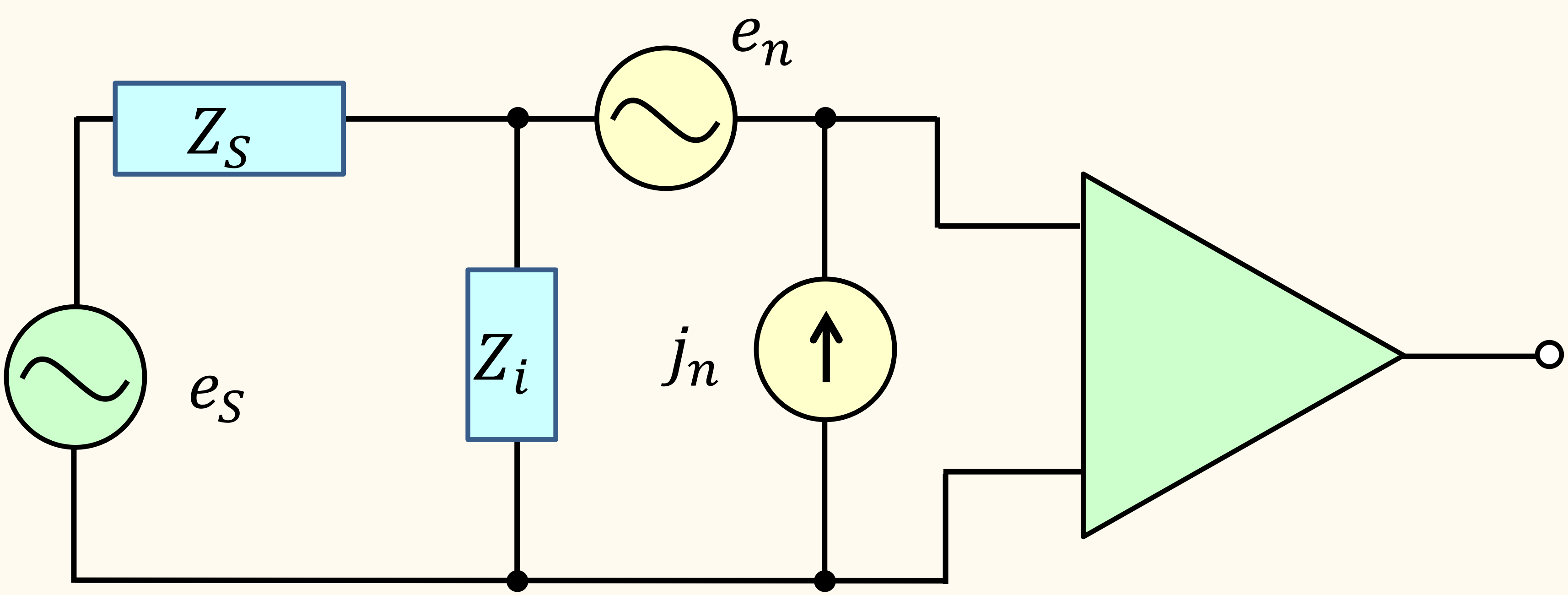
Signal to noise ratio: S/N ratio

Noise Figure: $$\mathrm{NF} = 10\log_{10}\frac{(S/N)_{\mathrm{in}}}{(S/N)_{\mathrm{out}}} = 10\log_{10}\frac{S_{\mathrm{in}}N_{\mathrm{out}}}{S_{\mathrm{out}}N_{\mathrm{in}}}$$

$$N_{\mathrm{out}} = G_{\mathrm{p}}\overline{e_{\mathrm{N}}^2}$$

$$\mathrm{NF} = 10\log_{10}\frac{S_{\mathrm{in}}G_{\mathrm{p}}\overline{e_{\mathrm{N}}^2}}{S_{\mathrm{in}}G_{\mathrm{p}}\overline{e_{\mathrm{R}}^2}} = 10\log_{10}\frac{\overline{e_{\mathrm{N}}^2}}{\overline{e_{\mathrm{R}}^2}} = 10\log_{10}\frac{\overline{e_n^2} + \overline{e_{\mathrm{R}}^2} + \overline{j_n^2}R^2}{\overline{e_{\mathrm{R}}^2}}$$

6.2.2 Noise impedance matching



Optimization of S/N ratio including the noise-source in the amplifier
(a care should be taken to the effect of noise to the object)

Noises from the signal source, amplifiers: repel as much as possible
Signals from the source: absorb …

Noise temperature method: not almighty

6.2.2 Noise impedance matching

Nyquist theorem:

$$\sqrt{\overline{J^2}\ \overline{V^2}} = 2k_{\mathrm{B}} T \Delta f$$ Noise temperature definition ($J(f), V(f)$)

Noise temperature and matched source impedance

$$T_{\mathrm{a}} = \frac{\sqrt{\overline{e_n^2}\ \overline{j_n^2}}}{2k_{\mathrm{B}}}, \quad R_{\mathrm{bs}} = \sqrt{\frac{\overline{e_n^2}}{\overline{j_n^2}}}$$

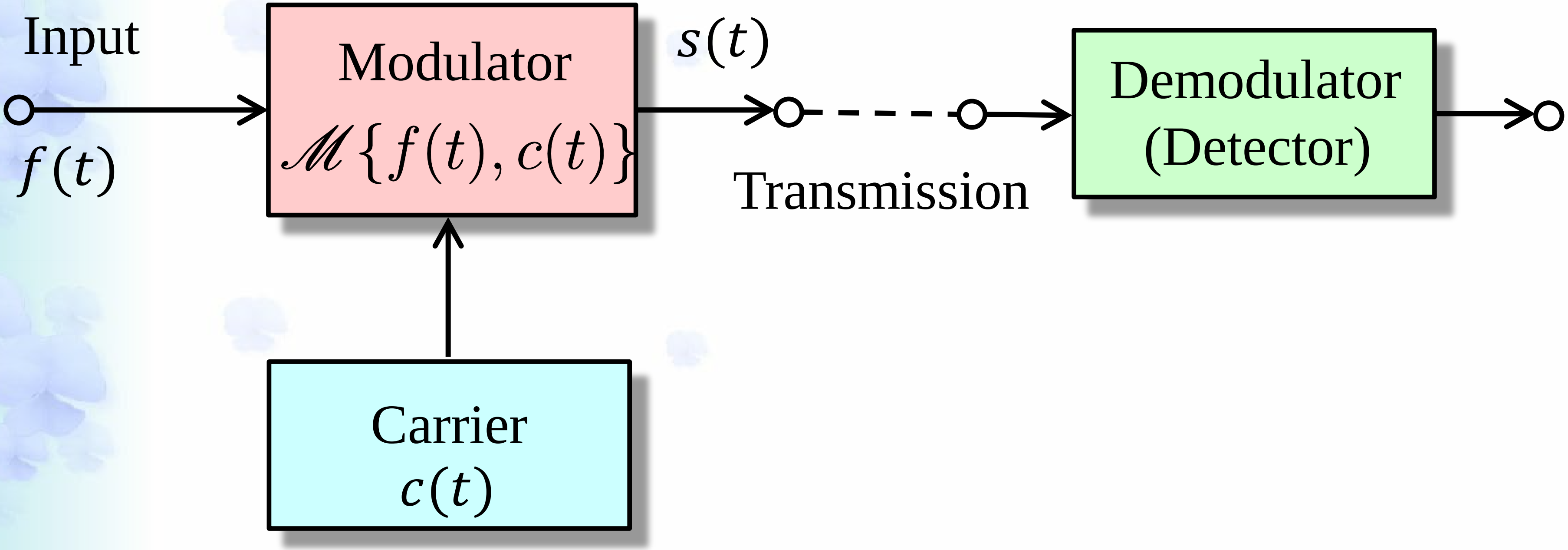
Output noise temperature:

$$T_{\mathrm{n}} = \left(1 + \frac{\mathrm{Re}(1/Z_{\mathrm{i}})}{\mathrm{Re}(1/Z_{\mathrm{s}})}\right) \frac{T_{\mathrm{a}}}{2\mathrm{Re}Z} \left(\frac{|Z|^2}{R_{\mathrm{bs}}} + R_{\mathrm{bs}}\right), \quad \frac{1}{Z} \equiv \frac{1}{Z_{\mathrm{i}}} + \frac{1}{Z_{\mathrm{s}}}$$

Minimize T_n: $Z_{\mathrm{i}} = \dfrac{1}{R_{\mathrm{bs}}^{-1} - Z_{\mathrm{s}}^{-1}}$ Noise matching condition

$$T_{\mathrm{n}} = \left(1 + \frac{\mathrm{Re}(1/Z_{\mathrm{i}})}{\mathrm{Re}(1/Z_{\mathrm{s}})}\right) T_{\mathrm{a}}$$

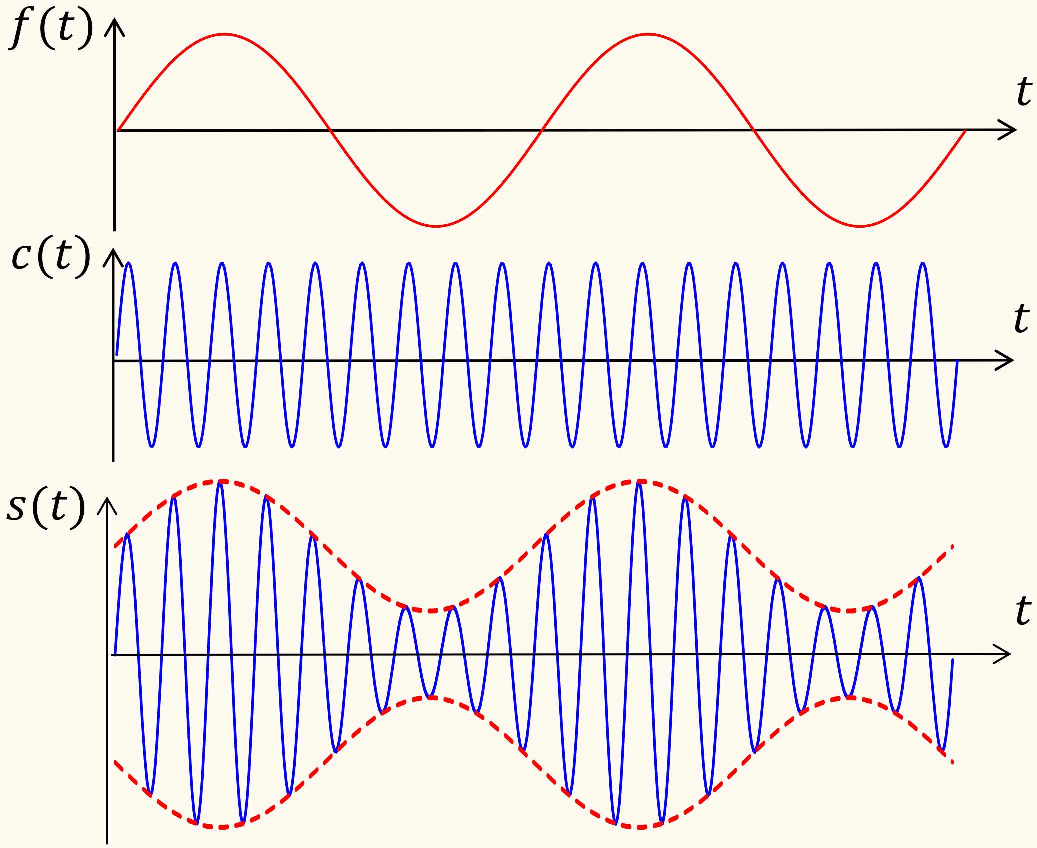
Electric communication
- Baseband communication
- Carrier communication



Modulation
- Amplitude modulation
- Frequency (Phase) modulation

→ Analog / Pulse

6.3.2 Amplitude modulation



$$c(t) = A\cos\omega_c t$$

$$s(t) = A[1 + mf(t)]\cos\omega_c t$$

m: Modulation index

$$0 < m \leq 1$$

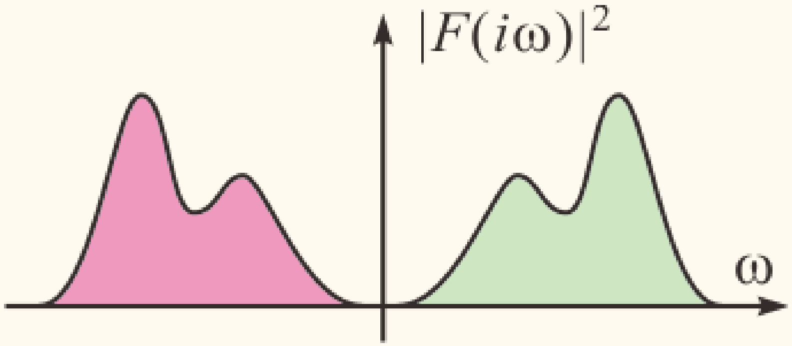
$$S(i\omega) = \int_{-\infty}^{\infty} s(t)e^{i\omega t}dt = \int_{-\infty}^{\infty} A[1 + mf(t)]\cos(\omega_c t)e^{i\omega t}dt$$

$$= A\left\{\pi[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{m}{2}[F(i(\omega - \omega_c)) + F(i(\omega + \omega_c))]\right\}$$

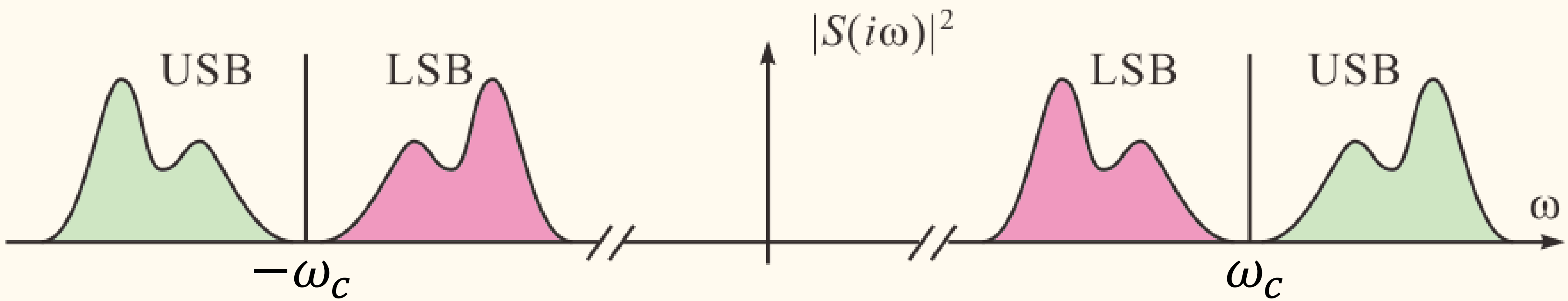
6.3.2 Amplitude modulation

$$S(i\omega) = \int_{-\infty}^{\infty} s(t)e^{i\omega t}dt = \int_{-\infty}^{\infty} A[i + mf(t)]\cos(\omega_c t)e^{i\omega t}dt$$
$$= A\left\{\pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)] + \frac{m}{2}[F(i(\omega-\omega_c)) + F(i(\omega+\omega_c))]\right\}$$

$$F(i\omega) = \mathscr{F}\{f(t)\}$$



$f(t)$: Real $\quad F(i\omega) = F^*(-i\omega)$

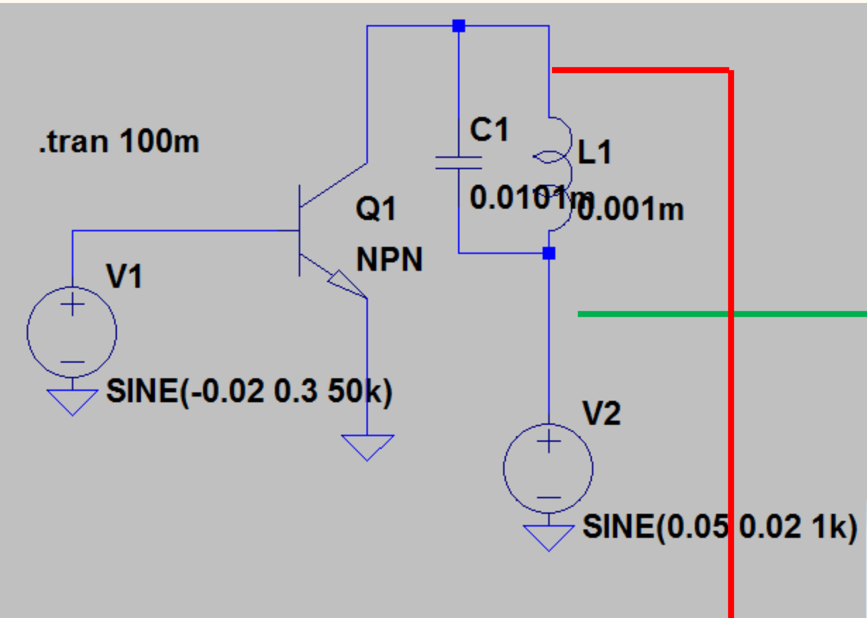


Upper side band (USB), Lower side band (LSB)

6.3.2 Amplitude modulation (circuit example)

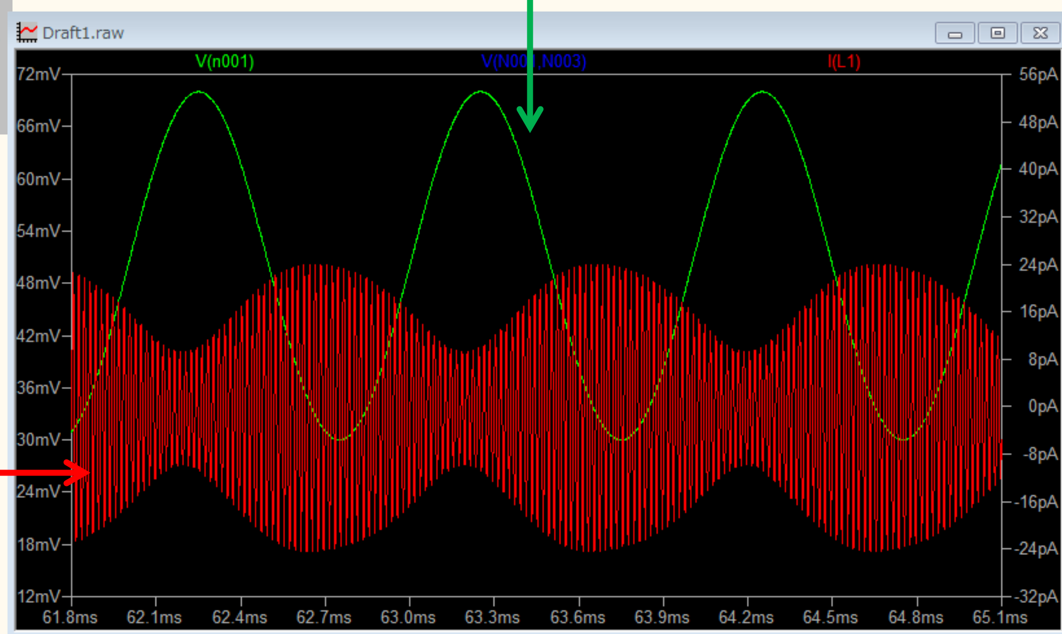
Collector modulation circuit

C-class amplification (non-linear) region



Modulation voltage

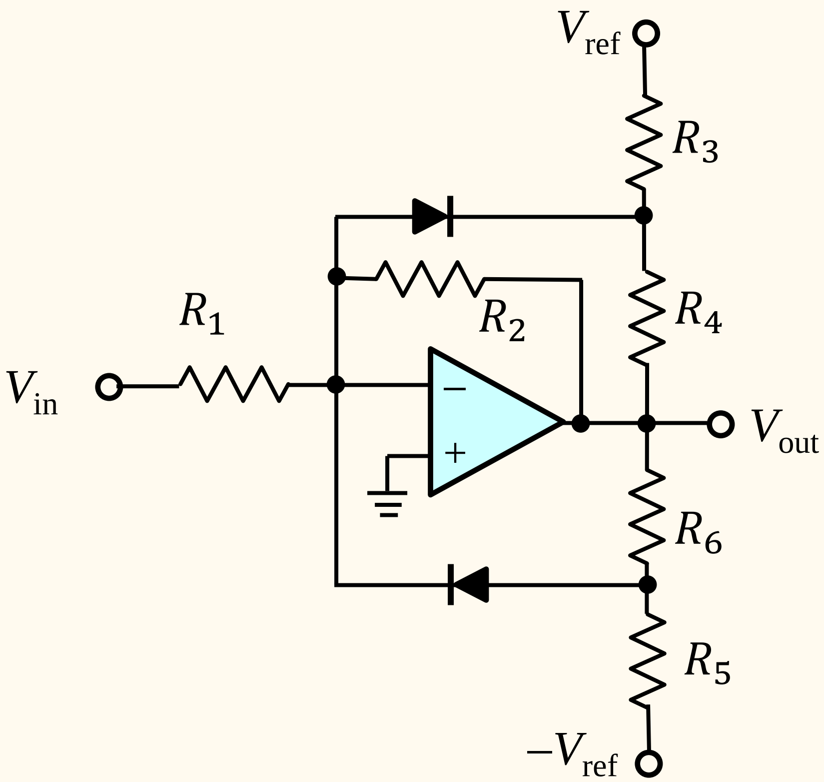
Current through the inductor

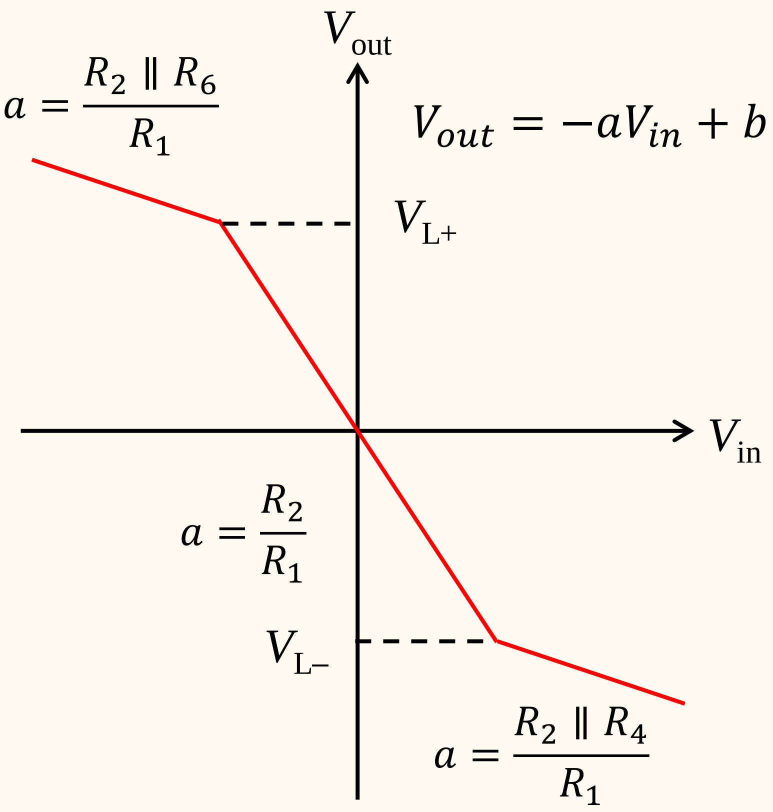


6.3.2 Amplitude modulation (circuit example2)

Modulation of oscillator circuit

Soft limiter circuit

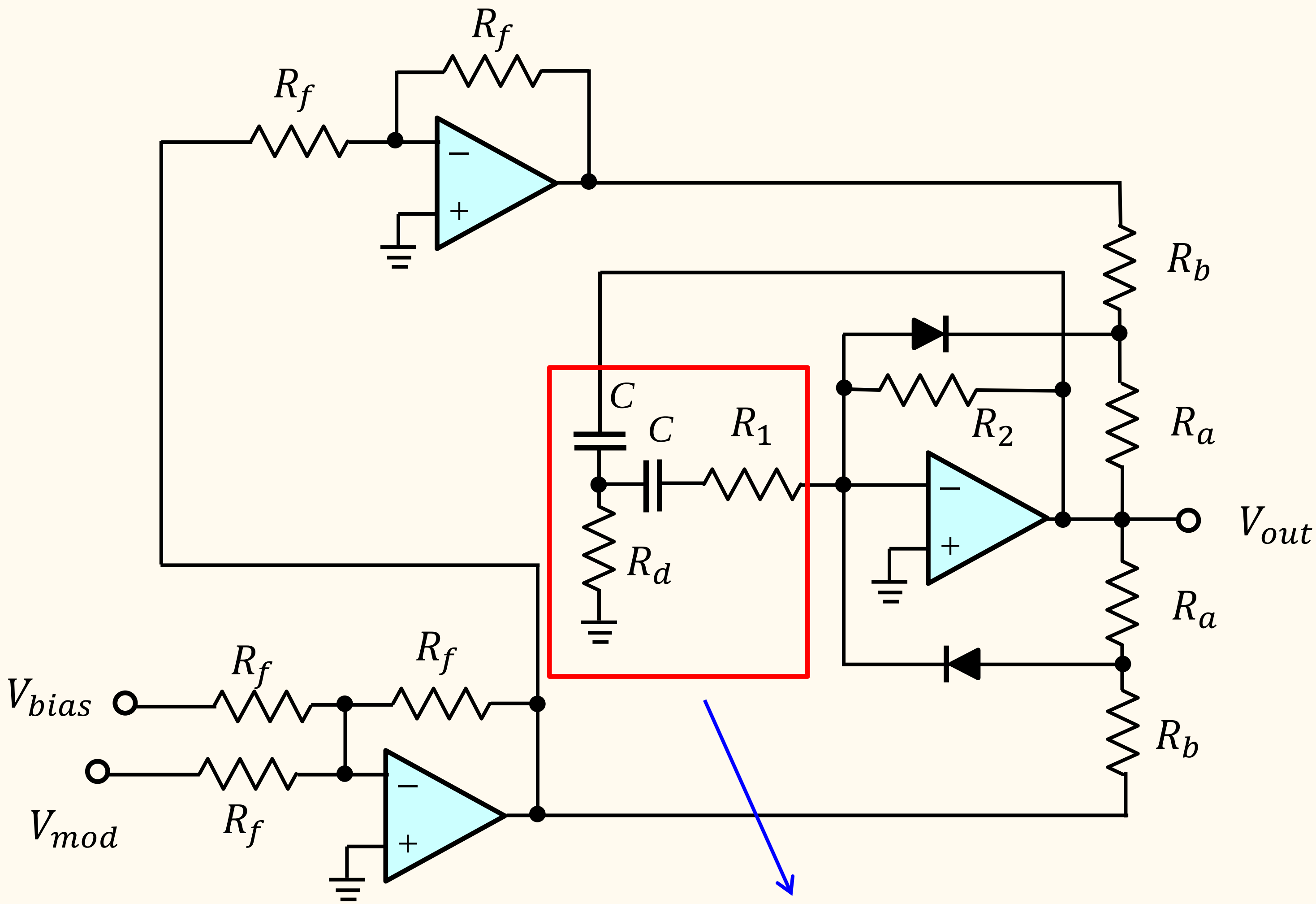




$$V_{L-} = -\frac{R_4}{R_3}V_{ref} - \left(1 + \frac{R_4}{R_3}\right)V_{th}$$

: controllable with V_{ref}

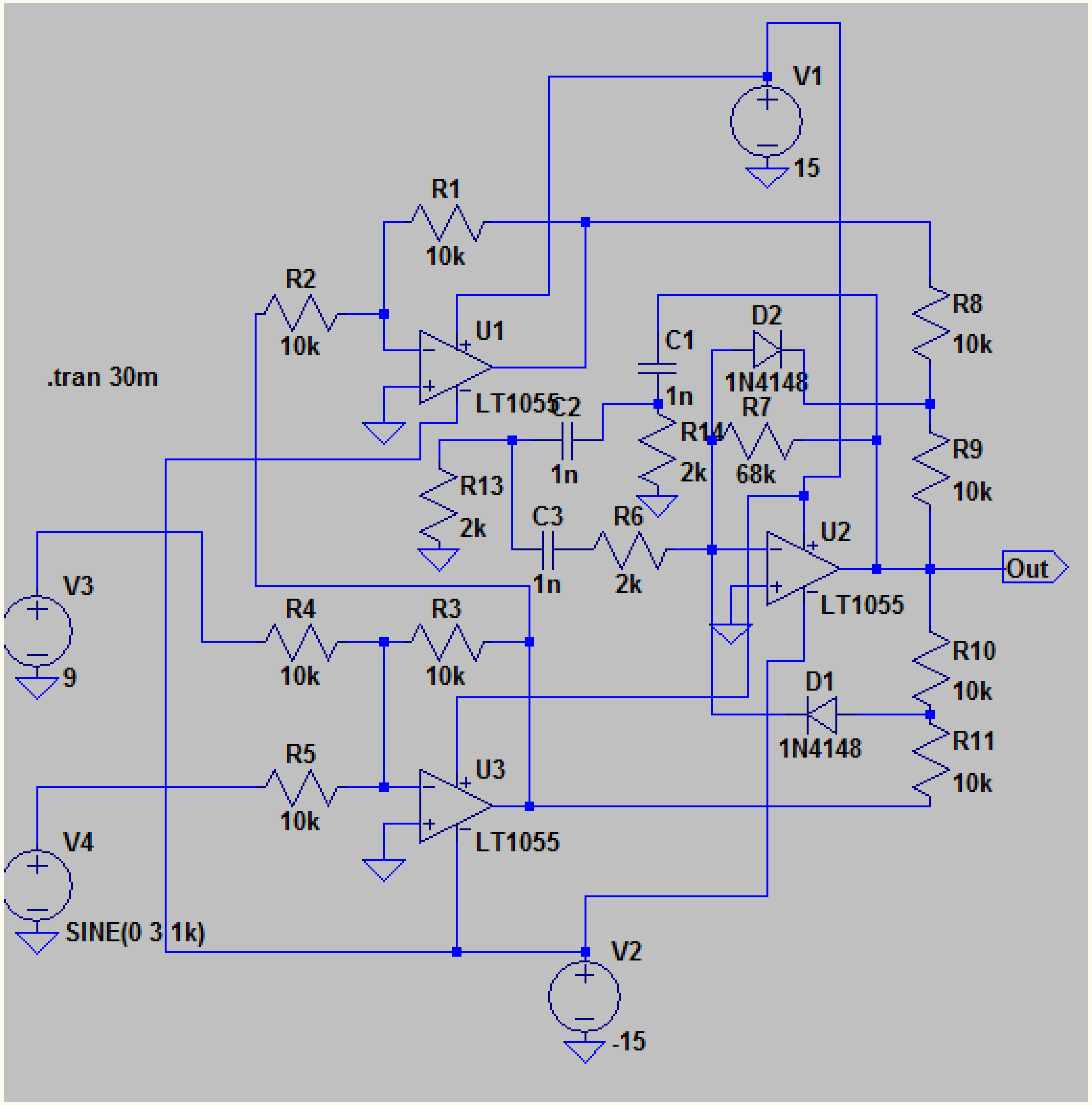
6.3.2 Amplitude modulation (circuit example2)



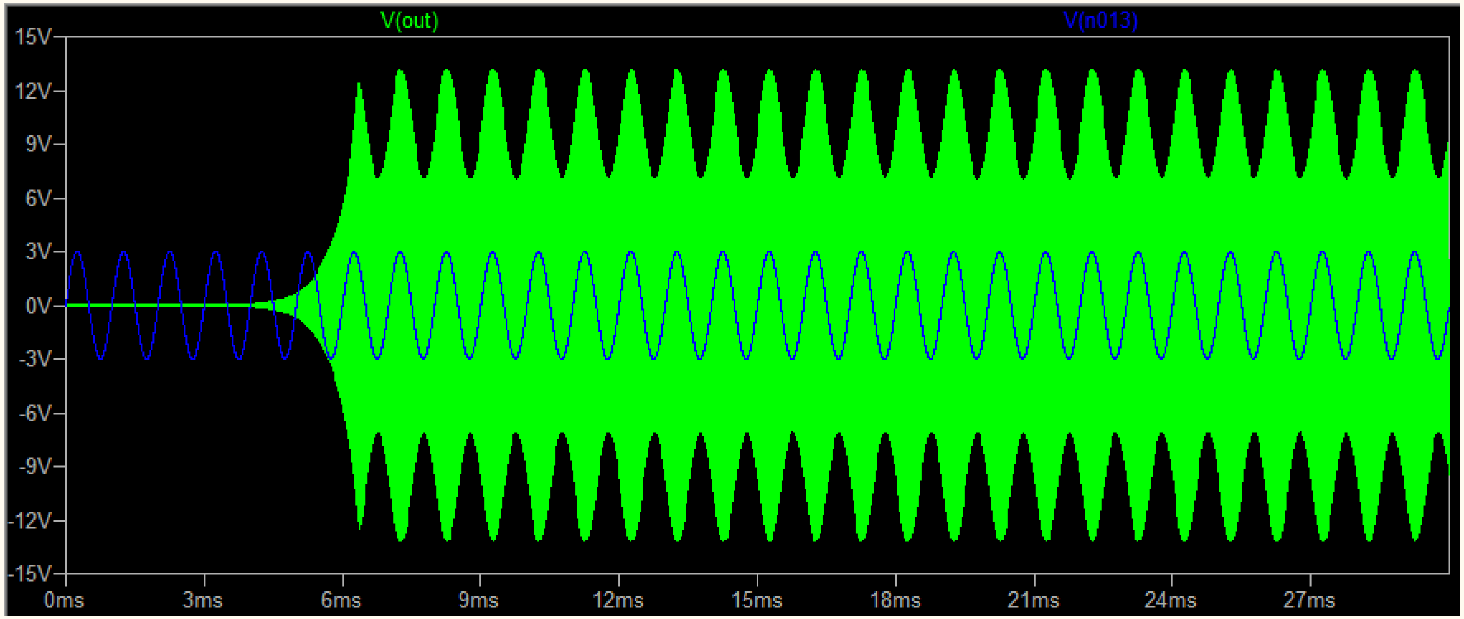
Delayed feedback → Oscillation

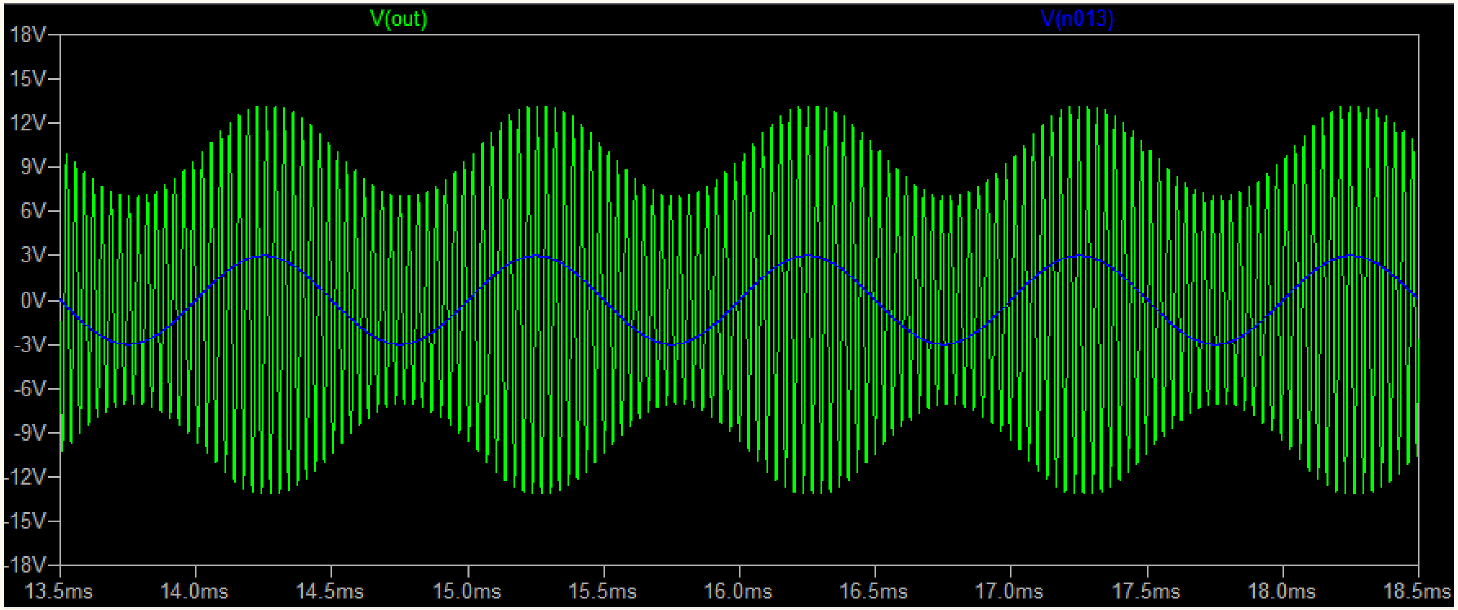
The amplitude is softly limited with the modulation voltage.

6.3.2 Amplitude modulation (circuit example2)

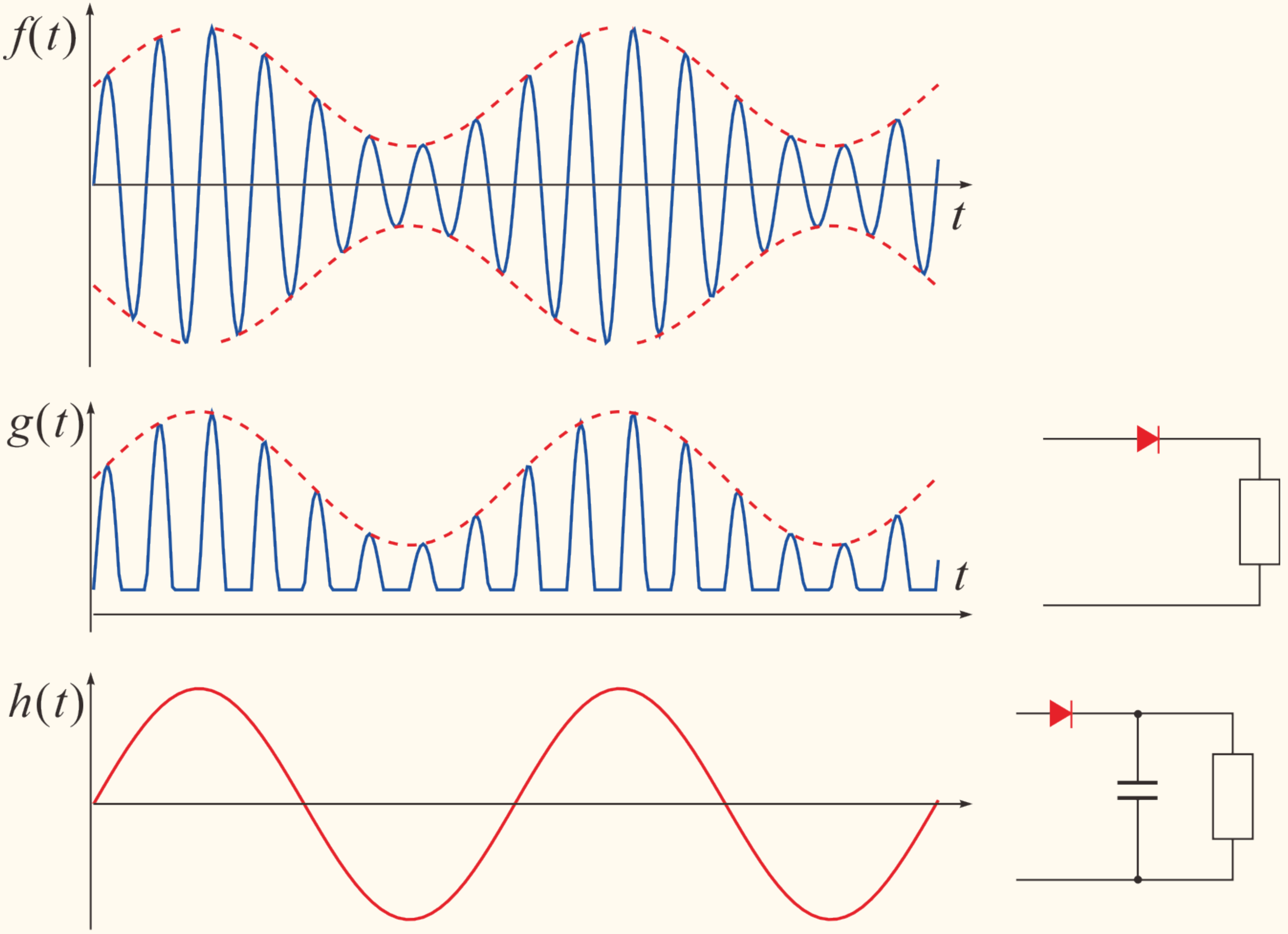


6.3.2 Amplitude modulation (circuit example2)

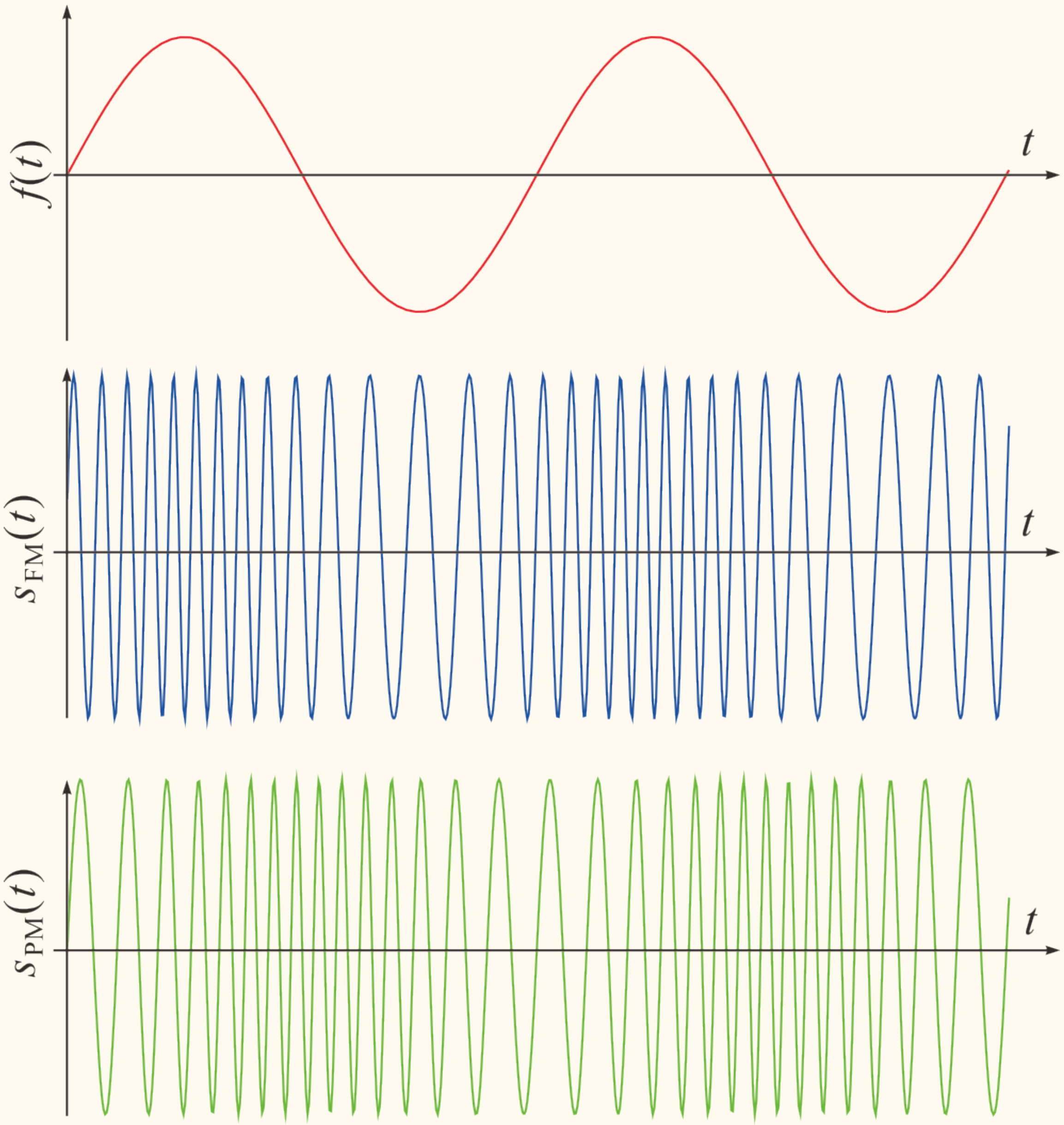




6.3.2 Amplitude modulation (demodulation)



6.3.3 Angle modulation



6.3.3 Angle modulation

$$s(t) = A\cos\theta_{\mathrm{i}}(t), \quad \theta_{\mathrm{i}}(t) = \omega_{\mathrm{c}}t + \phi[t, f(t)]$$

Differential angular frequency $\omega_{\mathrm{i}}(t) = \dfrac{d\theta_{\mathrm{i}}(t)}{dt} = \omega_{\mathrm{c}} + \dfrac{d\phi[t, f(t)]}{dt}$

$$\frac{d\phi[t, f(t)]}{dt} = k_f f(t) \quad (\mathrm{Frequency Modulation, FM}),$$

$$\phi[t, f(t)] = k_p f(t) \quad (\mathrm{Phase Modulation, PM})$$

$$s_{\mathrm{FM}}(t) = A\cos\left[\omega_{\mathrm{c}}t + k_f \int^t f(\tau)d\tau\right],$$

$$s_{\mathrm{PM}}(t) = A\cos[\omega_{\mathrm{c}}t + k_f f(t)]$$

Frequency ω component: only phase shift $\pi/2$:
No difference in signal outlook.

6.3.3 Angle modulation (Frequency modulation)

$$f(t) = A_p \cos \omega_p t$$

$$s_{\mathrm{FM}} = A\cos(\omega_{\mathrm{c}} t + \beta \sin \omega_p t) = A\mathrm{Re}\left[\exp(i\omega_{\mathrm{c}} t)\exp(i\beta \sin \omega_p t)\right]$$

$$\left(\beta \equiv \frac{k_f A_p}{\omega_p} = \frac{\Delta f}{f_p}\right)$$

$\sin \omega_p t$: Periodic function with $T = 2\pi/\omega_p$

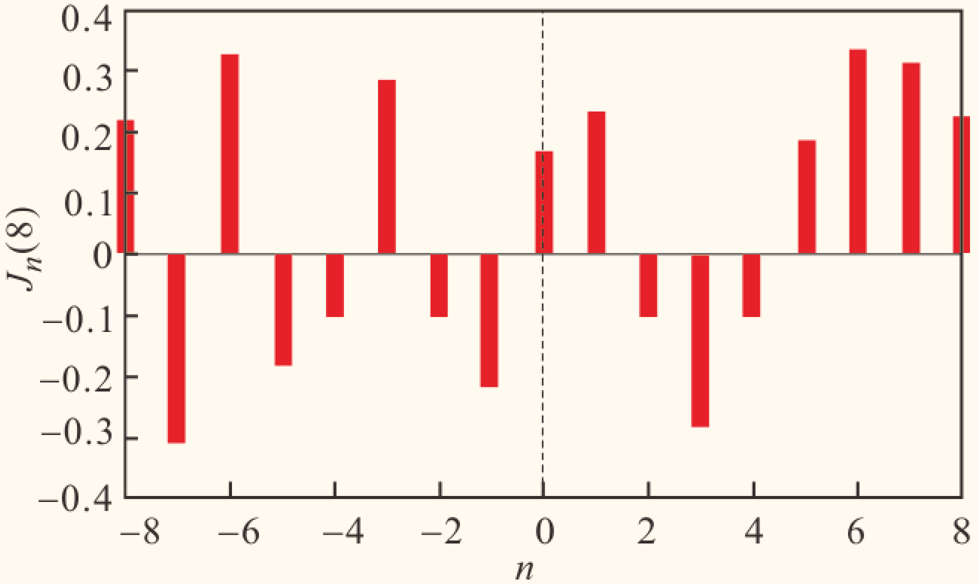
$$\exp(i\beta \sin \omega_p t) = \sum_{n=-\infty}^{\infty} c_n \exp(i\omega_p t),$$ Fourier series expansion

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} \exp(i\beta \sin \omega_p t)\exp(-in\omega_p t)dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[i(\beta \sin \theta - n\theta]d\theta = J_n(\beta)$$

First kind Bessel function

6.3.3 Angle modulation (Frequency modulation)



$$s_{\mathrm{FM}}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[(\omega_{\mathrm{c}} + n\omega_p)t]$$

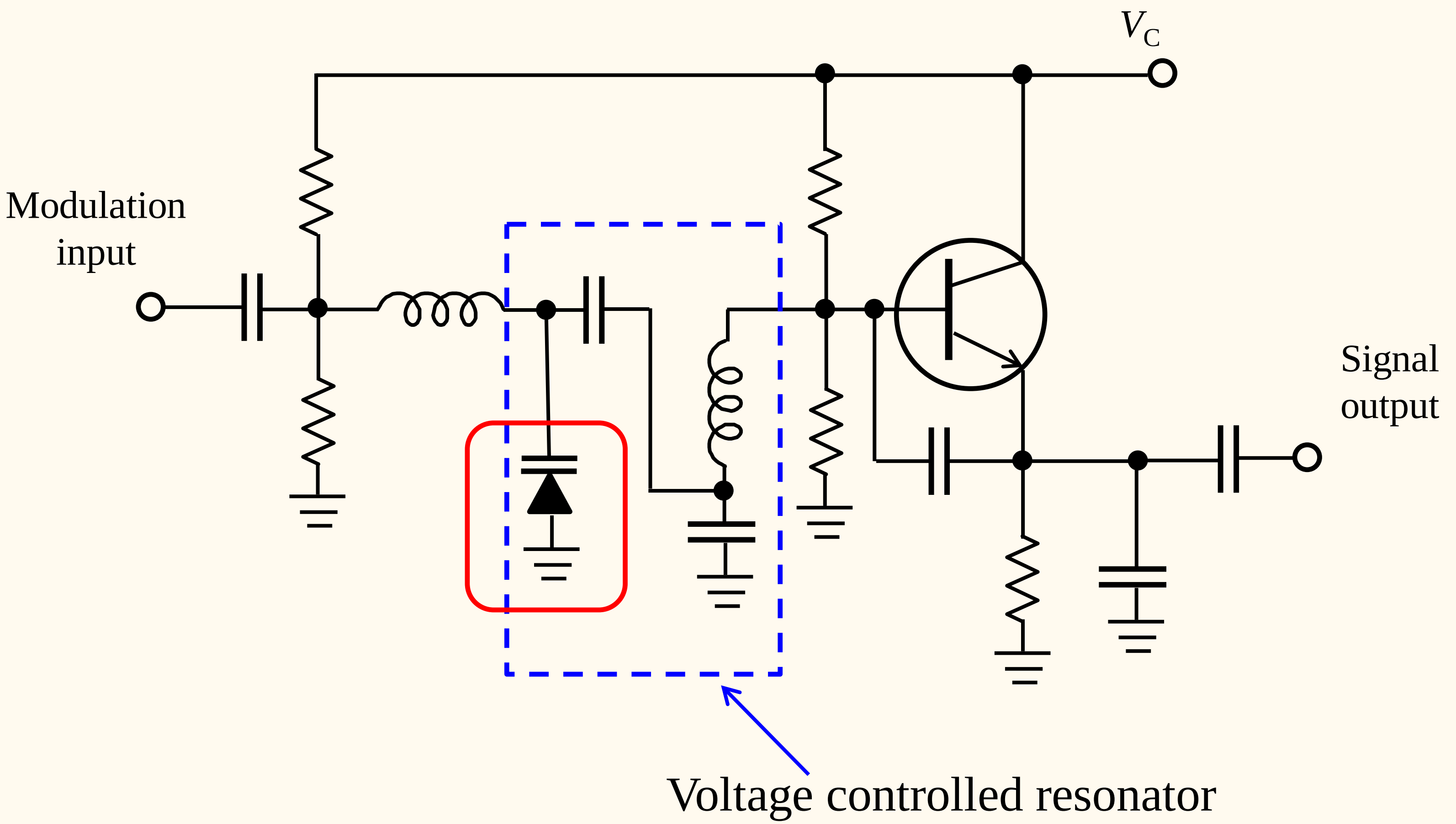
$$S_{\mathrm{FM}}(i\omega) = \pi A \sum_{n=-\infty}^{\infty} J_n(\beta)\{\delta[\omega - (\omega_{\mathrm{c}} + n\omega_p)] + \delta[\omega + (\omega_{\mathrm{c}} + n\omega_p)]\}$$

Actual band width:

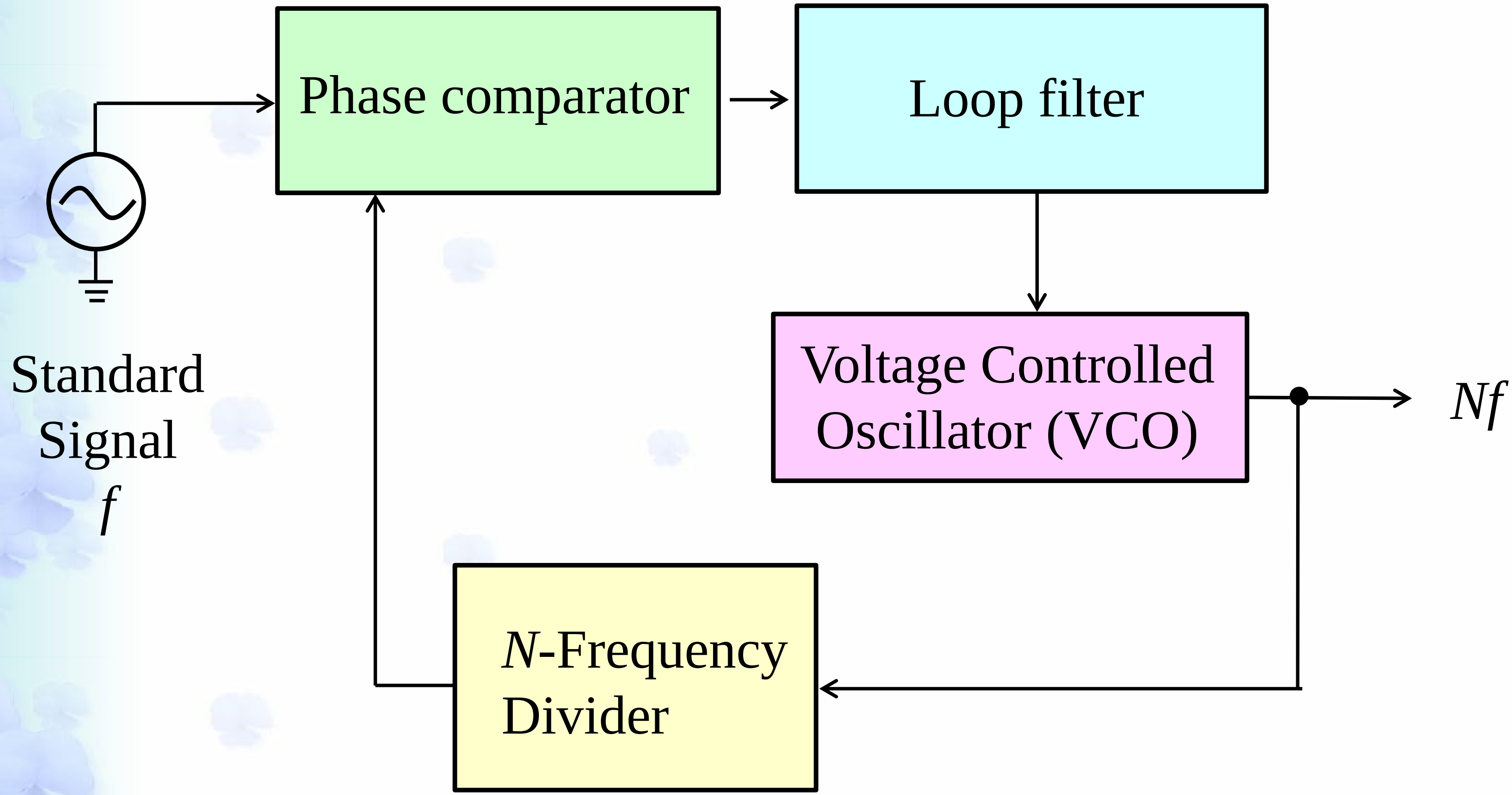
$$\omega_{\mathrm{bw}} = 2(\omega_{\mathrm{f}} + \xi\omega_{\mathrm{w}}) \qquad 1 \leq \xi \leq 2$$

6.3.3 Angle modulation (circuit example)

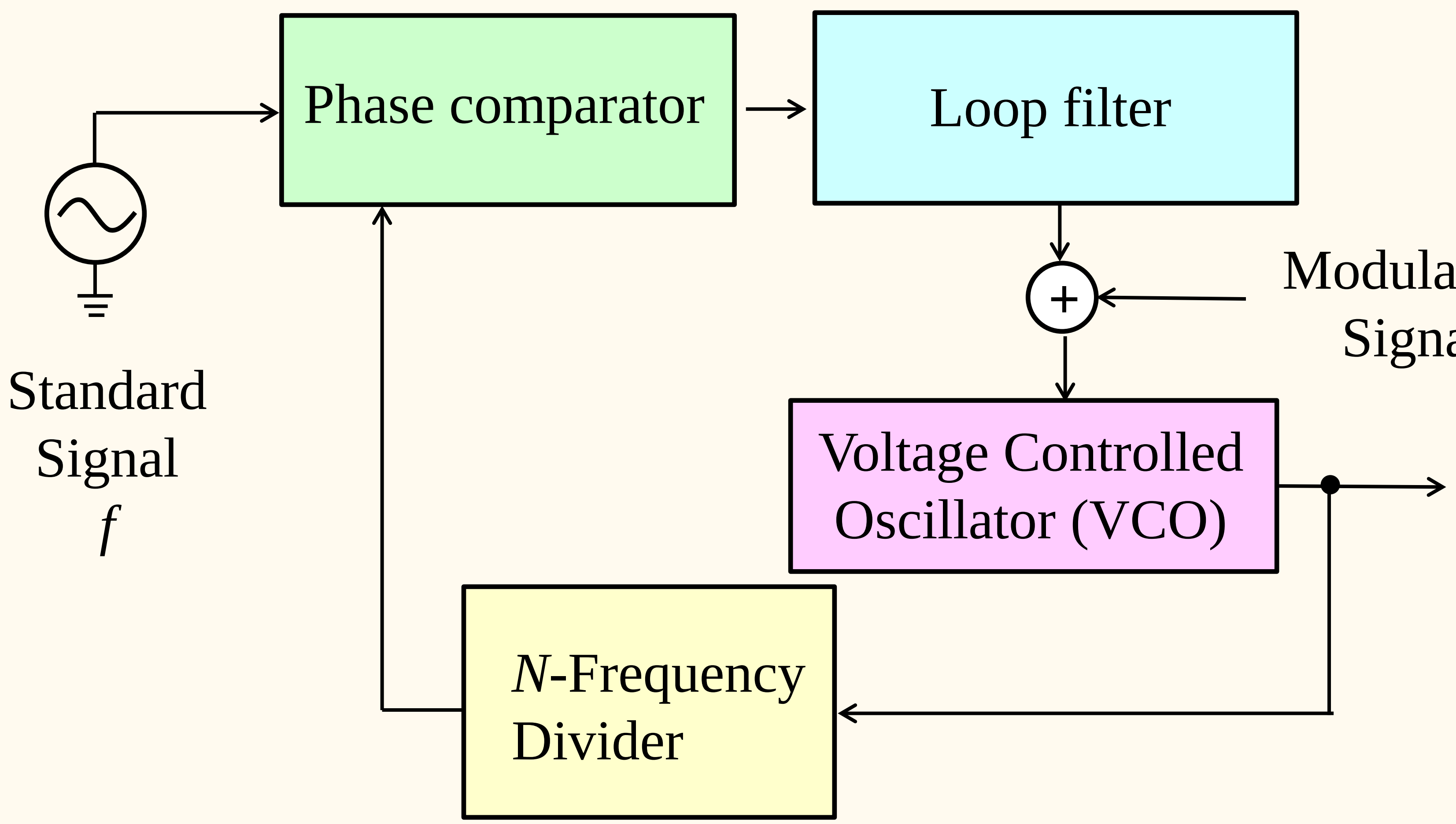
Voltage Controlled Oscillator (VCO)



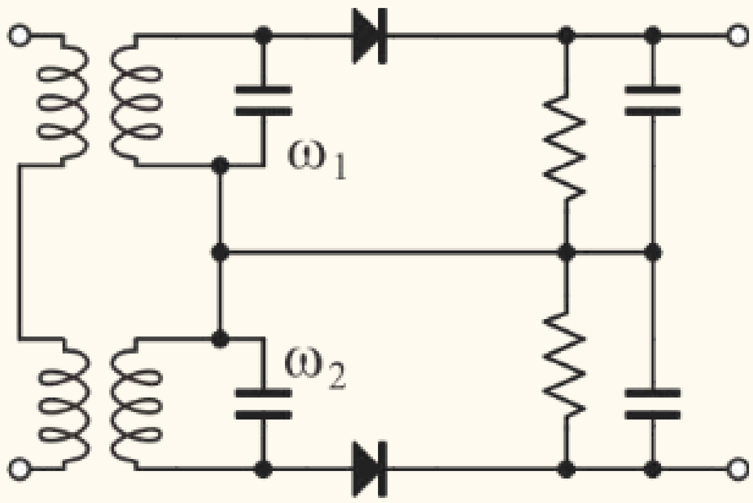
Phase Lock Loop (PLL)



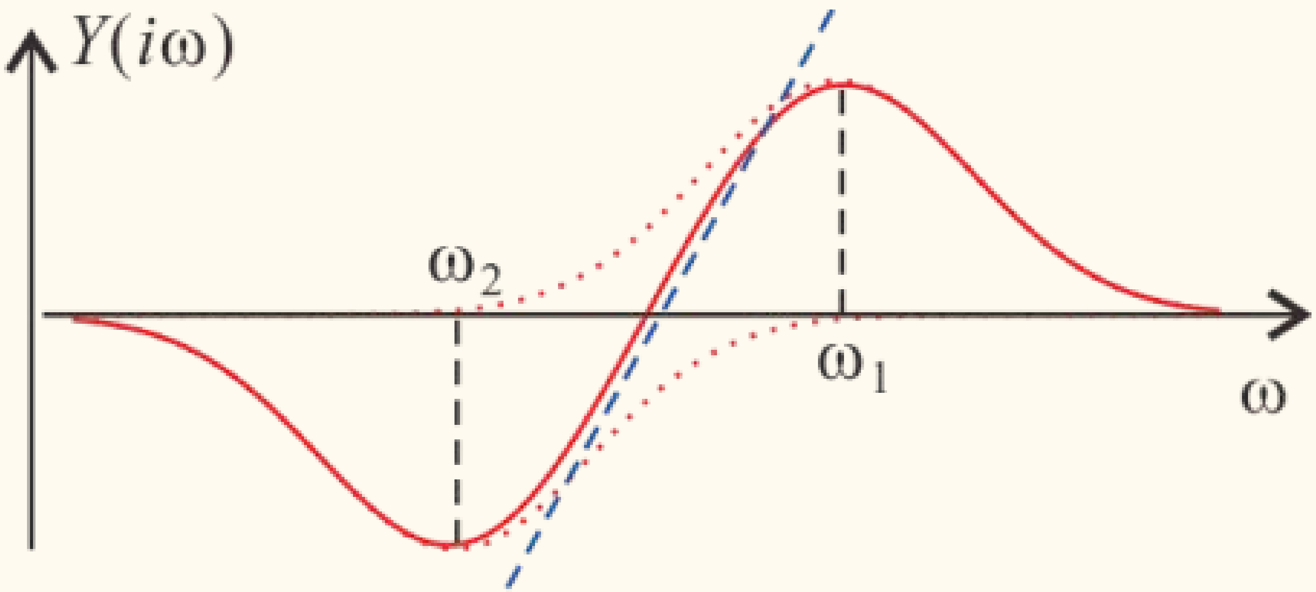
6.3.3 Angle modulation (circuit example)



6.3.4 Angle modulation (Frequency demodulation)

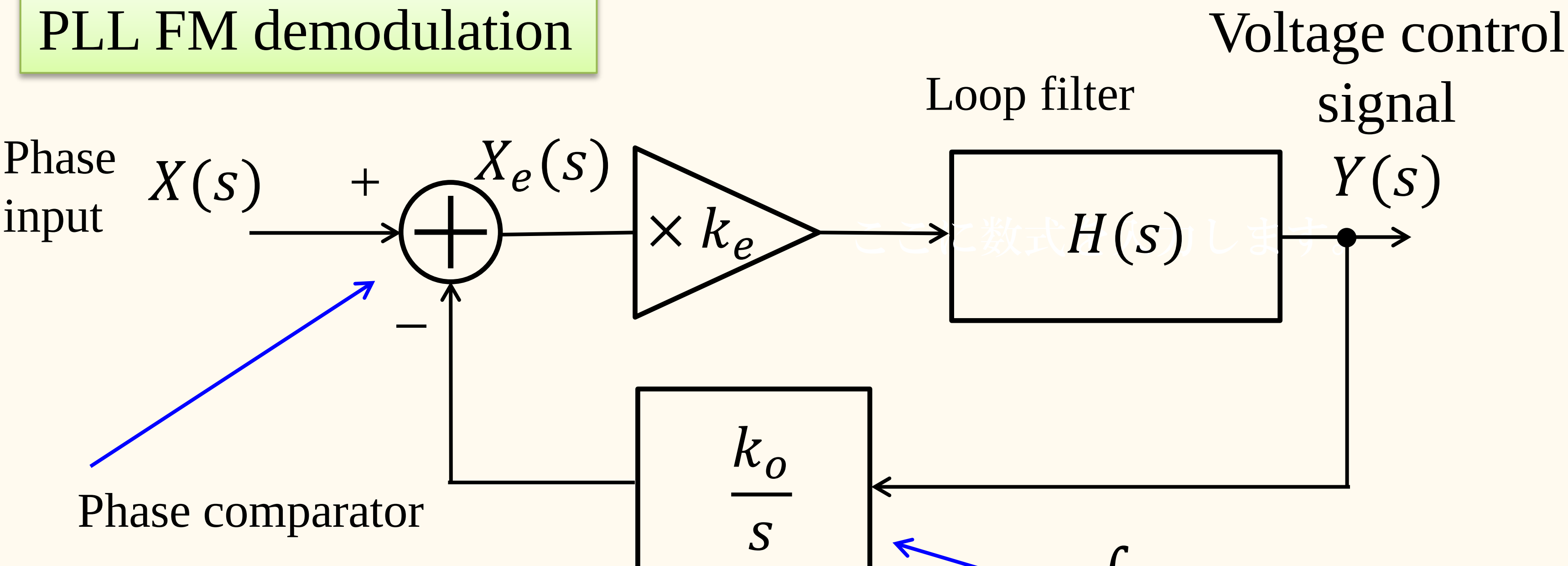


Double tuned circuit



6.3.4 Angle modulation (frequency demodulation)

PLL FM demodulation



$$k_o \int y(t)dt$$

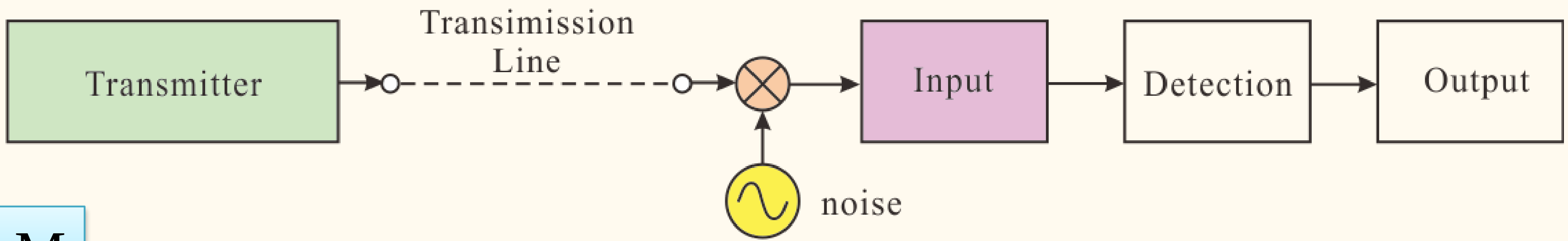
$$Y(s) = \frac{s k_e H(s)}{s + k_e k_o H(s)} X(s)$$

$g(t)$: Frequency modulation signal (original)

$$\phi(t) = k_f \int_{-\infty}^{t} g(\tau)d\tau, \quad sX(s) = k_f G(s)$$

$$\therefore\ Y(s) = \frac{k_f k_e H(s)}{s + k_e k_o H(s)} G(s) \approx \frac{k_f}{k_o} G(s)$$

6.3.5 Modulation and noise



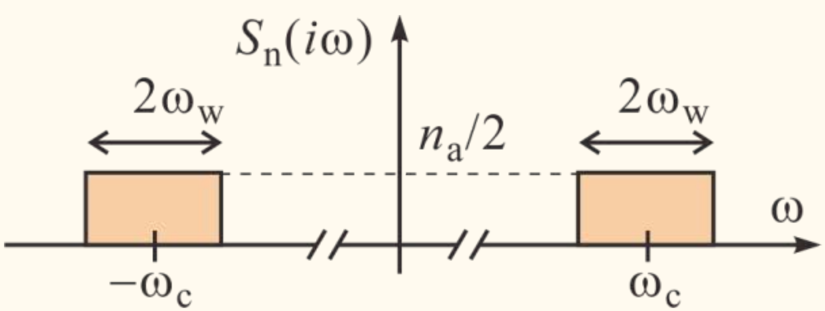
AM

Received signal $r(t) = A_r[1 + mf(t)]\cos\omega_c t + n_i(t)$

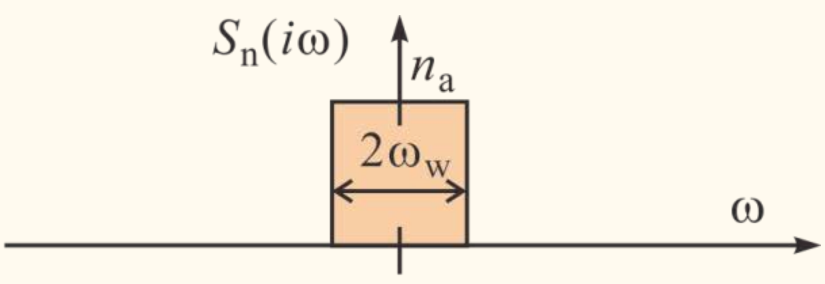
Demodulated output $g(t) = A_r m f(t) + n_o(t)$

Averaged signal power

received: $S_{pr} = \dfrac{A_r^2}{2} + \dfrac{(A_r m)^2}{2}\langle f^2\rangle$, output: $S_{po} = A_r^2 m^2 \langle f^2\rangle$



Received



Demodulated

6.3.5 Modulation and noise

ω_W : Noise bandwidth (assumption: white)

Noise power: $2 \times \frac{n_a}{2 \times 2\pi} \times 2\omega_w = \frac{n_a \omega_w}{\pi}$, $\frac{n_a \times 2\omega_w}{2\pi} = \frac{n_a \omega_w}{\pi}$

Received Demodulated

$$\left.\frac{S}{N}\right|_{\rm in} = \frac{\pi[A_r^2 + (A_r m)^2 \langle f^2 \rangle]}{2 n_a \omega_w}, \quad \left.\frac{S}{N}\right|_{\rm out} = \frac{\pi A_r^2 m^2 \langle f^2 \rangle}{n_a \omega_w} = 2\eta \left.\frac{S}{N}\right|_{\rm in}$$

$$\eta = \frac{m^2 \langle f^2 \rangle}{1 + m^2 \langle f^2 \rangle}$$:Power transmission efficiency

$$0 < m \leq 1 \ \rightarrow \eta < \frac{1}{2}$$

Input sinusoidal: $\langle f^2 \rangle = \frac{1}{2} \ \rightarrow \ \eta < \frac{1}{3}$

6.3.5 Modulation and noise

FM, PM

In phase — Out of phase

$$r(t) = \underbrace{A_r \cos[\omega_c t + \phi(t)]}_{\text{Signal}} + \underbrace{n_l(t)\cos\omega_c t - n_r(t)\sin\omega_c t}_{\text{Noise}}$$

$$= A_r \cos[\omega_c t + \phi(t)] + A_{\mathrm{n}}(t)\cos[\omega_c t + \phi_{\mathrm{n}}(t)]$$

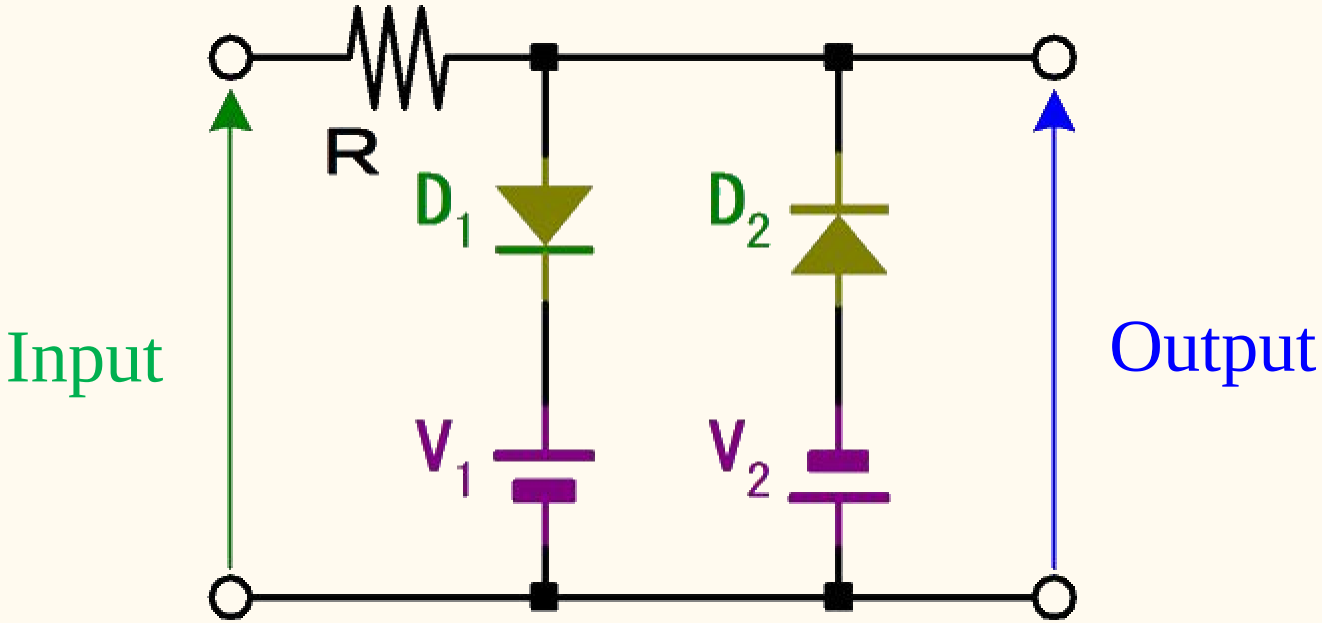
$$= V_r(t)\cos[\omega_c t + \theta(t)] \quad (\theta(t) = \phi(t) + \underbrace{\phi_{\mathrm{no}}(t)}_{\text{Phase noise}})$$

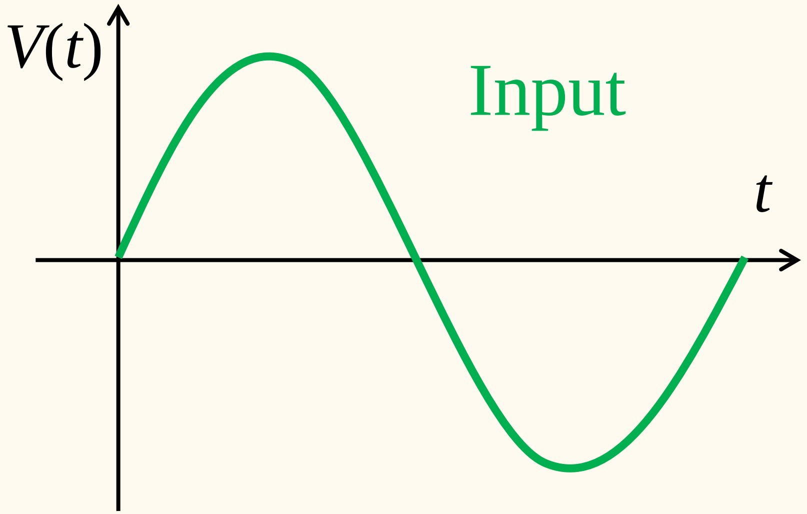
$$V_r(t) = \sqrt{A_r^2 + A_n^2(t) + 2A_r A_{\mathrm{n}}(t)\cos[\phi_{\mathrm{n}}(t) - \phi(t)]},$$

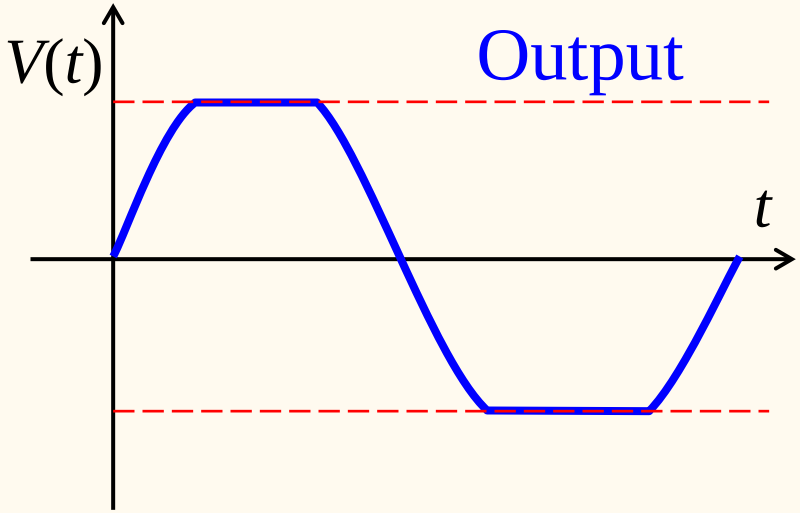
$$\phi_{\mathrm{no}}(t) = \arctan\frac{A_{\mathrm{n}}(t)\sin[\phi_{\mathrm{n}}(t) - \phi(t)]}{A_r + A_{\mathrm{n}}(t)\cos[\phi_{\mathrm{n}}(t) - \phi(t)]}$$

Time-dependent part in $V_r(t)$ can be cut with a limiter circuit.

6.3.5 Modulation and noise (Diode limitter)







6.3.5 Modulation and noise

$$A_r \gg A_\mathrm{n}(t)$$

$$\phi_\mathrm{no} \cong \arctan\left[\frac{A_\mathrm{n}(t)}{A_r}\sin[\phi_\mathrm{n}(t)-\phi(t)]\right] \cong \frac{A_\mathrm{n}(t)}{A_r}\sin[\phi_\mathrm{n}(t)-\phi(t)]$$

Noise: White, Power spectrum density $n_a/2$, Band width ω_B

Noise power: $N_i = \dfrac{n_a\omega_B}{2\pi}$ Signal power: $\dfrac{A_r^2}{2}$ $\dfrac{S_\mathrm{i}}{N_\mathrm{i}} = \dfrac{\pi A_r^2}{n_a\omega_\mathrm{B}}$

Phase modulation $\phi[t, f(t)] = k_p f(t)$

Averaged signal power: $k_p^2\langle f^2\rangle$

Averaged noise power: $N_\mathrm{oPM} \cong \dfrac{1}{A_r^2}\langle A_\mathrm{n}(t)^2 \sin^2[\phi_\mathrm{n}(t)-\phi(t)]\rangle$

$\phi_n(t)$: Uniform in [0, 2π] → ignored

$$N_\mathrm{oPM} \cong \frac{1}{A_r^2}\langle A_\mathrm{n}(t)^2 \sin^2\phi(t)\rangle = \frac{n_a\omega_\mathrm{w}}{\pi A_r^2}$$

6.3.5 Modulation and noise

$$f(t) = A_p \cos \omega_p t, \quad \beta \equiv k_p A_p \to S_o = \frac{\beta^2}{2}, \quad \omega_\mathrm{B} = 2(\beta + \xi)\omega_\mathrm{w} \ (1 \le \xi \le 2)$$

$$\frac{S_\mathrm{o}}{N_\mathrm{o}} = \frac{\beta^2}{2} \frac{\pi A_r^2}{n_a \omega_\mathrm{w}} = \frac{\beta^2}{2} \frac{\omega_\mathrm{B}}{\omega_\mathrm{w}} \frac{\pi A_r^2}{n_a \omega_\mathrm{B}} = \beta^2(\beta + \xi) \frac{S_i}{N_i}$$

Frequency modulation

Demodulated output $d\theta/dt$ Signal, power: $k_f f(t)$, $k_f^2 \langle f^2 \rangle$

$$N_\mathrm{oFM} = \left\langle \frac{dn_\mathrm{no}}{dt} \right\rangle = \frac{1}{A_r^2} \left\langle \frac{dn_l}{dt} \right\rangle = \frac{1}{A_r^2} \int_{-\omega_\mathrm{w}}^{\omega_\mathrm{w}} n_a \omega^2 \frac{d\omega}{2\pi} = \frac{n_a \omega_\mathrm{w}^3}{3\pi A_r^2}$$

$$\beta \equiv k_f A_p / \omega_\mathrm{w} \qquad \frac{S_\mathrm{o}}{N_\mathrm{o}} = 3\beta^2(\beta + \xi) \frac{S_\mathrm{i}}{N_\mathrm{i}}$$

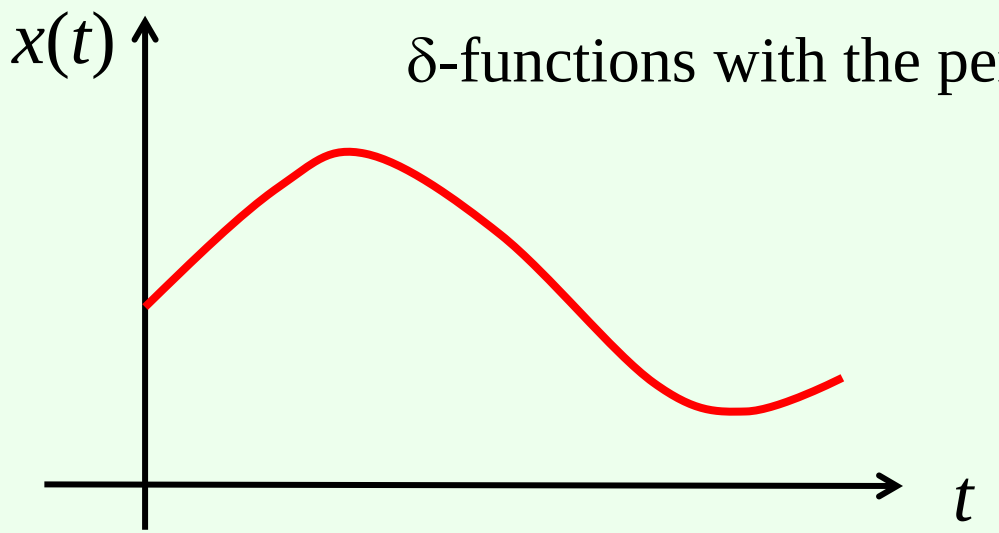
$$\left. \frac{S_\mathrm{o}}{N_\mathrm{o}} \right|_\mathrm{FM} = 3\beta^2 \left. \frac{S_\mathrm{o}}{N_\mathrm{o}} \right|_\mathrm{AM}, \qquad \left. \frac{S_\mathrm{o}}{N_\mathrm{o}} \right|_\mathrm{PM} = \beta^2 \left. \frac{S_\mathrm{o}}{N_\mathrm{o}} \right|_\mathrm{AM}$$

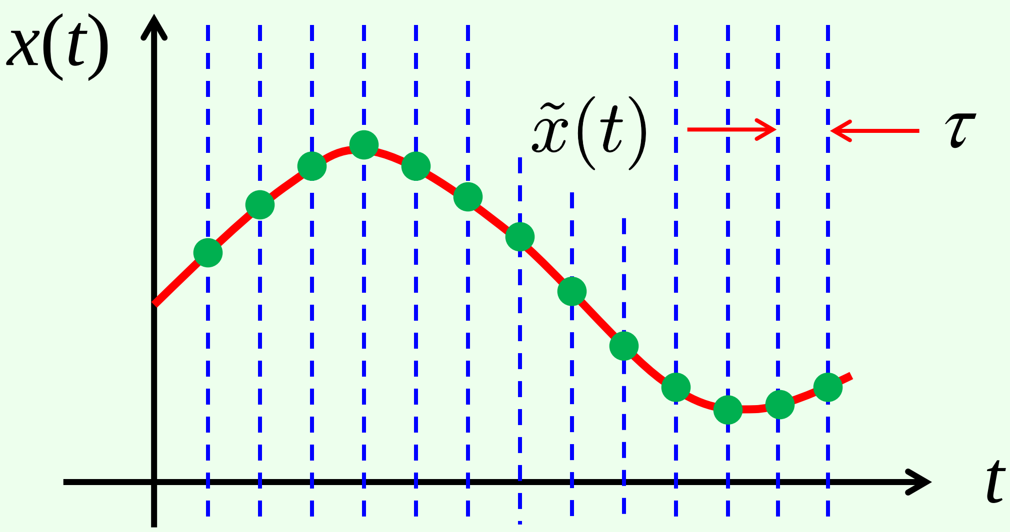
6.4 Discrete signal

6.4.1 Sampling theorem

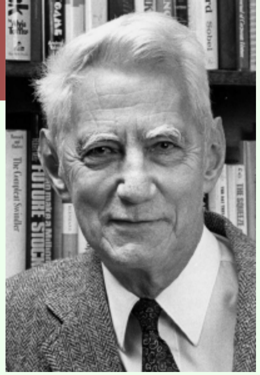
Sampled signal $\tilde{x}(t) = x(t)\delta_\tau(t)$

δ-functions with the period τ



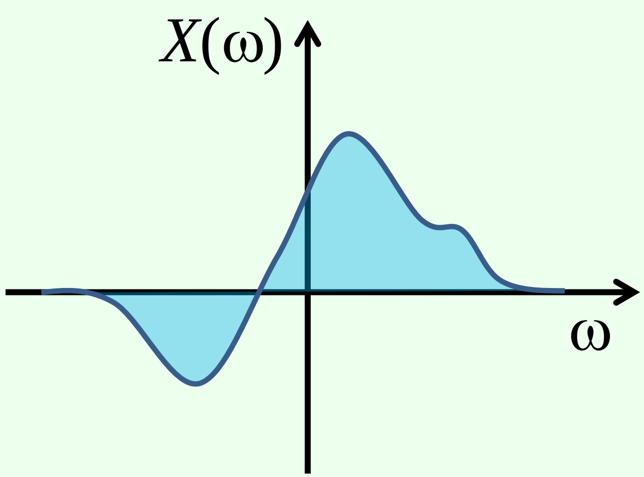


Isao Someya
1915-2007



Claude Shannon
1916-2001

1928 H. Nyquist
1949 C. Shannon
染谷勲



$$\delta_\tau(t) = \sum_{j=-\infty}^{\infty} \delta(t - j\tau) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{\tau}\int_{-\pi/\tau}^{\pi/\tau} \delta(s)ds\right] \exp\left(-in\frac{2\pi}{\tau}t\right)$$

$$= \frac{1}{\tau}\sum_{n=-\infty}^{\infty} \exp\left(-in\frac{2\pi}{\tau}t\right)$$

$$\mathscr{F}\{\delta_\tau(t)\} = \int_{-\infty}^{\infty}\left[\frac{1}{\tau}\sum_{-\infty}^{\infty} e^{-in(2\pi/\tau)t}\right] e^{i\omega t}dt = \frac{1}{\tau}\sum_{n=-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[i\left(\omega - n\frac{2\pi}{\tau}\right)t\right]dt$$

$$= \frac{2\pi}{\tau}\sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau}\right) = \frac{2\pi}{\tau}\delta_{2\pi/\tau}(\omega)$$

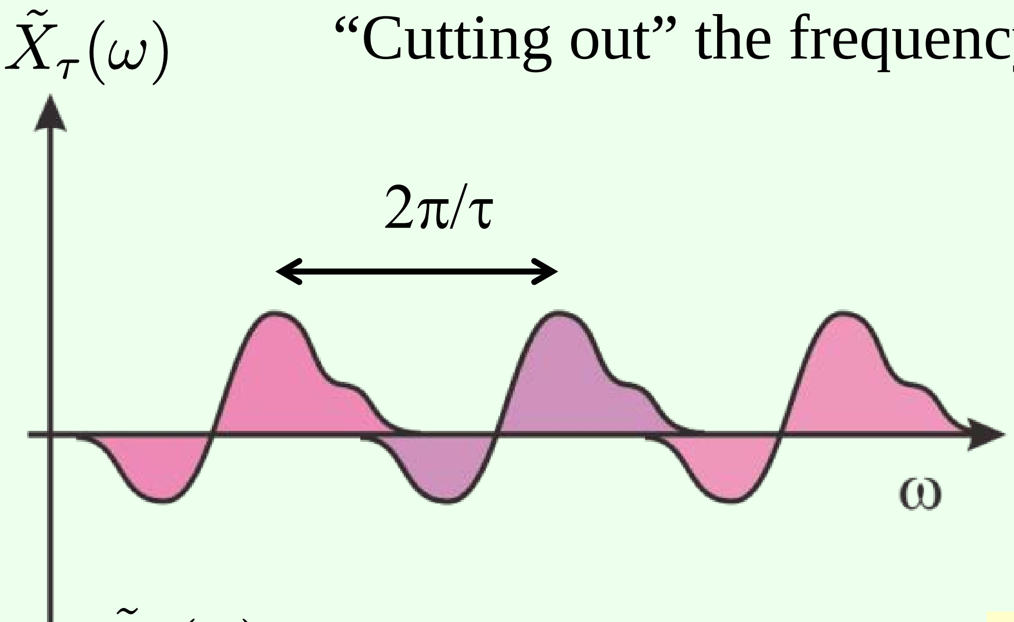
$$\mathscr{F}\{x(t)\} = X(\omega),\ \mathscr{F}\{\tilde{x}_\tau(t)\} = \tilde{X}_\tau(\omega)$$

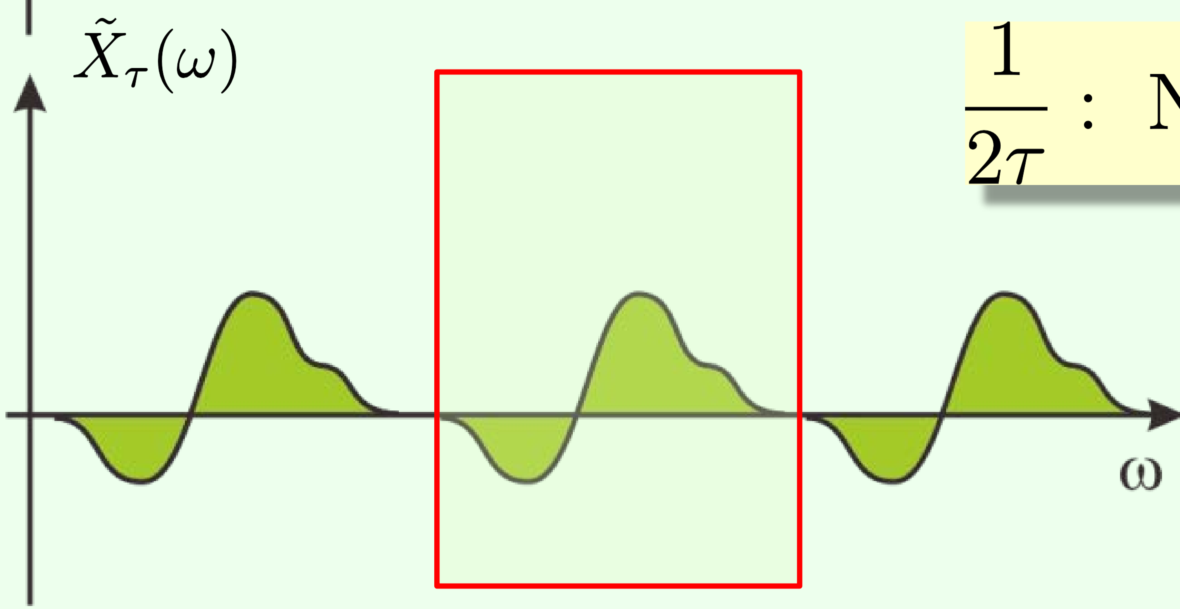
$$\tilde{X}_\tau(\omega) = \frac{1}{2\pi}X(\omega) * \frac{2\pi}{\tau}\delta_{2\pi/\tau}(\omega) = \frac{1}{\tau}X(\omega) * \sum_{n=-\infty}^{\infty}\delta\left(\omega - n\frac{2\pi}{\tau}\right)$$

$$= \frac{1}{\tau}\int_{-\infty}^{\infty} X(\omega')\left\{\sum_{n=-\infty}^{\infty}\delta\left(\omega - n\frac{2\pi}{\tau} - \omega'\right)\right\}d\omega' = \frac{1}{\tau}\sum_{n=-\infty}^{\infty} X\left(\omega - n\frac{2\pi}{\tau}\right)$$

6.4.1 Sampling theorem

"Cutting out" the frequency spectrum





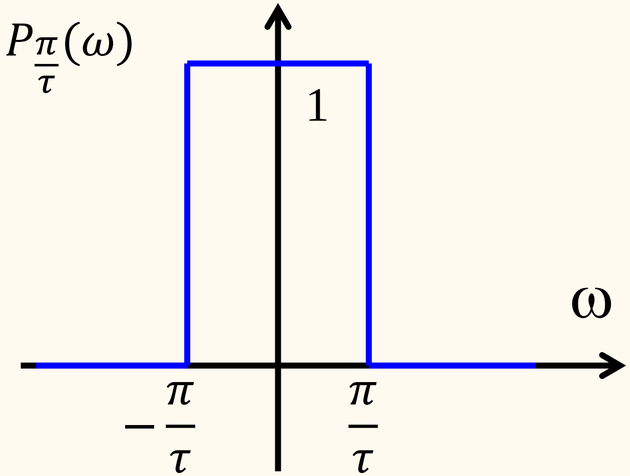
ω_h: Highest frequency in $\tilde{X}_\tau(\omega)$

$$\frac{2\pi}{\tau} > 2\omega_h, \quad \tau < \frac{\pi}{\omega_h}$$

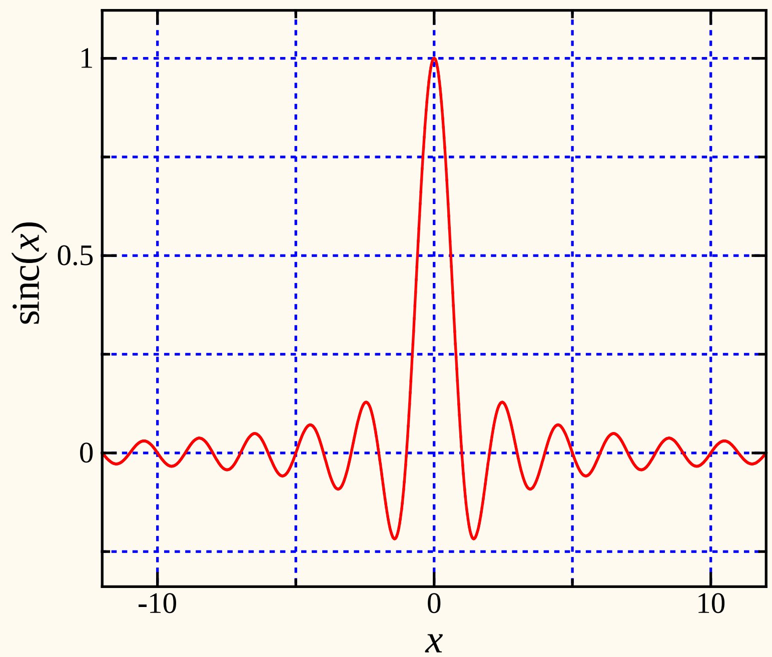
$$\frac{1}{2\tau} \; : \; \text{Nyquist frequency}$$

6.4.1 Sampling theorem: Reconstructing signal

$$P_{\pi/\tau}(\omega) = \begin{cases} 1 & |\omega| \le \dfrac{\pi}{\tau}, \\ 0 & |\omega| > \dfrac{\pi}{\tau} \end{cases}$$

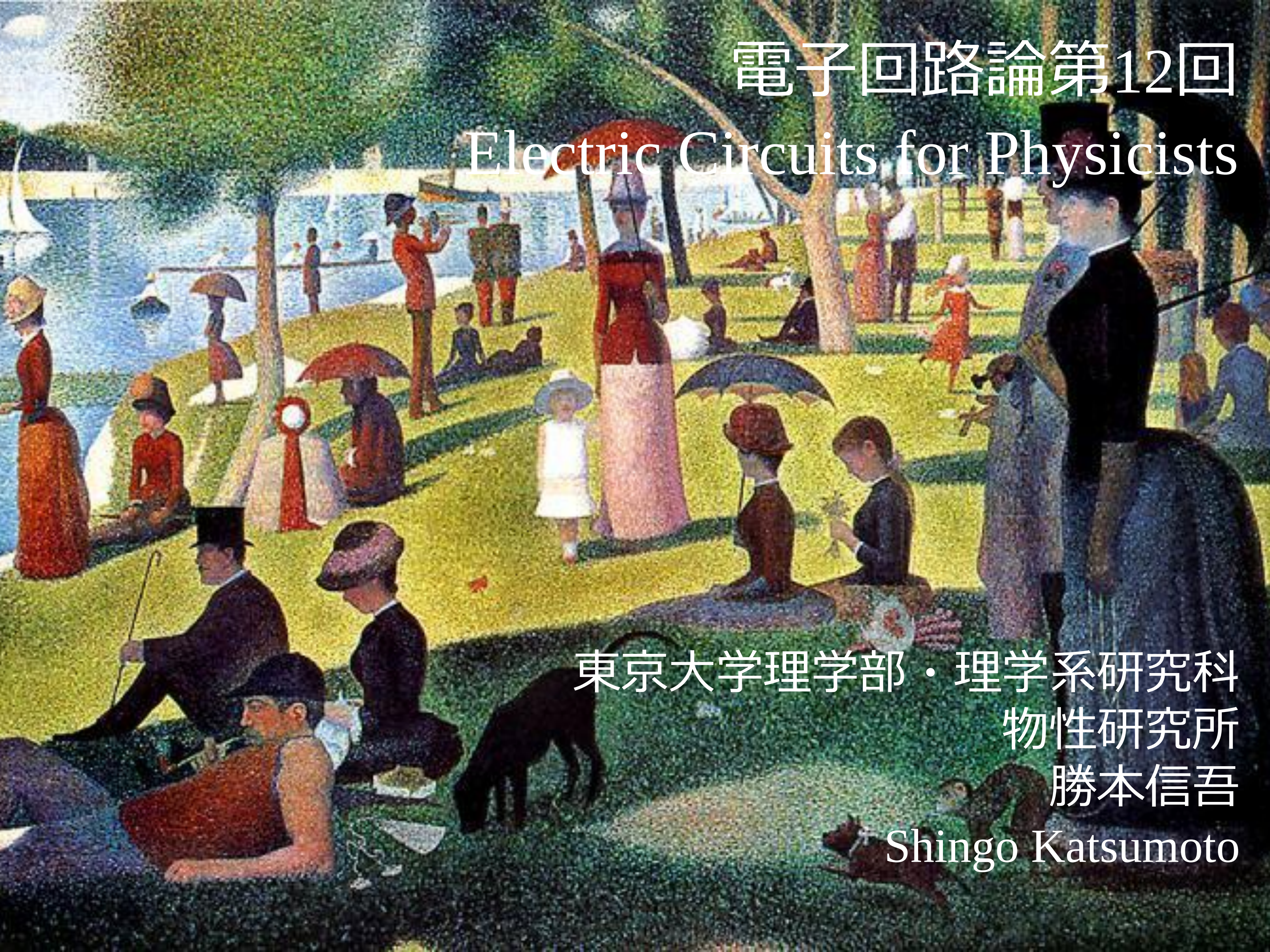


$$x(t) = \mathscr{F}^{-1}\{\tau P_{\pi/\tau}(\omega)\tilde{X}_\tau(\omega)\}$$



$$x(t) = \tau\frac{1}{\tau}\mathrm{sinc}\left(\frac{t}{\tau}\right) * \tilde{x}_\tau(t) = \mathrm{sinc}\left(\frac{t}{\tau}\right) * \sum_{n=-\infty}^{\infty} x(t)\delta(t-n\tau)$$

$$= \int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{s}{\tau}\right)\sum_{n=-\infty}^{\infty} x(t-s)\delta(t-n\tau-s)ds = \sum_{n=-\infty}^{\infty} \mathrm{sinc}\left(\frac{t-n\tau}{\tau}\right)x(n\tau)$$



電子回路論第12回

Electric Circuits for Physicists

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto


Slopestyle

Outline

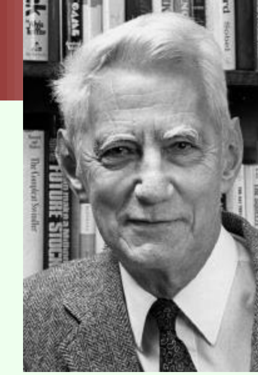
- 6.4 Discrete signal
  - 6.4.1 Sampling theorem
  - 6.4.2 Pulse amplitude modulation (PAM)
  - 6.4.3 Discrete Fourier transform
  - 6.4.4 z-transform
  - 6.4.5 Transfer function of discrete time signal

Ch.7 Digital signals and circuits

- 7.2 Logic gates
- 7.3 Implementation of logic gates
- 7.4 Circuit implementation and simplification of logic operation

6.4 Discrete signal

Isao Someya
1915-2007



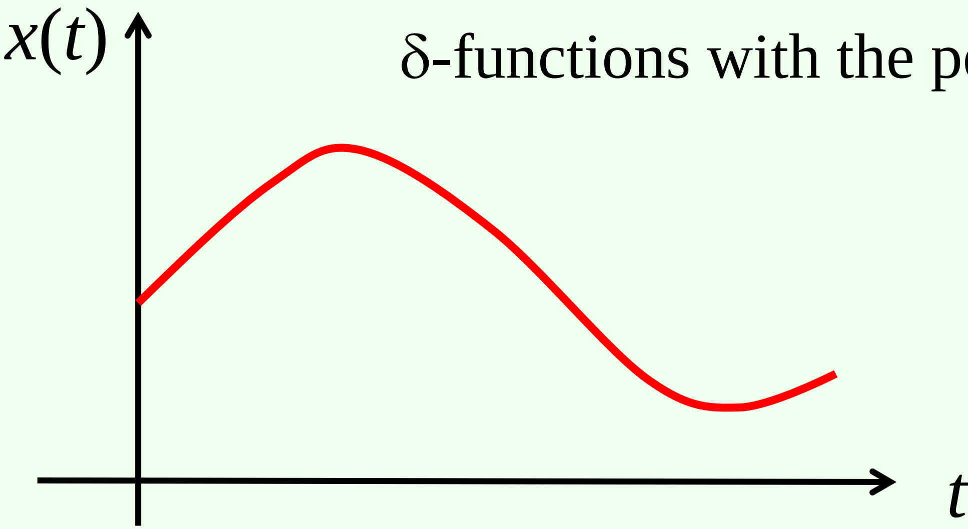
Claude Shannon
1916-2001

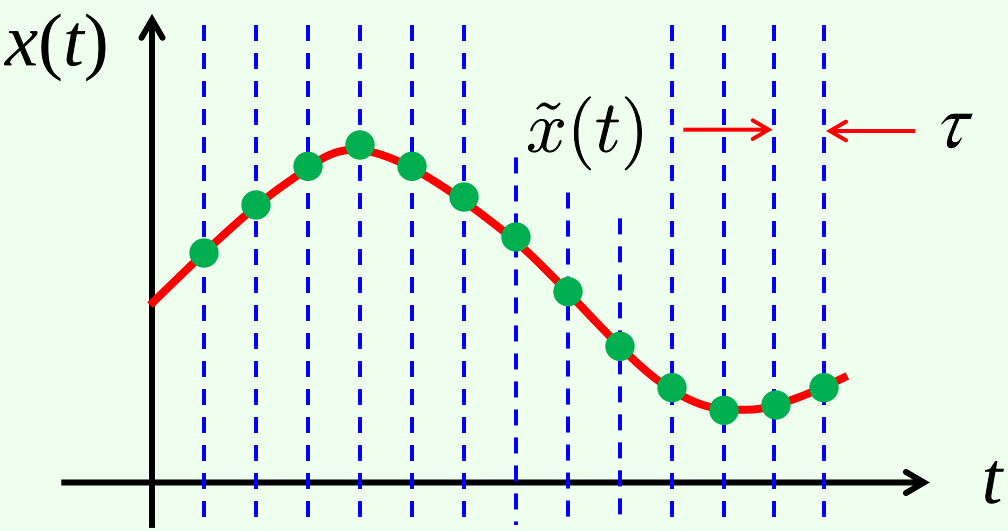
6.4.1 Sampling theorem

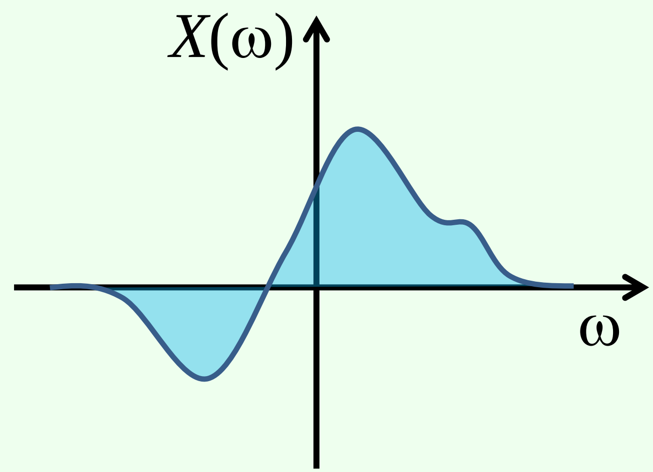
Sampled signal $\tilde{x}(t) = x(t)\underline{\delta_\tau(t)}$

δ-functions with the period τ

1928 H. Nyquist
1949 C. Shannon
染谷勲







6.4.1 Sampling theorem

$$\delta_\tau(t) = \sum_{j=-\infty}^{\infty} \delta(t - j\tau) = \sum_{n=-\infty}^{\infty} \left[\frac{1}{\tau} \int_{-\pi/\tau}^{\pi/\tau} \delta(s) ds \right] \exp\left(-in \frac{2\pi}{\tau} t \right)$$

$$= \frac{1}{\tau} \sum_{n=-\infty}^{\infty} \exp\left(-in \frac{2\pi}{\tau} t \right)$$

$$\mathscr{F}\{\delta_\tau(t)\} = \int_{-\infty}^{\infty} \left[\frac{1}{\tau} \sum_{-\infty}^{\infty} e^{-in(2\pi/\tau)t} \right] e^{i\omega t} dt = \frac{1}{\tau} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left[i \left(\omega - n\frac{2\pi}{\tau} \right) t \right] dt$$

$$= \frac{2\pi}{\tau} \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau} \right) = \frac{2\pi}{\tau} \delta_{2\pi/\tau}(\omega)$$

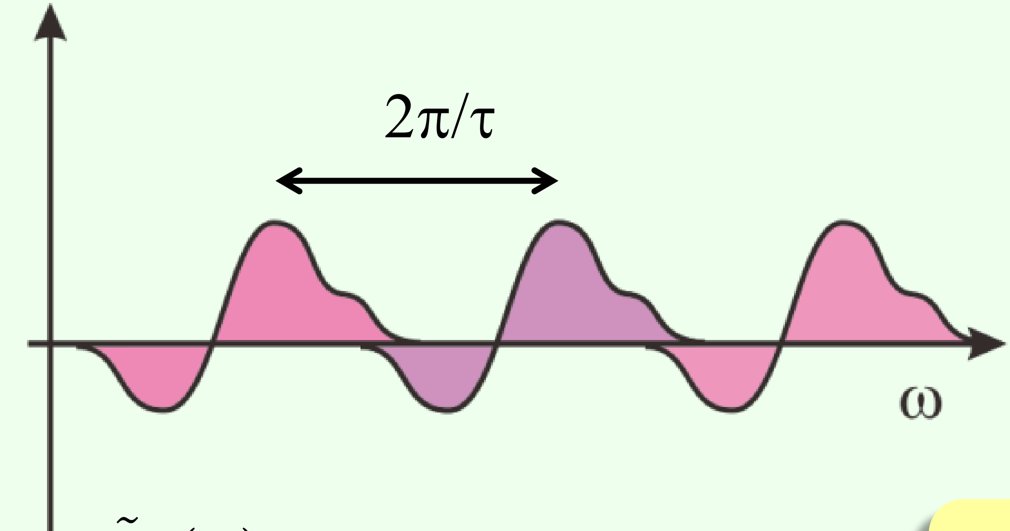
$$\mathscr{F}\{x(t)\} = X(\omega), \ \mathscr{F}\{\tilde{x}_\tau(t)\} = \tilde{X}_\tau(\omega)$$

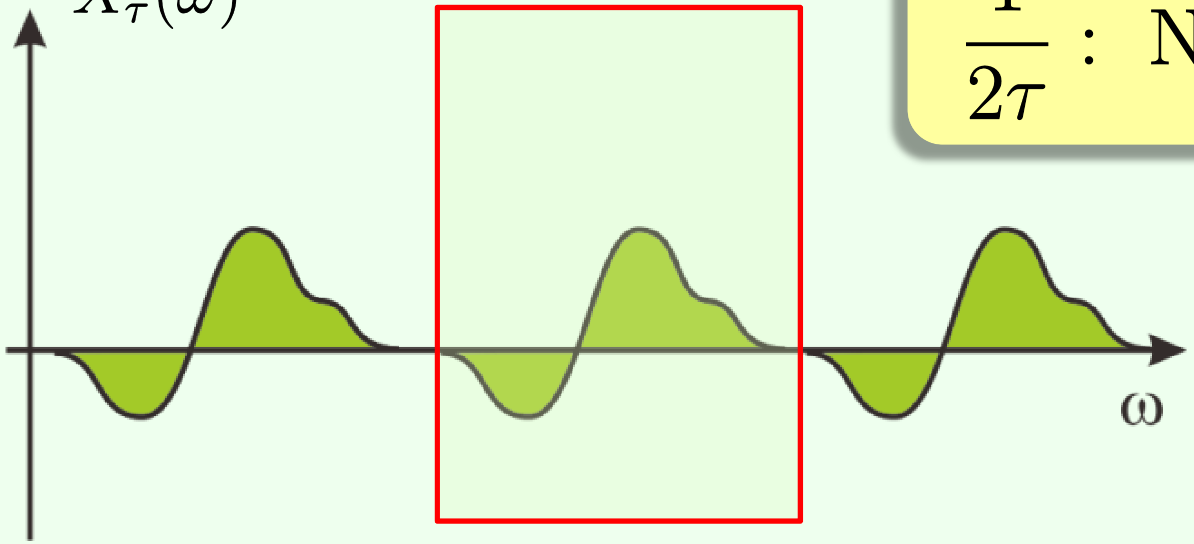
$$\tilde{X}_\tau(\omega) = \frac{1}{2\pi} X(\omega) * \frac{2\pi}{\tau} \delta_{2\pi/\tau}(\omega) = \frac{1}{\tau} X(\omega) * \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau} \right)$$

$$= \frac{1}{\tau} \int_{-\infty}^{\infty} X(\omega') \left\{ \sum_{n=-\infty}^{\infty} \delta\left(\omega - n\frac{2\pi}{\tau} - \omega' \right) \right\} d\omega' = \frac{1}{\tau} \sum_{n=-\infty}^{\infty} X\left(\omega - n\frac{2\pi}{\tau} \right)$$

6.4.1 Sampling theorem

"Cutting out" the frequency spectrum





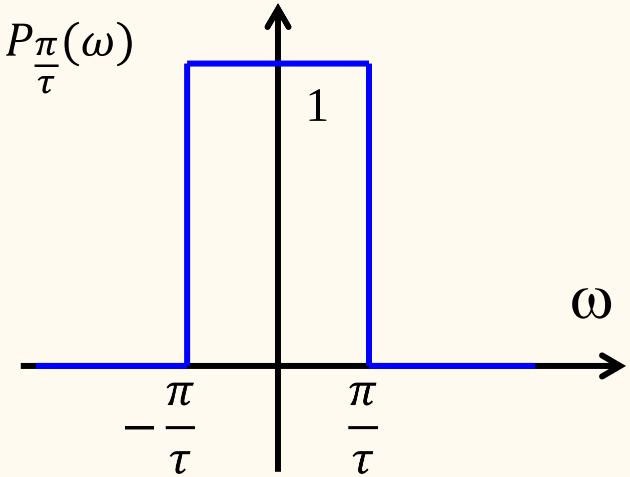
ω_h: Highest frequency in $\tilde{X}_\tau(\omega)$

$$\frac{2\pi}{\tau} > 2\omega_h, \quad \tau < \frac{\pi}{\omega_h}$$

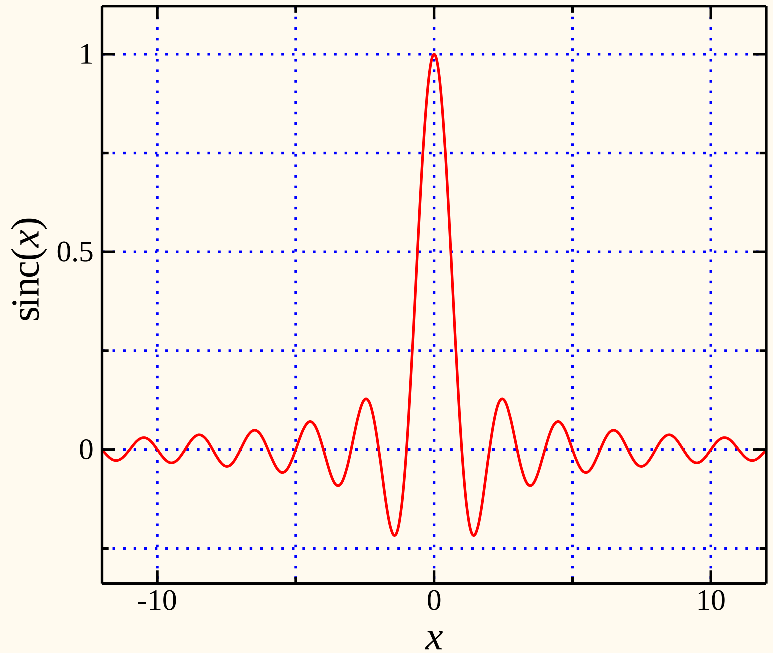
$$\frac{1}{2\tau} : \text{ Nyquist frequency}$$

6.4.1 Sampling theorem: Reconstructing signal

$$P_{\pi/\tau}(\omega) = \begin{cases} 1 & |\omega| \le \dfrac{\pi}{\tau}, \\ 0 & |\omega| > \dfrac{\pi}{\tau} \end{cases}$$



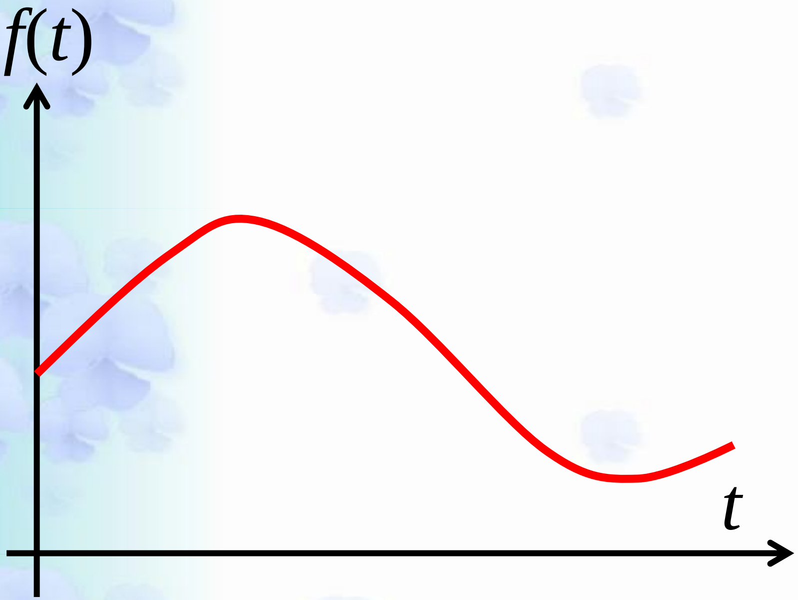
$$x(t) = \mathscr{F}^{-1}\{\tau P_{\pi/\tau}(\omega)\tilde{X}_\tau(\omega)\}$$

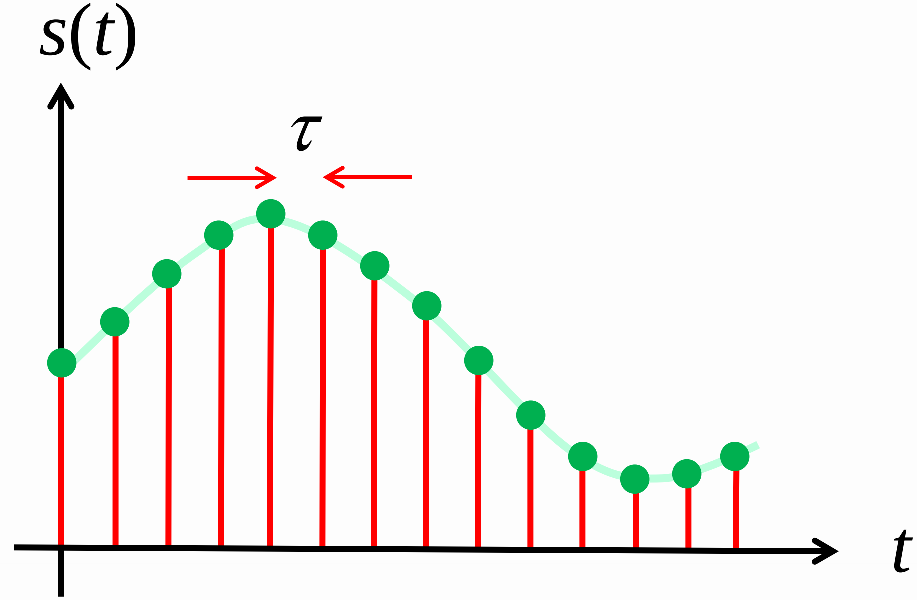


$$x(t) = \tau\frac{1}{\tau}\mathrm{sinc}\left(\frac{t}{\tau}\right) * \tilde{x}_\tau(t) = \mathrm{sinc}\left(\frac{t}{\tau}\right) * \sum_{n=-\infty}^{\infty} x(t)\delta(t-n\tau)$$

$$= \int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{s}{\tau}\right)\sum_{n=-\infty}^{\infty} x(t-s)\delta(t-n\tau-s)ds = \sum_{n=-\infty}^{\infty}\mathrm{sinc}\left(\frac{t-n\tau}{\tau}\right)x(n\tau)$$

6.4.2 Pulse amplitude modulation (PAM)





Carrier: $\delta_\tau(t)$ $\quad s(t) = f(t)\delta_\tau(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\tau)$

Demodulation = Reconstruction of continuous signal from sampled data.

$$f(t) = \mathscr{F}^{-1}\{P_{\pi/\tau}(\omega)\mathscr{F}\{s(t)\}\}$$

Demodulation of PAM and a trick in the sampling theorem

In the sampling theorem, though we only have discrete-time data, we can reconstruct complete original signal.

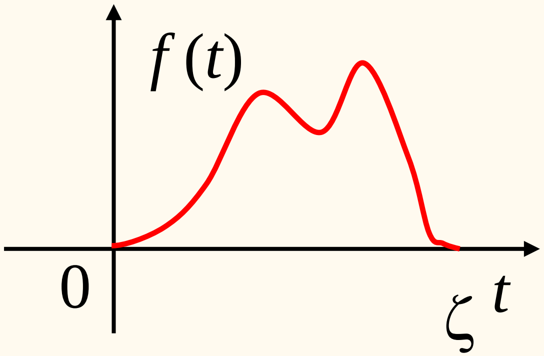
↑

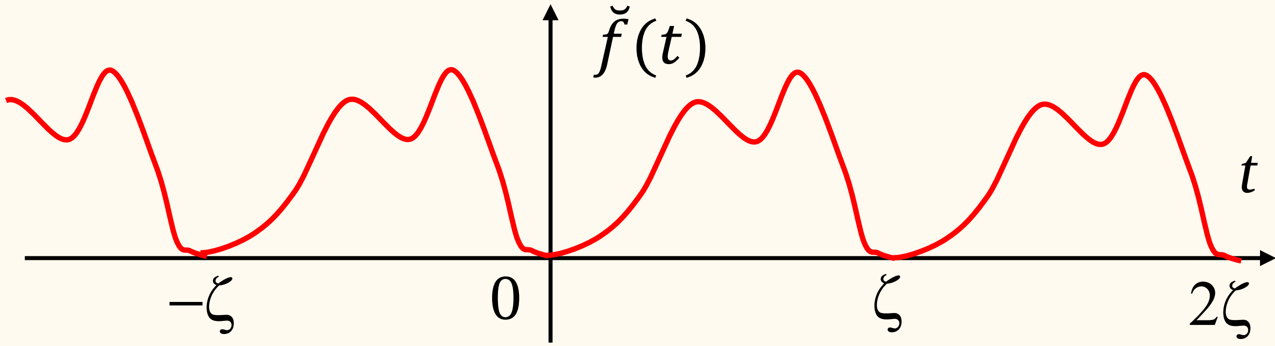
Assumption: we have data in infinite period $[-\infty, +\infty]$.

However in actual situations we can never have such data.

Need to consider handling data in a finite period.

6.4.3 Discrete Fourier transform





Assumption:

$F(\omega) = \mathscr{F}\{f(t)\}$, not zero in $\omega \in \left(-\frac{\pi}{\tau}, \frac{\pi}{\tau}\right)$

$$N = \frac{\zeta}{\tau} \in \mathbb{N}$$

can be assumed without loosing generality

$$\breve{f}(t) = \sum_{n=-\infty}^{\infty} f(t - n\zeta), \quad \breve{F}(\omega) = \sum_{n=-\infty}^{\infty} F\left(\omega + n\frac{2\pi}{\zeta}\right)$$

$$\left(\breve{f}(t) = (f * \delta_\zeta)(t) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi)\delta(t - n\zeta - \xi)d\xi\right)$$

Fourier expansion: $$\breve{f}(t) = \frac{1}{\zeta} \sum_{n=-\infty}^{\infty} F\left(n\frac{2\pi}{\zeta}\right) \exp\left(2n\pi i\frac{t}{\zeta}\right)$$

6.4.3 Discrete Fourier transform

$$n = l + mN \qquad \sum_{n=-\infty}^{\infty} \to \sum_{l=0}^{N-1} \sum_{m=-\infty}^{\infty}$$

Discreteness:

$$t = j\tau \quad j \in \mathbb{Z}$$

$$\breve{f}(j\tau) = \frac{1}{\zeta} \sum_{l=0}^{N-1} \sum_{m=-\infty}^{\infty} F\left[(l+mN)\frac{2\pi}{\zeta}\right] \exp\left[(l+mN)2\pi i \frac{j\tau}{\zeta}\right]$$

$$= \frac{1}{N\tau} \sum_{l=0}^{N-1} \sum_{m=-\infty}^{\infty} F\left(\frac{2\pi l}{\zeta} + m\frac{2\pi}{\tau}\right) \exp\left(2\pi i \frac{lj}{N}\right)$$

$$= \frac{1}{N\tau} \sum_{l=0}^{N-1} \breve{F}\left(l\frac{2\pi}{\zeta}\right) \exp\left(2\pi i \frac{lj}{N}\right)$$

Twiddle factor: $W_N \equiv \exp\left(-i\frac{2\pi}{N}\right)$

$$\eta \equiv \frac{2\pi}{\zeta} \qquad \breve{f}(j\tau) = \frac{1}{N\tau} \sum_{l=0}^{N-1} \breve{F}(l\eta) W_N^{-lj}$$

6.4.3 Discrete Fourier transform

$$\forall n, m \in \mathbb{Z} \quad W_N^{n+mN} = W_N^n,$$

Twiddle factor:

$$\frac{1}{N}\sum_{n=0}^{N-1} W_N^{nm} = \begin{cases} 1 & \text{for} \quad m = 0, \\ 0 & \text{for} \quad m \neq 0. \end{cases}$$

$$\tau \sum_{j=0}^{N-1} \breve{f}(j\tau) W_N^{mj} = \sum_{j=0}^{N-1} \left[\frac{1}{N} \sum_{l=0}^{N-1} \breve{F}(l\eta) W_N^{(m-l)j} \right] = \breve{F}(m\eta)$$

$$f_n \equiv \breve{f}(n\tau), \quad F_k \equiv \frac{1}{\tau}\breve{F}(k\eta)$$

Discrete Fourier transform:
(DFT)

$$F_k = \sum_{n=0}^{N-1} f_n W_N^{kn},$$

$$f_n = \frac{1}{N}\sum_{k=0}^{N-1} F_k W_N^{-kn}.$$

6.4.3 Discrete Fourier transform

$$\boldsymbol{F} = {}^t\{F_i\},\ \boldsymbol{W} = \{W_N^{ij}\},\ \boldsymbol{f} = {}^t\{f_i\}$$

$$\boldsymbol{F} = \boldsymbol{W}\boldsymbol{f}, \quad \boldsymbol{f} = \frac{1}{N}\boldsymbol{W}^*\boldsymbol{F}$$

$${}^t\boldsymbol{W}^*\boldsymbol{W} = N\boldsymbol{I}_N \quad i.e., \frac{1}{\sqrt{N}}\boldsymbol{W} \ : \text{unitary}$$

6.4.4 z-transform

Discrete Laplace transform: z-transform

$$\tilde{f}_\tau(t) = \sum_{n=0}^{\infty} f(n\tau)\delta(t - n\tau) \quad (t \geq 0)$$

$$\mathscr{L}\{\tilde{f}_\tau(t)\}(s) = \mathscr{L}\left\{\sum_{n=0}^{\infty} f(n\tau)\delta(t - n\tau)\right\}$$

$$= \sum_{n=0}^{\infty} f(n\tau)\mathscr{L}\{\delta(t - n\tau)\} = \sum_{n=0}^{\infty} f(n\tau)\exp(-sn\tau)$$

$$z = \exp(s\tau),\ f_n = f(n\tau),\ F(z) = \mathscr{L}\{\tilde{f}_\tau(t)\}$$

$$F(z) = \sum_{n=0}^{\infty} f_n z^{-n} = \mathscr{Z}[\tilde{f}_\tau(t)]$$

one-sided z-transform

6.4.4 z-transform

f_n	$F(z)$	conversion area
$\delta(n)$	1	z-plane
1	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
n	$\dfrac{z^{-1}}{(1-z^{-1})^2}$	$\lvert z\rvert > 1$
n^k	$\left(-z\dfrac{d}{dz}\right)^k \dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
a^n	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$\sin(n\omega\tau)$	$\dfrac{\sin(\omega\tau)z^{-1}}{1-2\cos(\omega\tau)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$e^{-n\alpha\tau}\cos(n\omega\tau)$	$\dfrac{1-e^{-\alpha\tau}\cos(\omega\tau)z^{-1}}{1-2e^{-\alpha\tau}\cos(\omega\tau)z^{-1}+e^{-2\alpha\tau}z^{-2}}$	$\lvert z\rvert > e^{-\alpha\tau}$

6.4.4 z-transform

Property	Signal	z-transform
linearity	$af_n + bg_n$	$aF(z) + bG(z)$
z-domain scaling	$f_{\alpha n}$	$F(z^{1/\alpha})$
time shift	f_{n+k}	$z^k \left[F(z) - \sum_{l=0}^{k-1} f(l) z^l \right]$
time shift II	f_{n-k}	$z^{-k} F(z)$
scaling	$e^{\mp \alpha n} f_n$	$F(e^{\pm \alpha} z)$
scaling II	$a^n x_n$	$F(a^{-1} z)$
product with index	$n f_n$	$-z \frac{d}{dz} F(z)$
differentiation	$n^k f_n$	$\left(-z \frac{d}{dz} \right)^n F(z)$
integration	$\frac{f_n}{n+a}$	$z^a \int_z^\infty \xi^{-a+1} F(\xi) d\xi$
convolution	$f_n * g_n$	$F(z) \cdot G(z)$
product	$f_n \cdot g_n$	$\frac{1}{2\pi i} \oint_c F(\xi) G\left(\frac{z}{\xi} \right) \xi^{-1} d\xi$

6.4.5 Transfer function for discrete time signal

$$\tilde{f}_\tau(t) = f(t)\delta_\tau(t) = \sum_{k=-\infty}^{\infty} f_k \delta(t - k\tau)$$

h_n: (impulse) response to $\delta(n\tau)$, response to discrete signal $f_n = f\ (n\tau)$

$$g_n = \mathscr{R}\{\tilde{f}_\tau(n\tau)\} = \mathscr{R}\left\{\sum_{k'=-\infty}^{\infty} f(k'\tau)\delta[(n-k')\tau]\right\}$$

$$= \sum_{k'=-\infty}^{\infty} f_{k'} h_{n-k'} = \sum_{k=-\infty}^{\infty} h_k f_{n-k}$$

$$G(z) = \mathscr{Z}[g_n] = \mathscr{Z}\left[\sum_{k=0}^{\infty} h_k f_{n-k}\right] = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{\infty} h_k f_{n-k}\right) z^{-n}$$

$$= \sum_{k=0}^{\infty} h_k \sum_{n=0}^{\infty} f_{n-k} z^{-n} = \sum_{k=0}^{\infty} h_k z^{-k} F(z)$$

$$H(z) = \mathscr{Z}[h_n] = \sum_{k=0}^{\infty} h_k z^{-k}$$: Transfer function

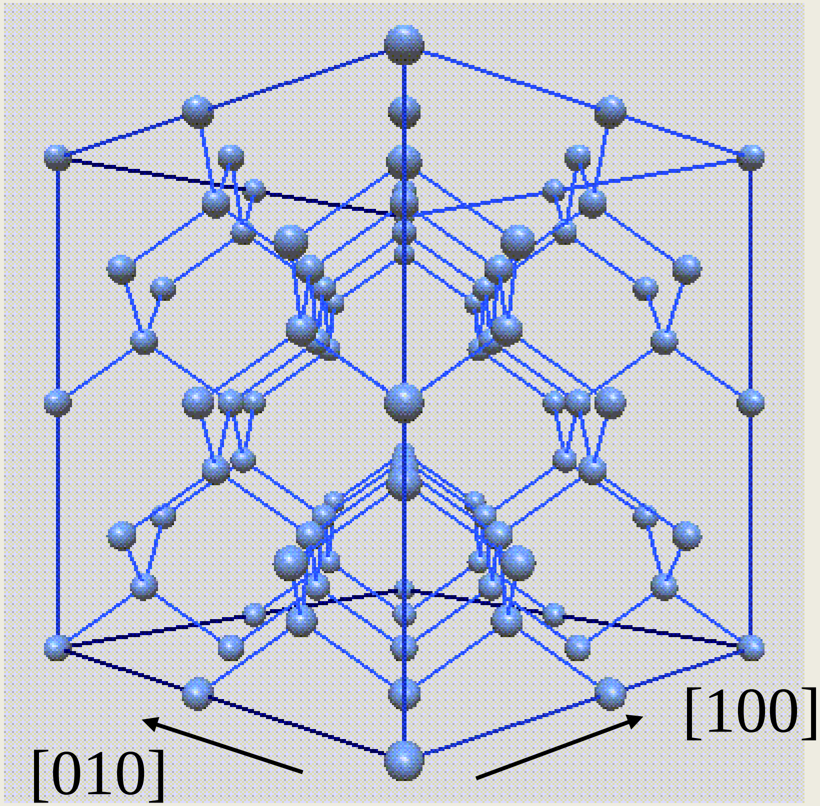
$$G(z) = H(z)F(z)$$

Crystal lattice and X-ray diffraction

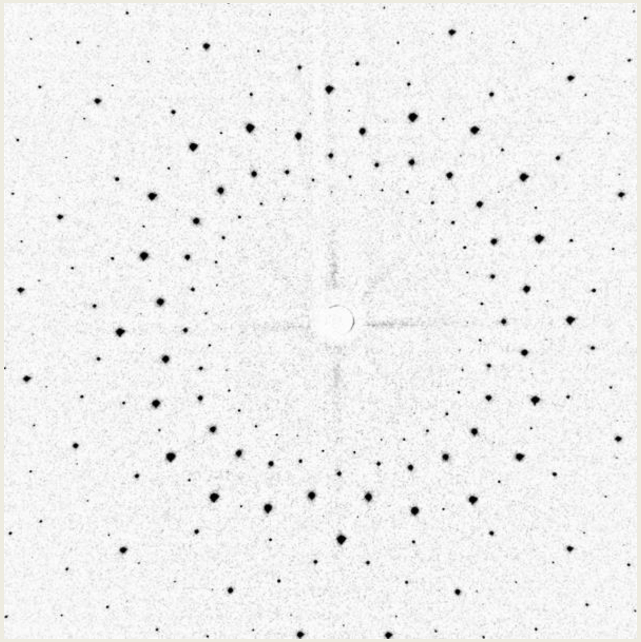
Max von Laue
1879-1960

Laue pattern

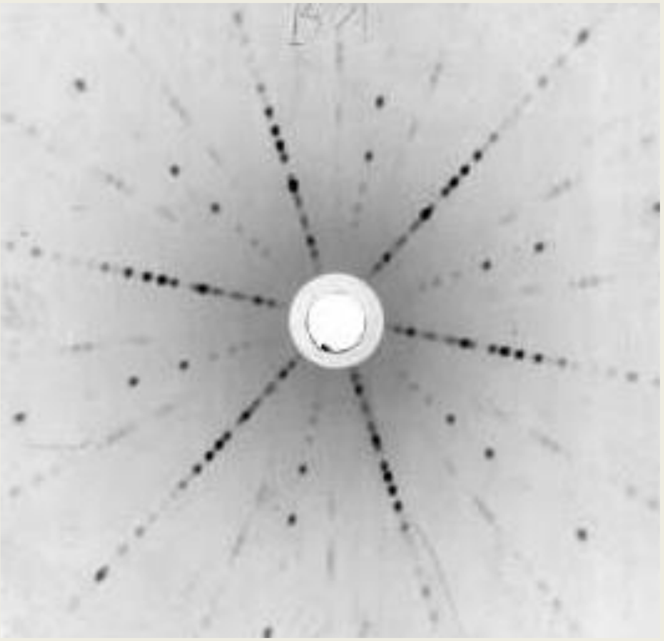
Diamond lattice



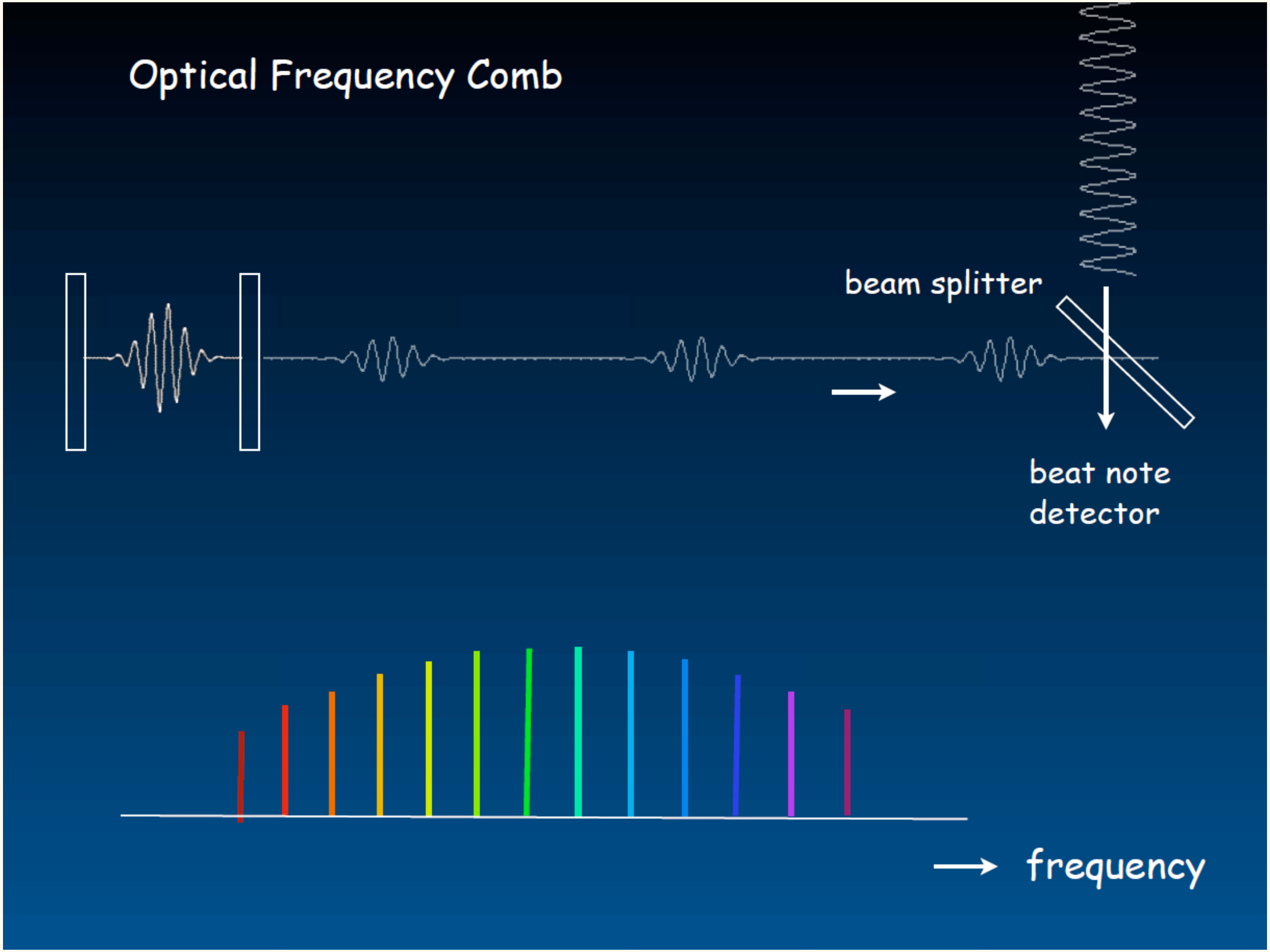
[100]



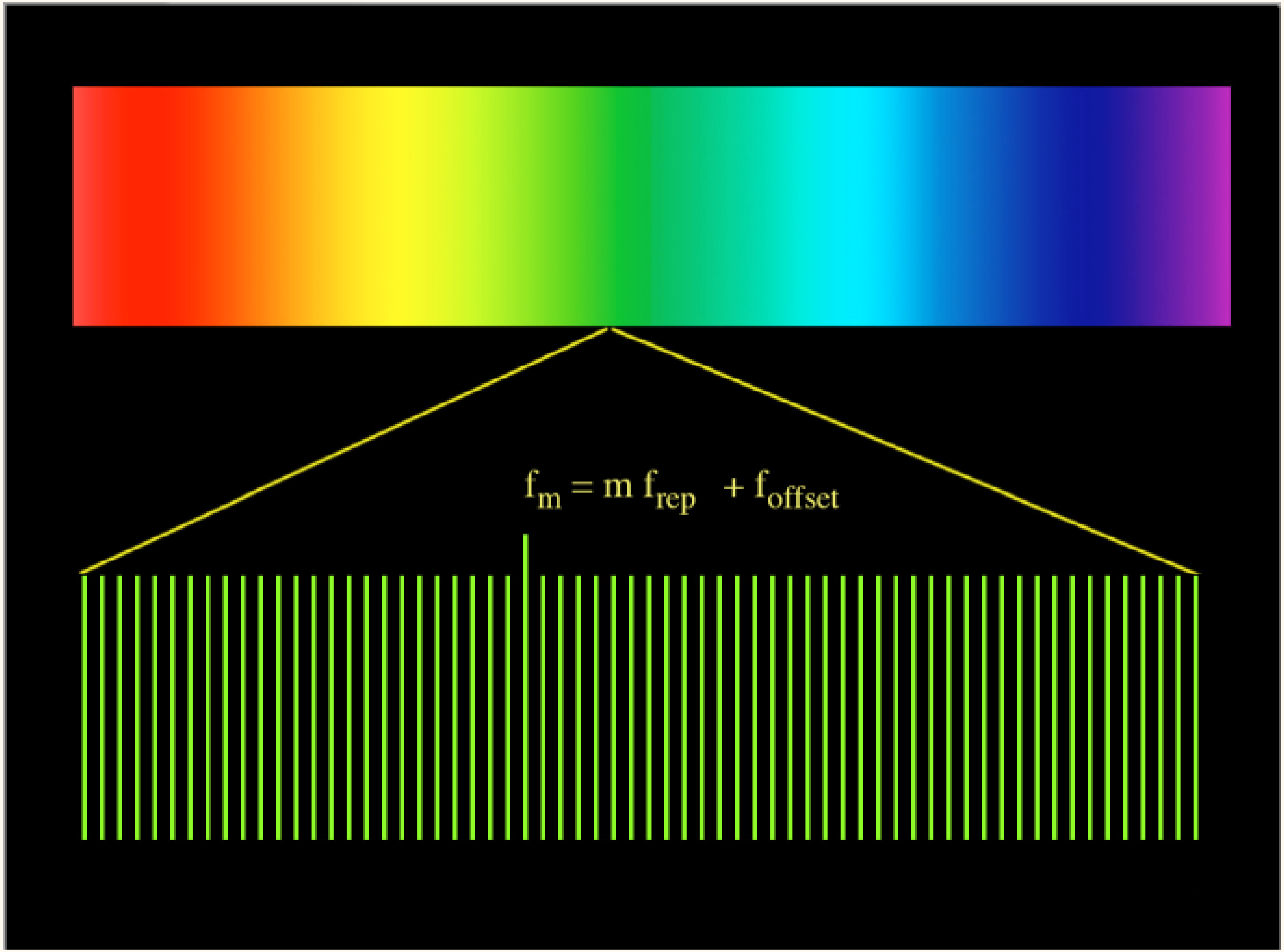
[111]



Optical Frequency Comb

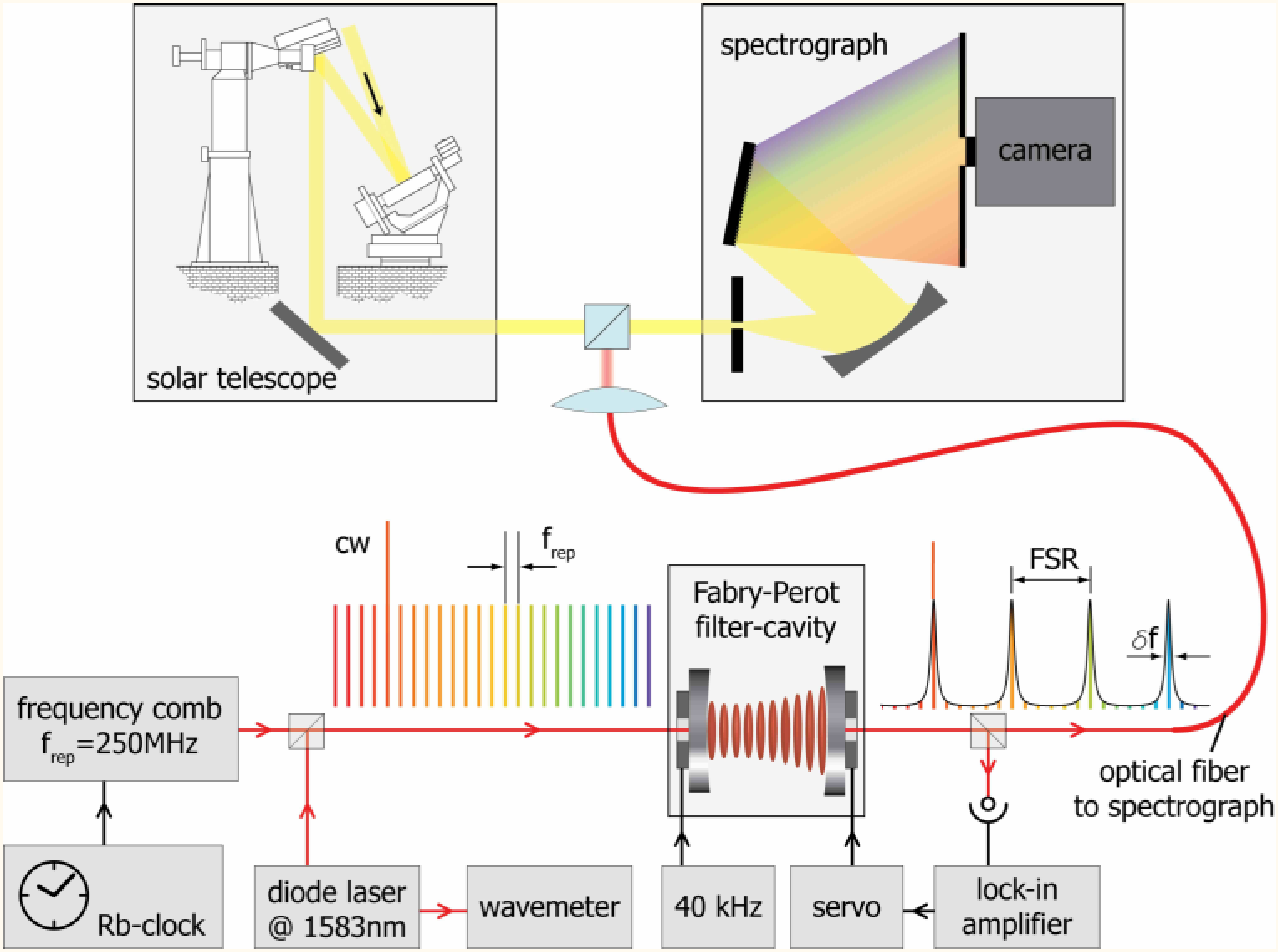


Frequency Comb



Theodor Hänsch, Max-Planck Institute, Science 2008

Measurement of the Doppler effect in cosmic expansion

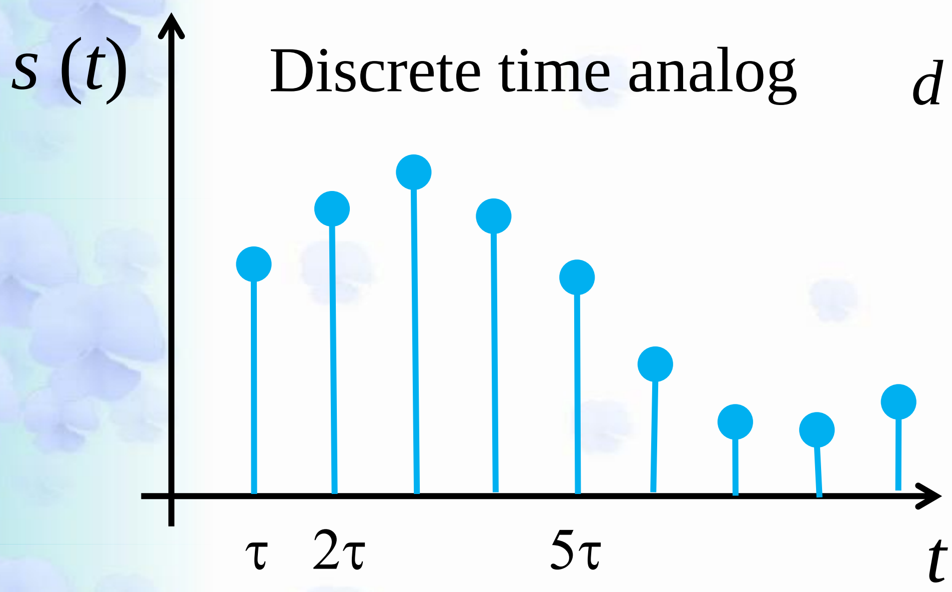


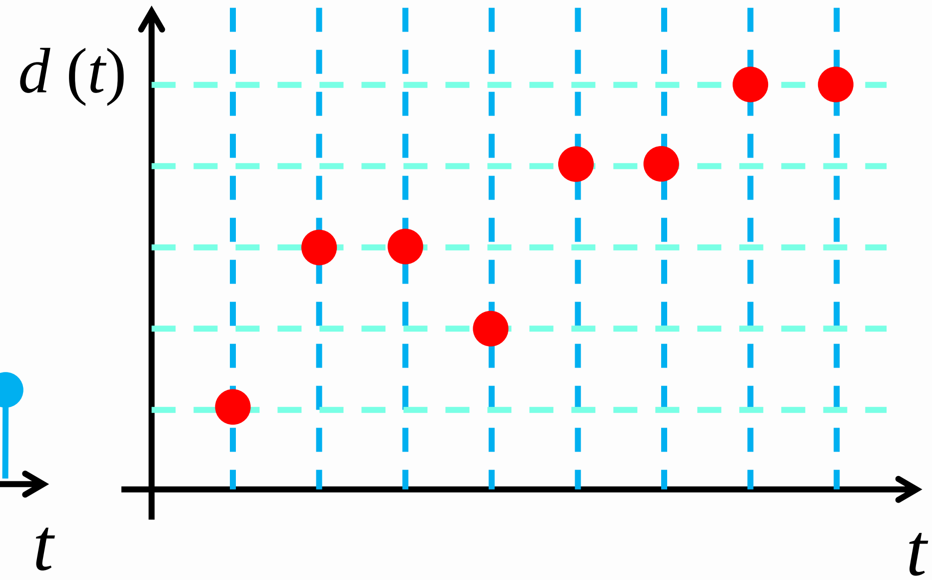
Byzantine mosaic

Chapter 7

Digital signal and circuits

Chartres Blue
(Stained glass)





Value discretized → Digital signal

Signal unit : 0 xor 1 (bit)

Boolean algebra : F xor T

Voltage level : L xor H

Multiple bit → binary operation → parallel signal

7.2 Logic gates

Digital signal=logic value $\rightarrow$ Logic operation : logic gates

De Morgan's laws: $\overline{x+y} = \bar{x}\cdot\bar{y},\ \overline{x\cdot y} = \bar{x}+\bar{y}$

		input					output			
t		t_1	t_2	$\cdots$	t_m		t_1	t_2	$\cdots$	t_m
Ch.	1	0	1	$\cdots$	f_{1m}	1	1	1	$\cdots$	q_{1m}
	2	1	0	$\cdots$	f_{2m}	2	0	1	$\cdots$	q_{2m}
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	n	0	1	$\cdots$	f_{nm}	l	0	1	$\cdots$	f_{lm}

Combinational logic $\rightarrow$ Truth table
Sequential logic $\rightarrow$ Timing chart

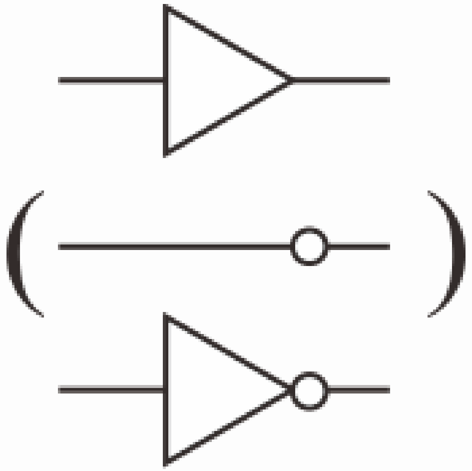
7.2.1 Combinational logic: Single input gates

Truth table

input	buffer	not
0	0	1
1	1	0

Circuit symbol

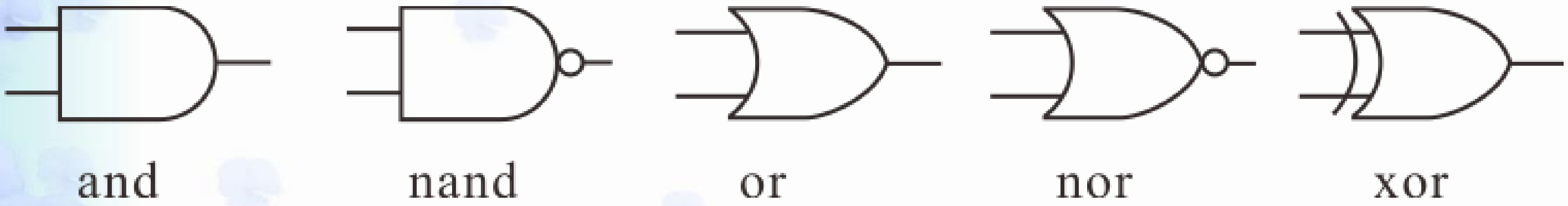
buffer



not

7.2.2 Combinational logic: Double input gates

input1	input 2	and	or	xor	nand
0	0	0	0	0	1
1	0	0	1	1	1
0	1	0	1	1	1
1	1	1	1	0	0



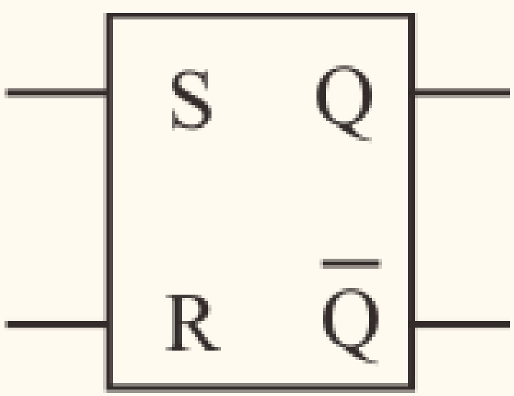
7.2.3 Sequential logic: Flip-Flop (FF)

RS (reset-set) Flip-Flop (FF)

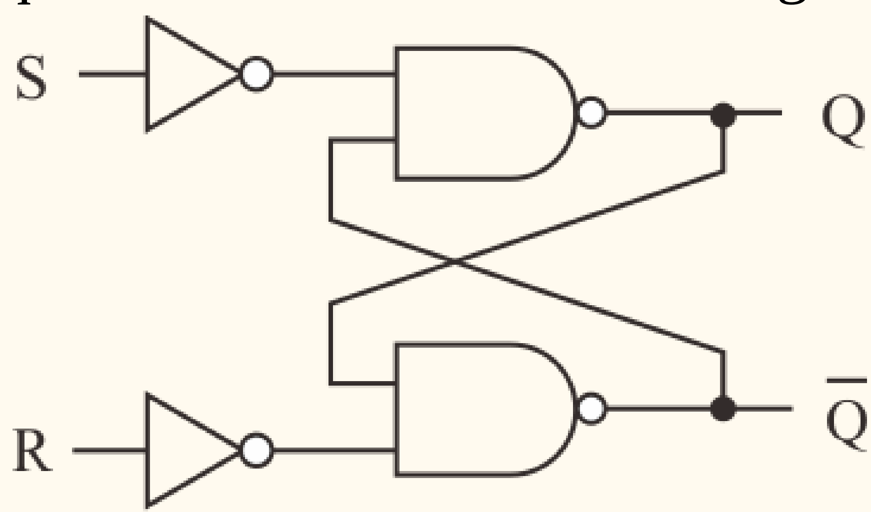
Truth table

S	R	Q	$\bar{Q}$	Response
0	0	Q	$\bar{Q}$	no change
0	1	0	1	reset
1	0	1	0	set
1	1	undetermined		

Symbol



Equivalent circuit with discrete gates

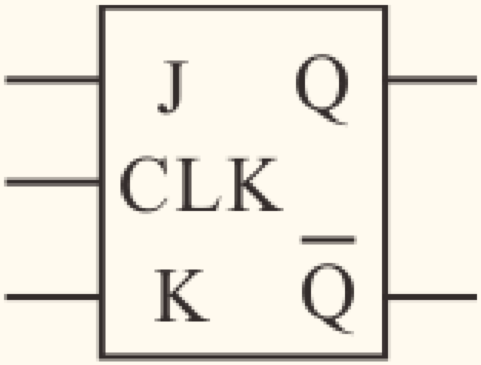


JK Flip-Flop

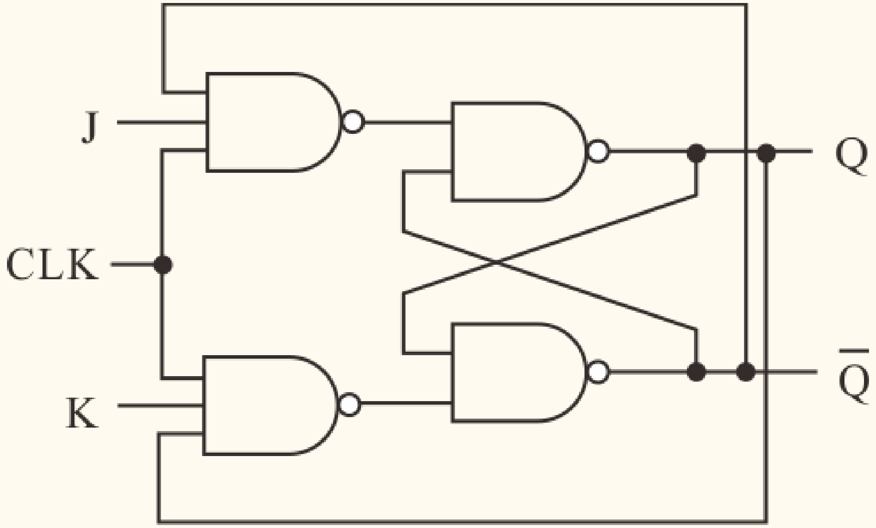
Truth table

J	K	Q	Q for the next CLK
0	0	0	0
0	0	1	1
0	1	–	0
1	0	–	1
1	1	0	1
1	1	1	0

Symbol



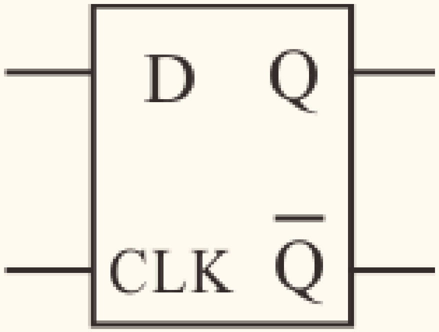
Equivalent circuit with discrete gates



7.2.3 Sequential logic: D-FF, T-FF

D-FF

Symbol

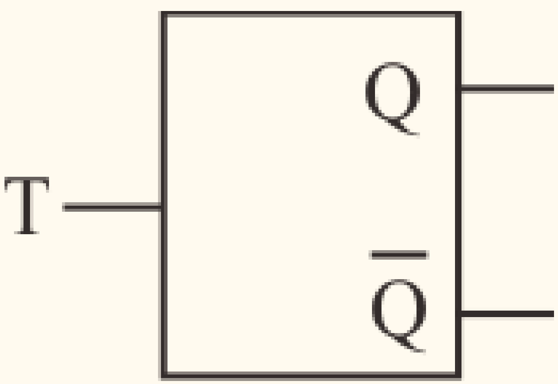


Truth table

D	CLK	Q
0	↑	0
1	↑	1
–	↓	Q (hold)

T-FF

Symbol

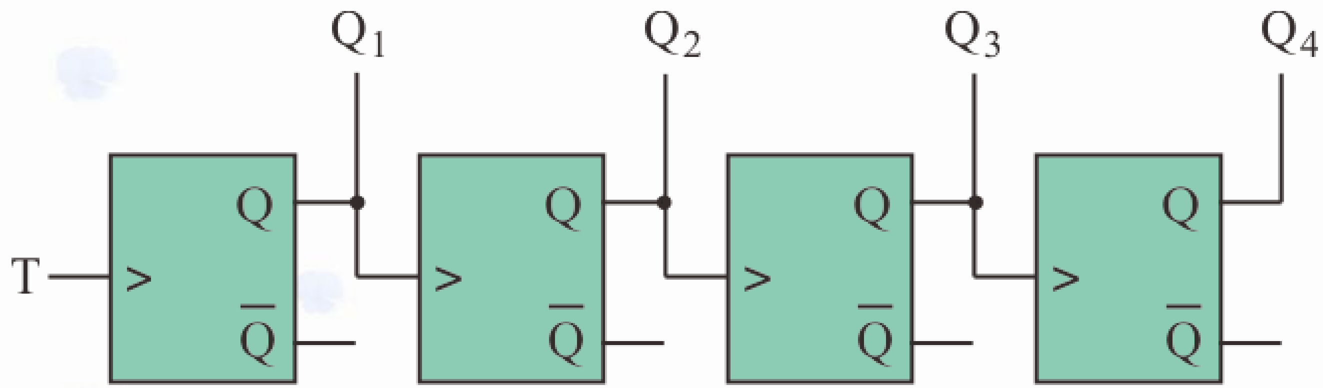


Truth table

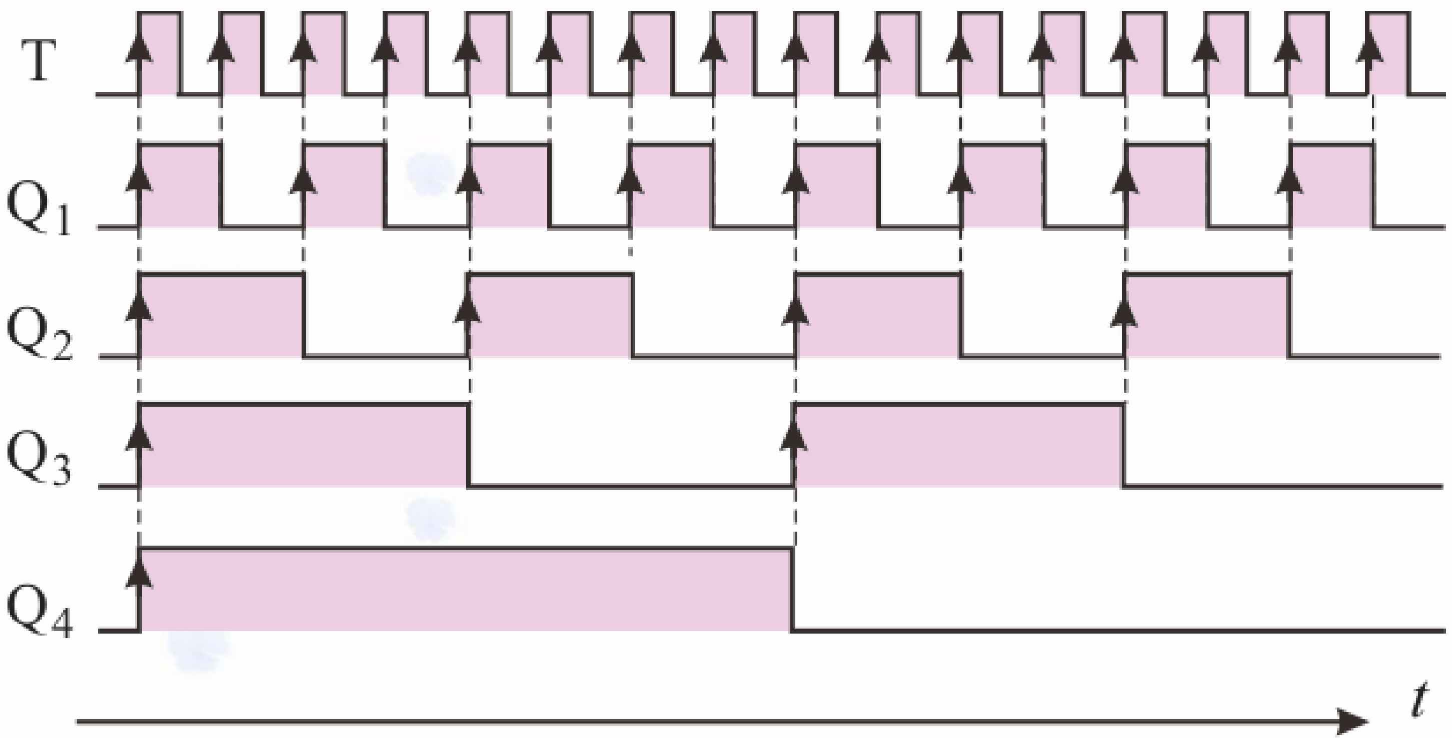
T	Q	Q
↓	0	0
↓	1	1
↑	0	1
↑	1	0

7.2.4 Sequential logic: Counters

Unsynchronized counter (ripple counter)



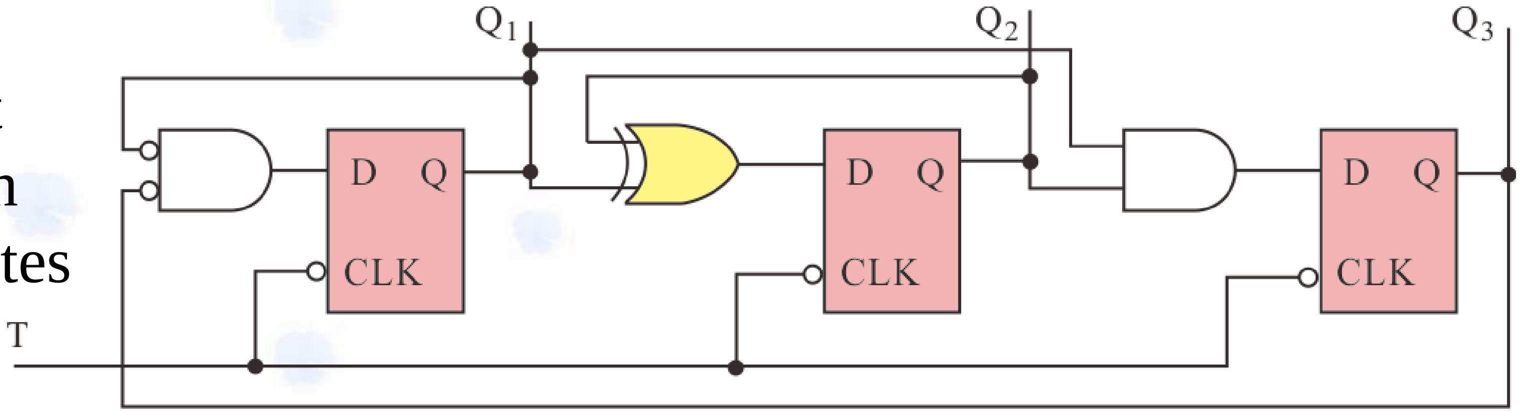
Timing chart



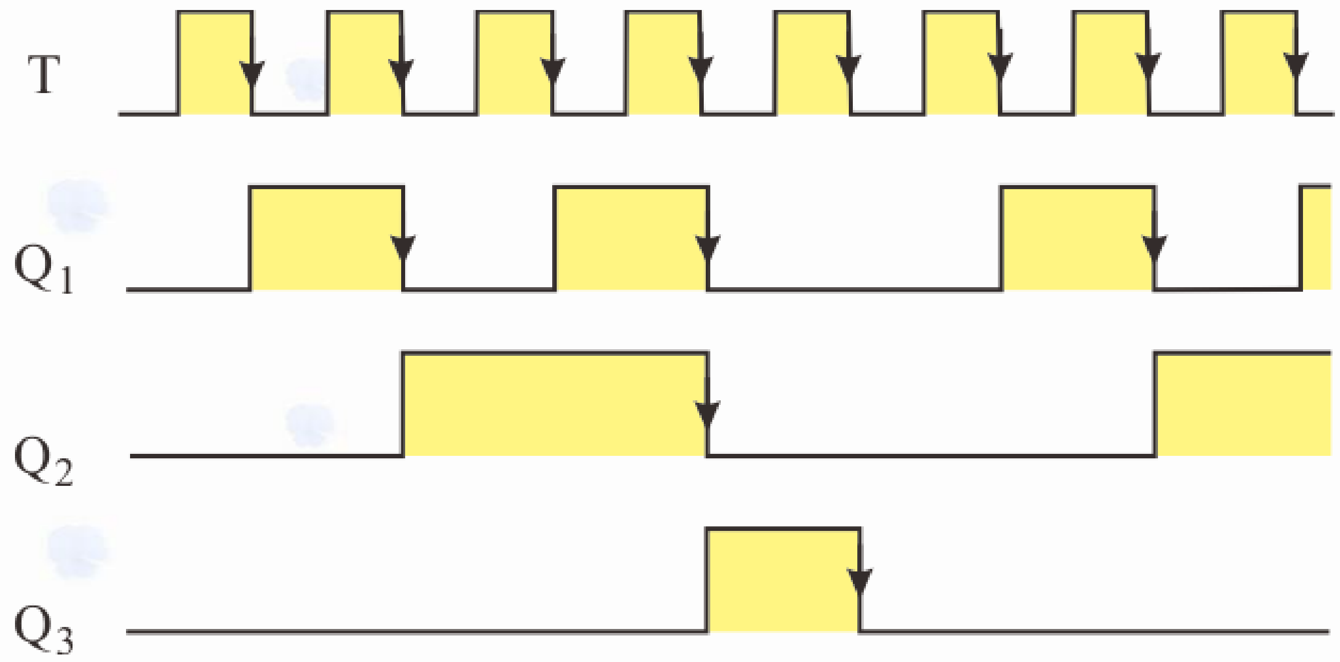
7.2.4 Sequential logic: Counters

Synchronized counter

Equivalent circuit with discrete gates

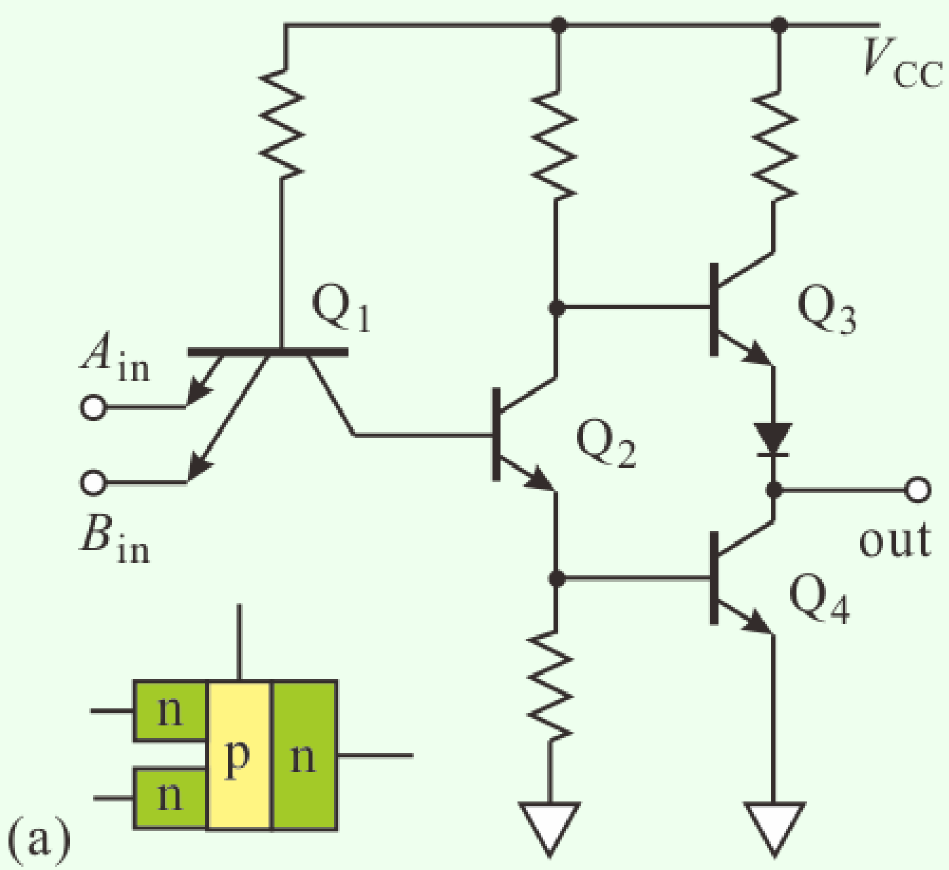


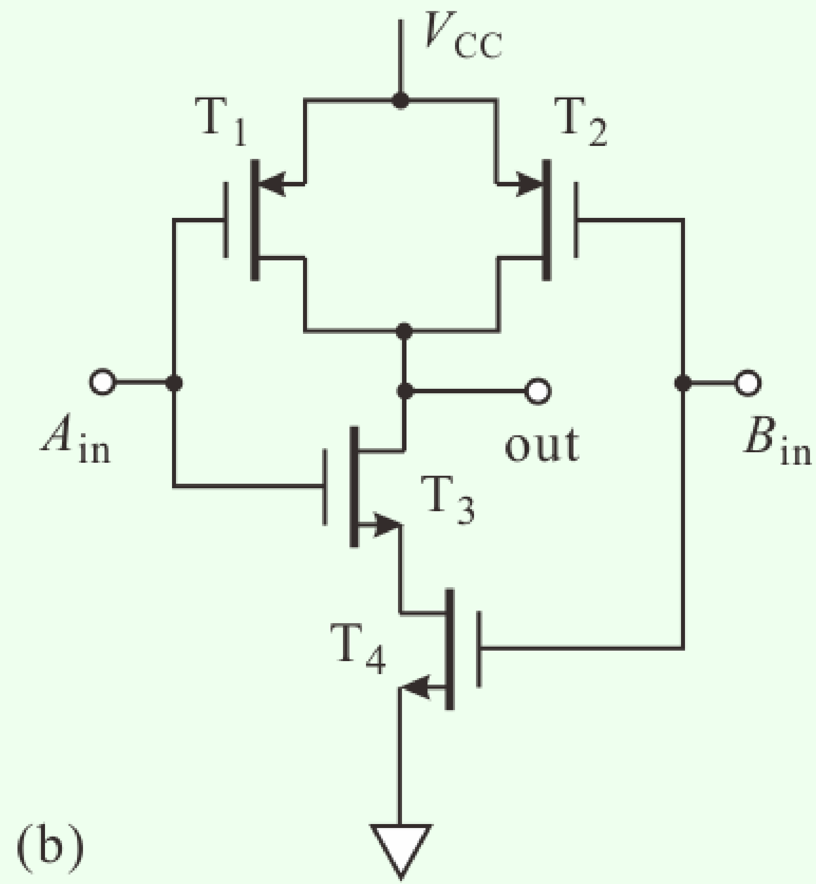
Timing chart



7.3 Implementation of logic gates

NAND gates



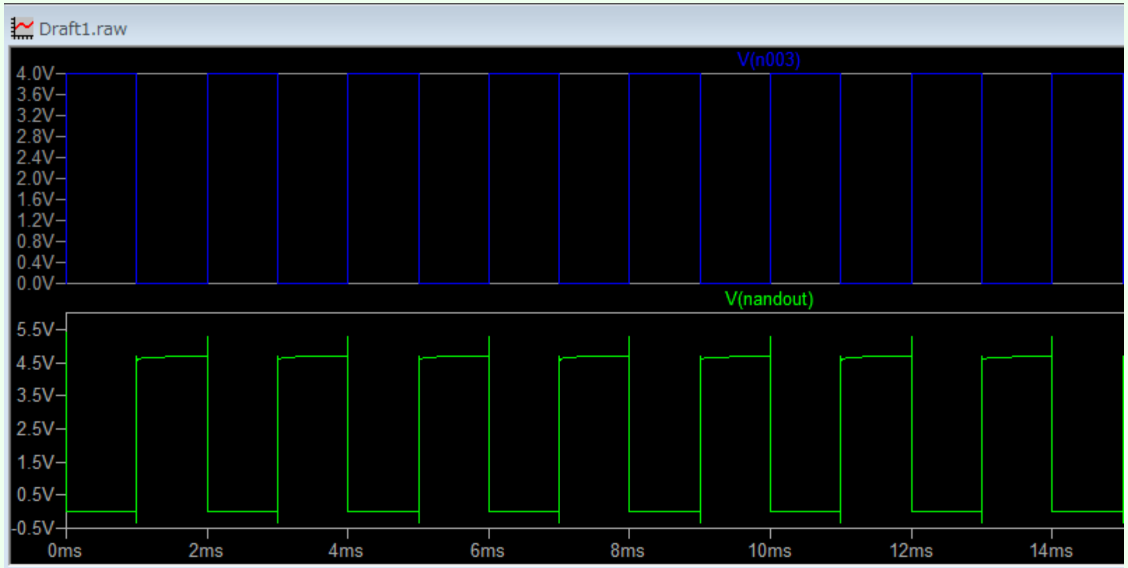


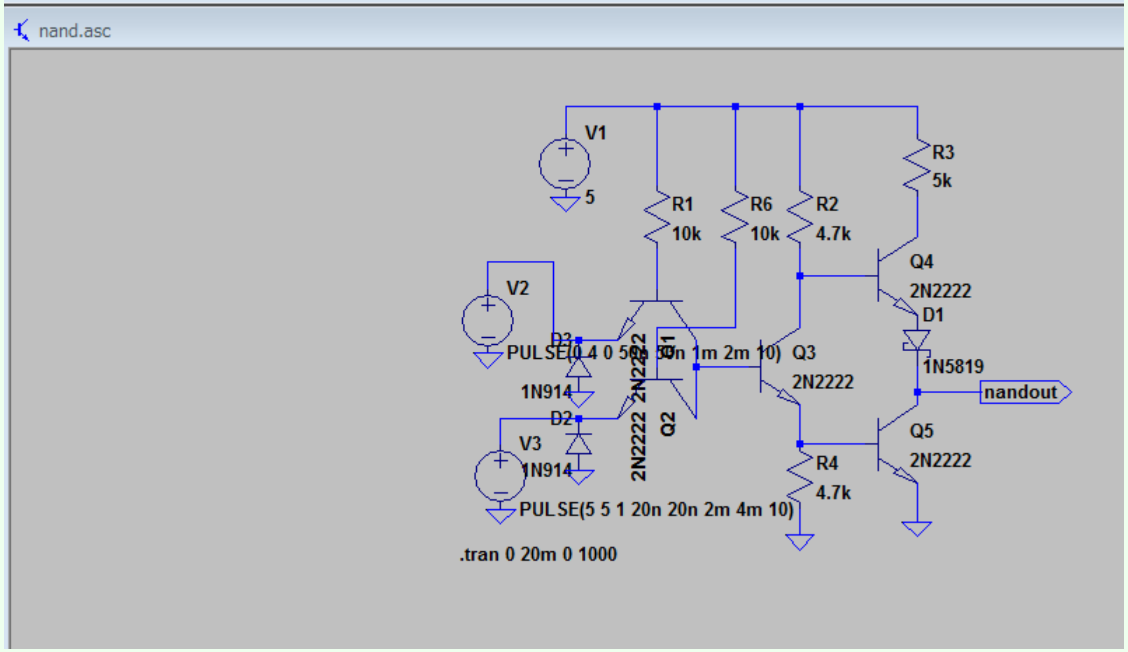
TTL (transistor-transistor logic)

CMOS (complimentary MOS)

7.3 Implementation of logic gates

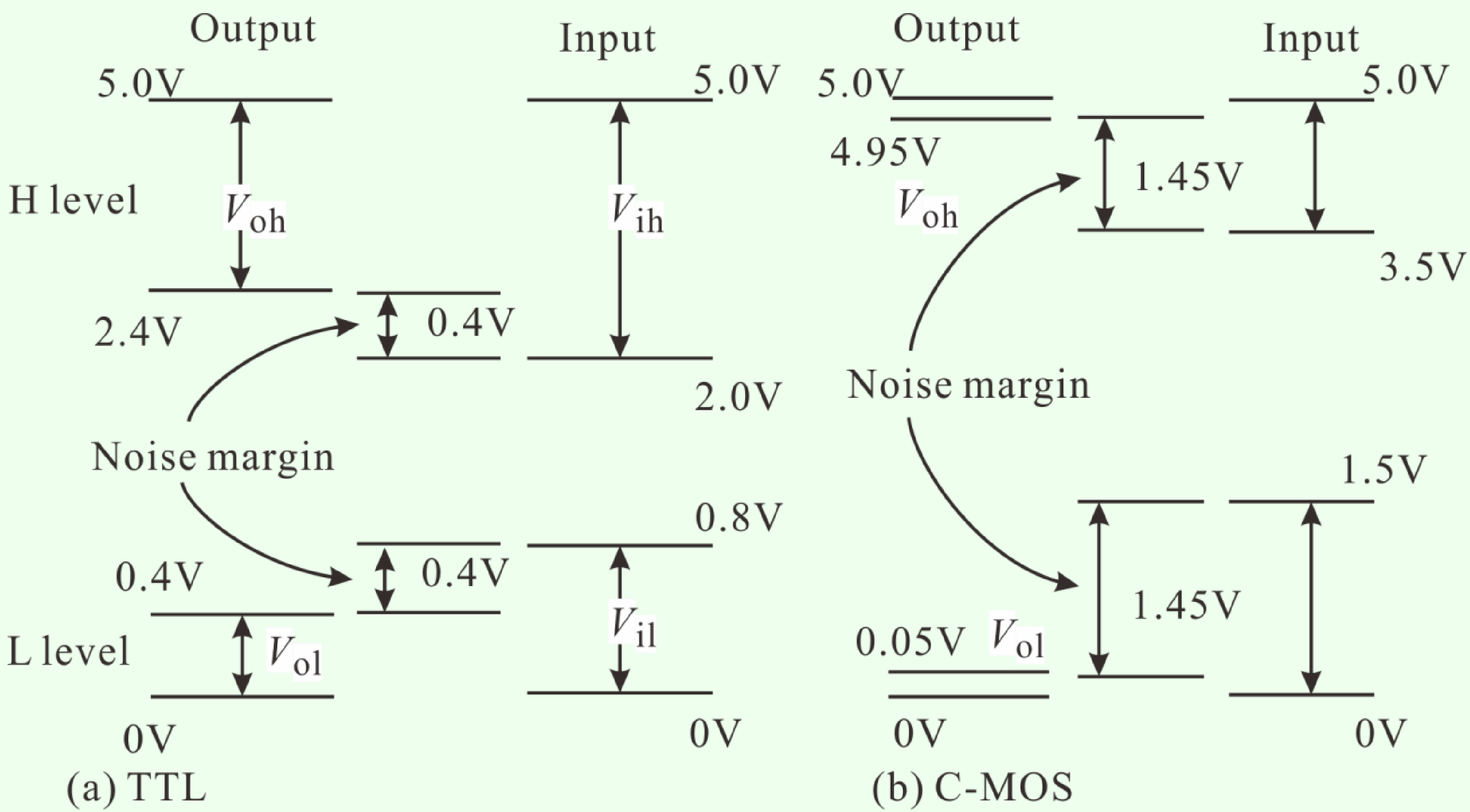
LT Spice simulation



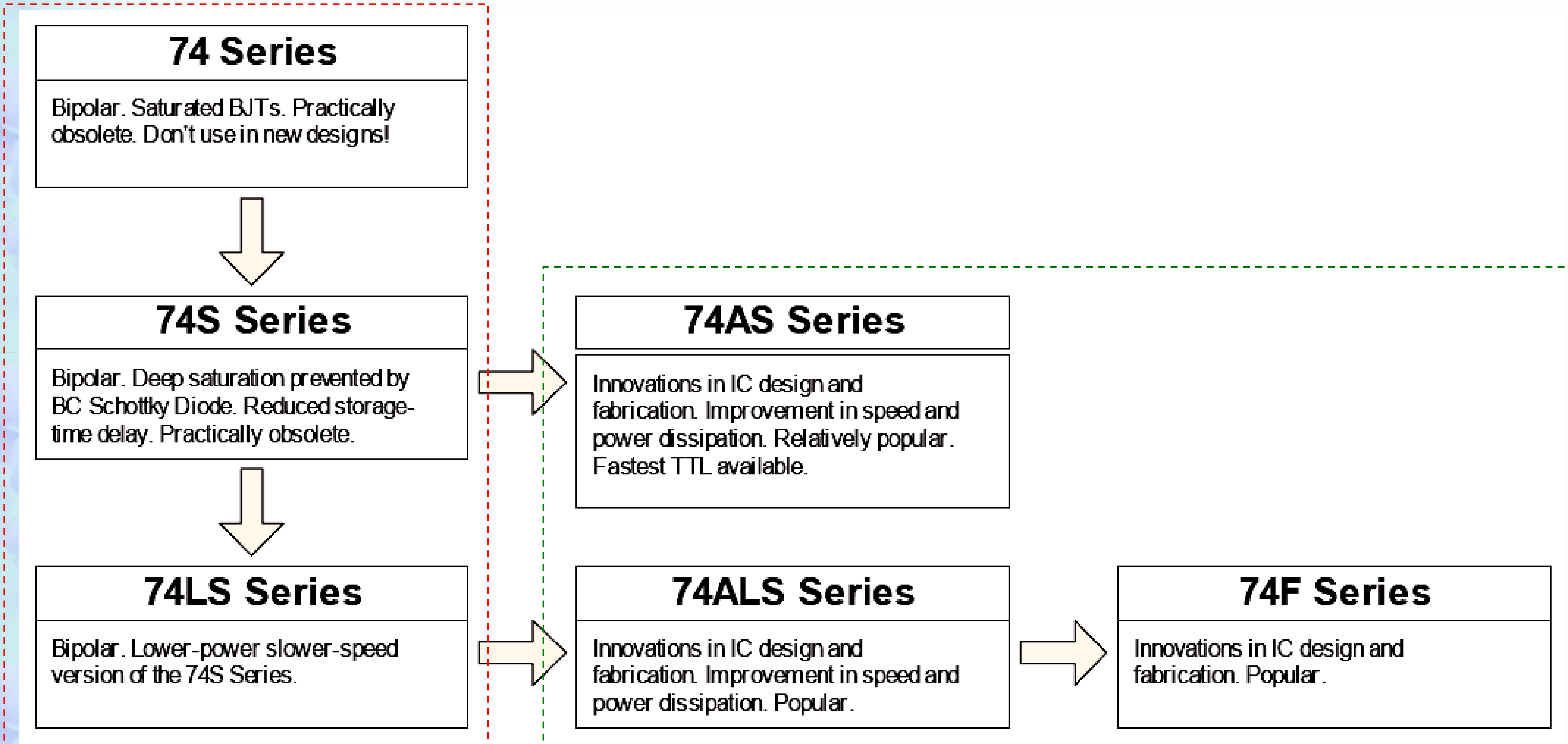


7.3 Implementation of logic gates

Voltage levels diagram



TTL logic family evolution



Legacy: don’t use in new designs

Widely used today

CMOS logic family evolution

obsolete

4000 Series

CMOS. Wide supply voltage range. High noise margin. Low speed. Weak output drive. Practically obsolete.

74C Series

CMOS. Pin-compatible with TTL devices. Low speed. Obsolete. Replaced by HC/HCT family.

74HC/HCT Series

CMOS. Drastic increase in speed. Higher output drive capability. HCT input voltage levels compatible with TTL.

General trend:

- Reduction of dynamic losses through successively decreasing supply voltages:

$12V \rightarrow 5V \rightarrow 3.3V \rightarrow 2.5V \rightarrow 1.8V$

CD4000 LVC/ALVC/AVC

- Power reduction is one of the keys to progressive growth of integration

74AC/ACT Series

CMOS. Functionally compatible, but not pin-compatible to TTL. Improved noise immunity and speed. ACT inputs are TTL compatible.

74AHC/AHCT Series

CMOS. Improved speed, lower power, lower drive capability.

BiCMOS Logic

CMOS/Bipolar. Combine the best features of CMOS and bipolar. Low power high speed. Bus interfacing applications (74BCT, 74ABT)

74LVC/ALVC/LV/AVC

CMOS. Reduced supply voltage.
LVC: 5V/3.3V translation
ALVC: Fast 3.3V only
AVC: Optimised for 2.5V, down to 1.2V

Summary

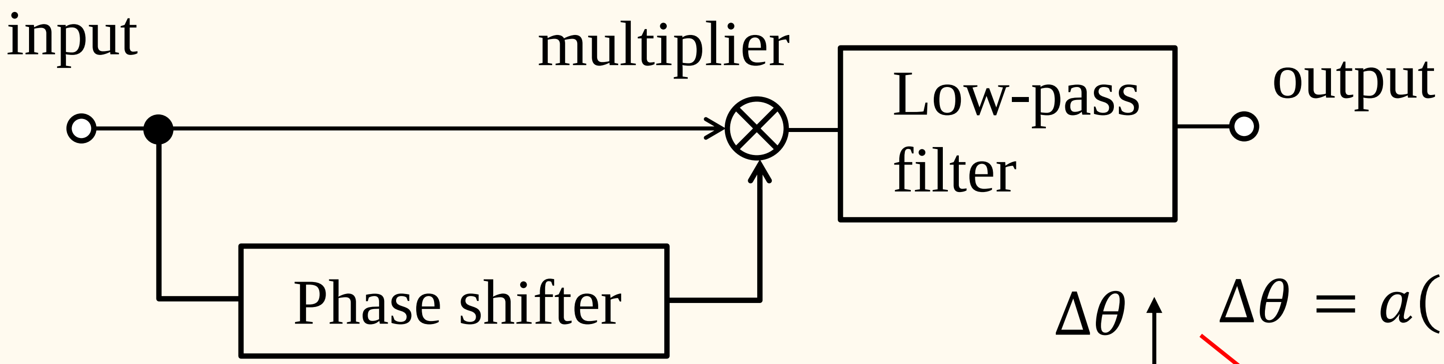
TTL

Logic Family	T_{PD}	$T_{rise/fall}$	$V_{IH,min}$	$V_{IL,max}$	$V_{OH,min}$	$V_{OL,max}$	Noise Margin
74	22ns		2.0V	0.8V	2.4V	0.4V	0.4V
74LS	15ns		2.0V	0.8V	2.7V	0.5V	0.3V
74F	5ns	2.3ns	2.0V	0.8V	2.7V	0.5V	0.3V
74AS	4.5ns	1.5ns	2.0V	0.8V	2.7V	0.5V	0.3V
74ALS	11ns	2.3ns	2.0V	0.8V	2.5V	0.5V	0.3V
ECL	1.45ns	0.35ns	-1.165V	-1.475V	-1.025V	-1.610V	0.135V
4000	250ns	90ns	3.5V	1.5V	4.95V	0.05V	1.45V
74C	90ns		3.5V	1.5V	4.5V	0.5V	1V
74HC	18ns	3.6ns	3.5V	1.0V	4.9V	0.1V	0.9V
74HCT	23ns	3.9ns	2.0V	0.8V	4.9V	0.1V	0.7V
74AC	9ns	1.5ns	3.5V	1.5V	4.9V	0.1V	1.4V
74ACT	9ns	1.5ns	2.0V	0.8V	4.9V	0.1V	0.7V
74AHC	3.7ns		3.85V	1.65V	4.4V	0.44V	0.55V

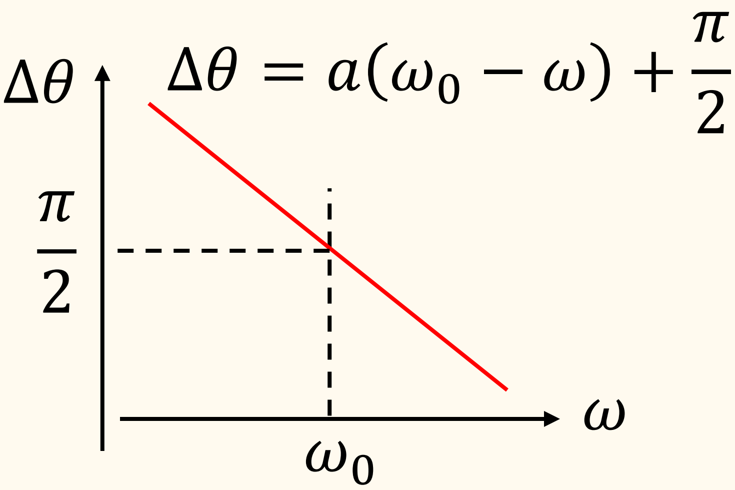
CMOS

Exercise F-1

Show that the following circuit works as a demodulator of frequency modulation (FM) signal (quadrature demodulator).



Here the phase shifter gives the shift proportional to the frequency difference between input and the career frequency ω_0. The shift at ω_0 is $\pi/2$ as shown in the right (this can be achieved with resonant circuits). The low-pass filter cuts components with frequencies as high as ω_0.



Exercise F-1

(hint) Assume the original signal $f(t)$ is much slower than the carrier $A\cos(\omega_0 t)$. Then the input can be approximated as

$$s(t) = A\cos\{[\omega_0 + k_f f(t)]t\}.$$

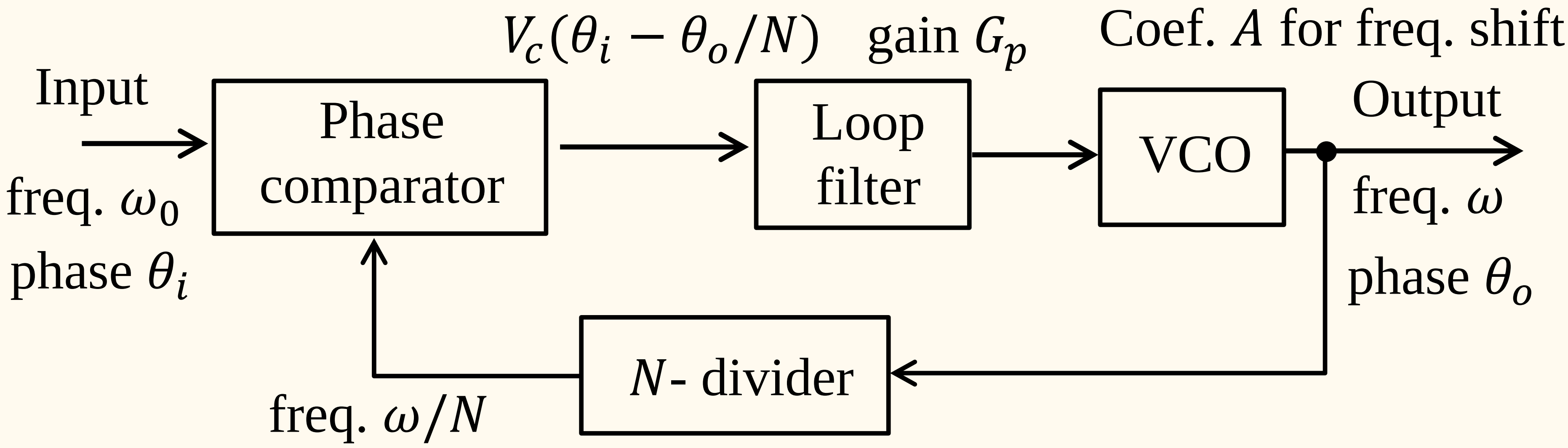
Then the phase shifter output is

$$q_{\mathrm{ps}}(t) = A\sin\{[\omega_0 + k_f f(t)]t - ak_f f(t)\}.$$

Taking product and high-frequency filtering gives …
(use $ak_f f(t) \ll 1$).

Exercise F-2

In the following phase lock loop (PLL) circuit, the initial ($t = 0$) oscillation frequency of voltage-controlled oscillator (VCO) ω deviates from $N\omega_0$ by $\Delta\omega$. Obtain the relaxation time of ω to $N\omega_0$.



(hint) Here we can put $\theta_i = 0$ hence input $= V_i \sin \omega_0 t$ without loosing generality. Similarly output $= V_o \sin[N\omega_0 t + \theta_o(t)]$. Now $\omega = N\omega_0 + d\theta_o/dt$ and it is easy to write $d\theta_o/dt$ with A, G_p, V_c, $\theta_o(t)$ and a constant.

Exercise F-3

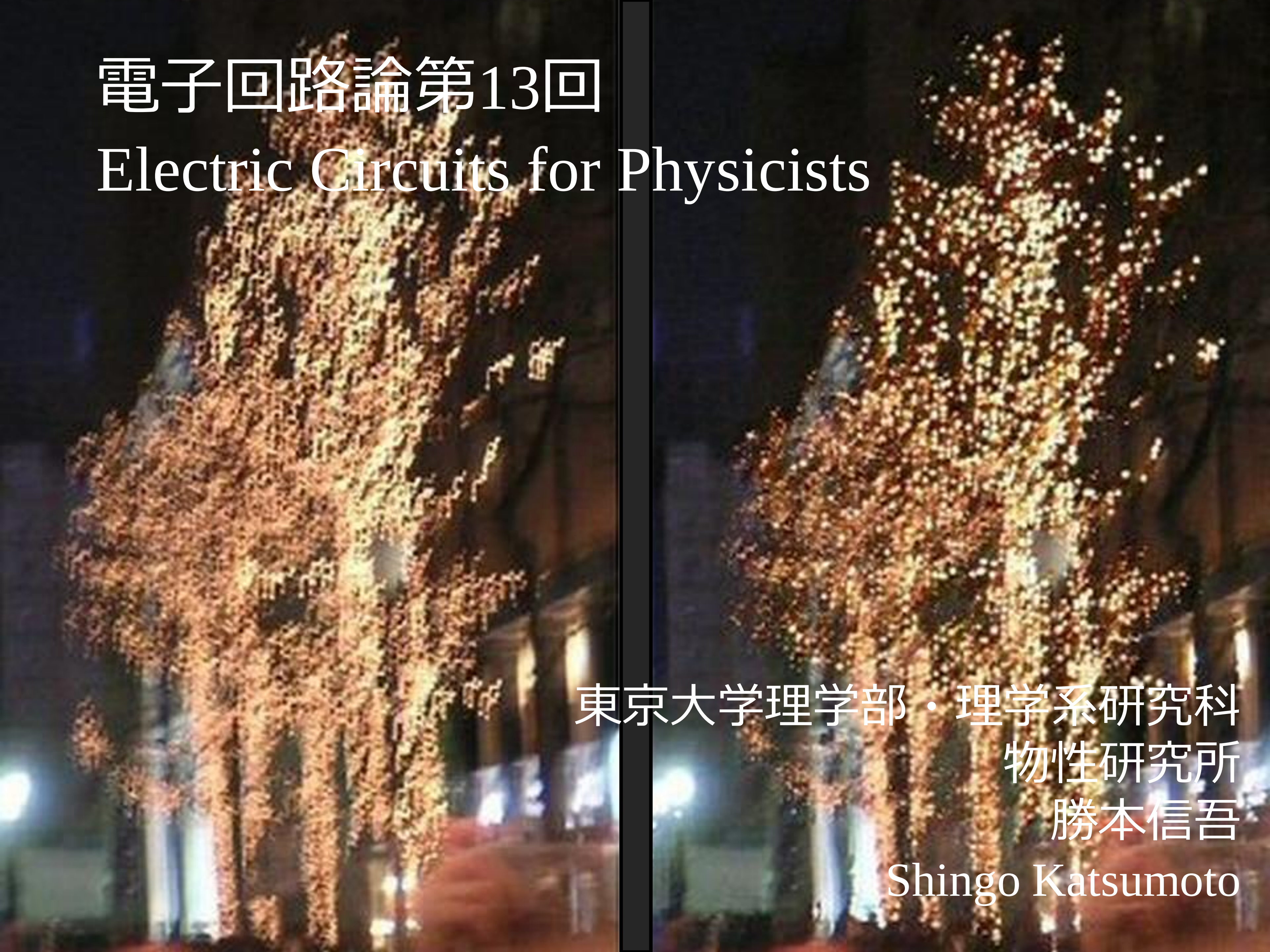
Solve the difference equation below with z-transform.

$$\begin{cases} x(n) - 2x(n-1) = n & (n \geq 0) \\ x(n) = 0 & (n < 0) \end{cases}$$

(hint) z-transform of n is $\dfrac{z}{(z-1)^2}$ as in the table (slide no.14).

Then z-transform of $x(n)$: $X(z)$ is easily obtained. Inverse z-transform gives $x(n)$.

Answer sheet submission deadline: 11th Jan. 2017.



電子回路論第13回
Electric Circuits for Physicists

東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Outline

7.2 Sequential digital circuit

7.3 Implementation of logic gates

7.4 Circuit implementation and simplification of logic operation

7.5 DA/AD converter circuits

7.6 Digital filters (digital signal processing)

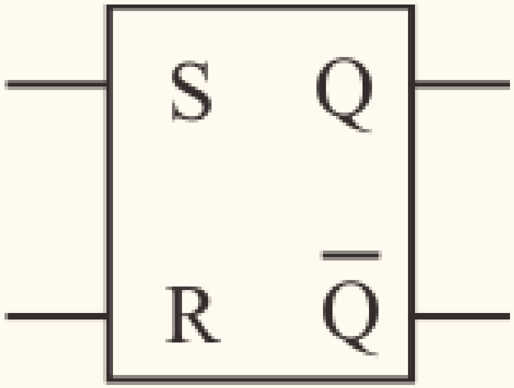
7.2.3 Sequential logic: Flip-Flop (FF)

RS (reset-set) Flip-Flop (FF)

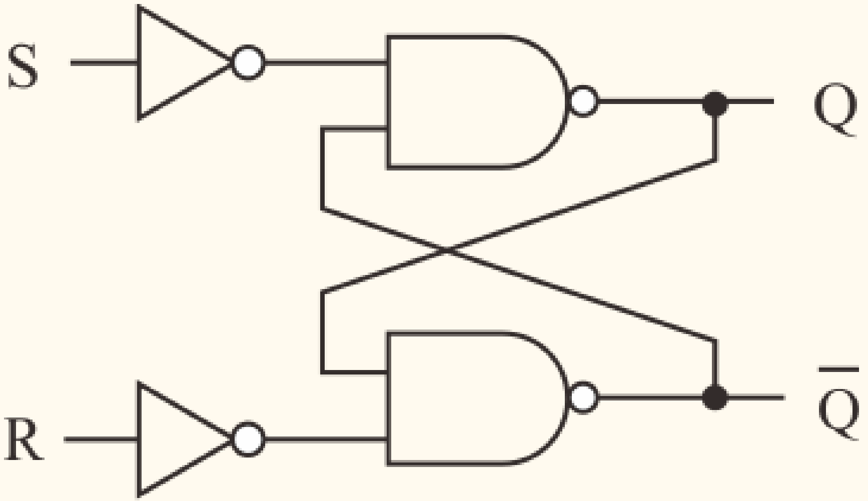
Truth table

S	R	Q	$\bar{Q}$	Response
0	0	Q	$\bar{Q}$	no change
0	1	0	1	reset
1	0	1	0	set
1	1	undetermined		

Symbol



Equivalent circuit with discrete gates



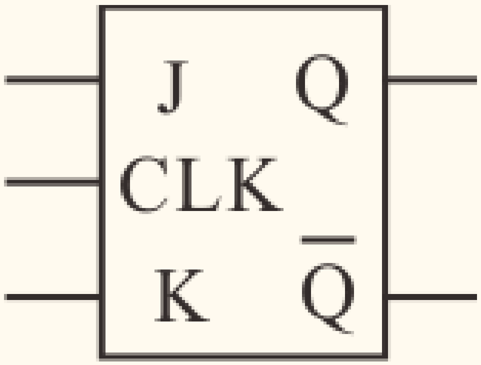
7.2.3 Sequential logic: Flip-Flop (FF)

JK Flip-Flop

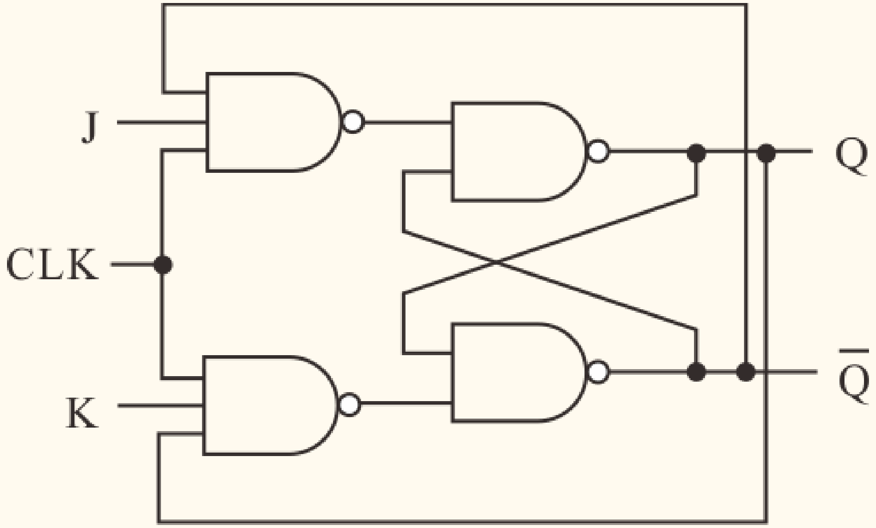
Truth table

J	K	Q	Q for the next CLK
0	0	0	0
0	0	1	1
0	1	–	0
1	0	–	1
1	1	0	1
1	1	1	0

Symbol



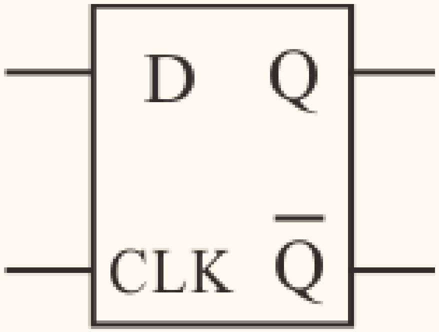
Equivalent circuit with discrete gates



7.2.3 Sequential logic: D-FF, T-FF

D-FF

Symbol

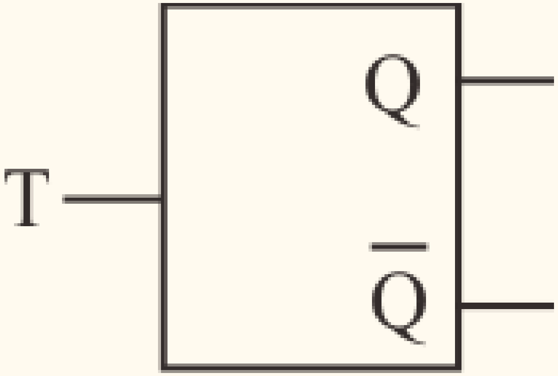


Truth table

D	CLK	Q
0	↑	0
1	↑	1
–	↓	Q (hold)

T-FF

Symbol

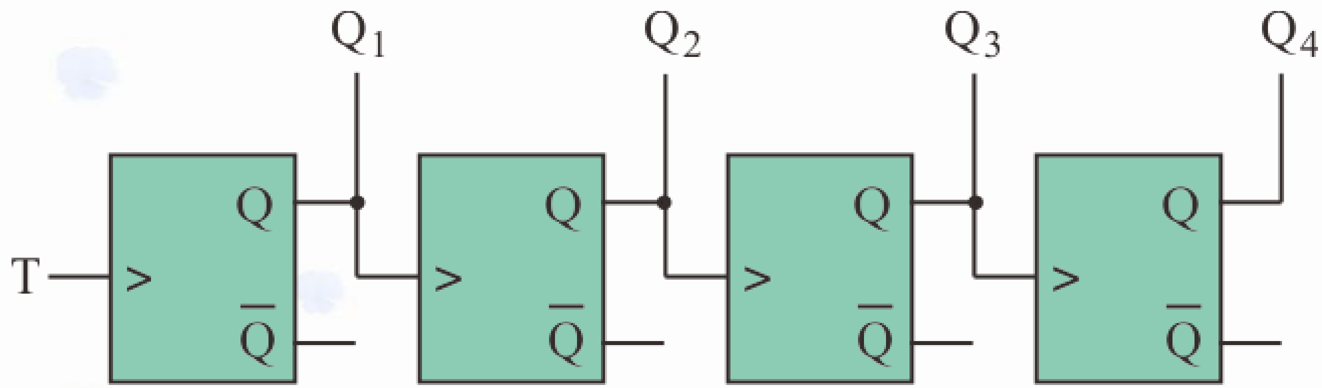


Truth table

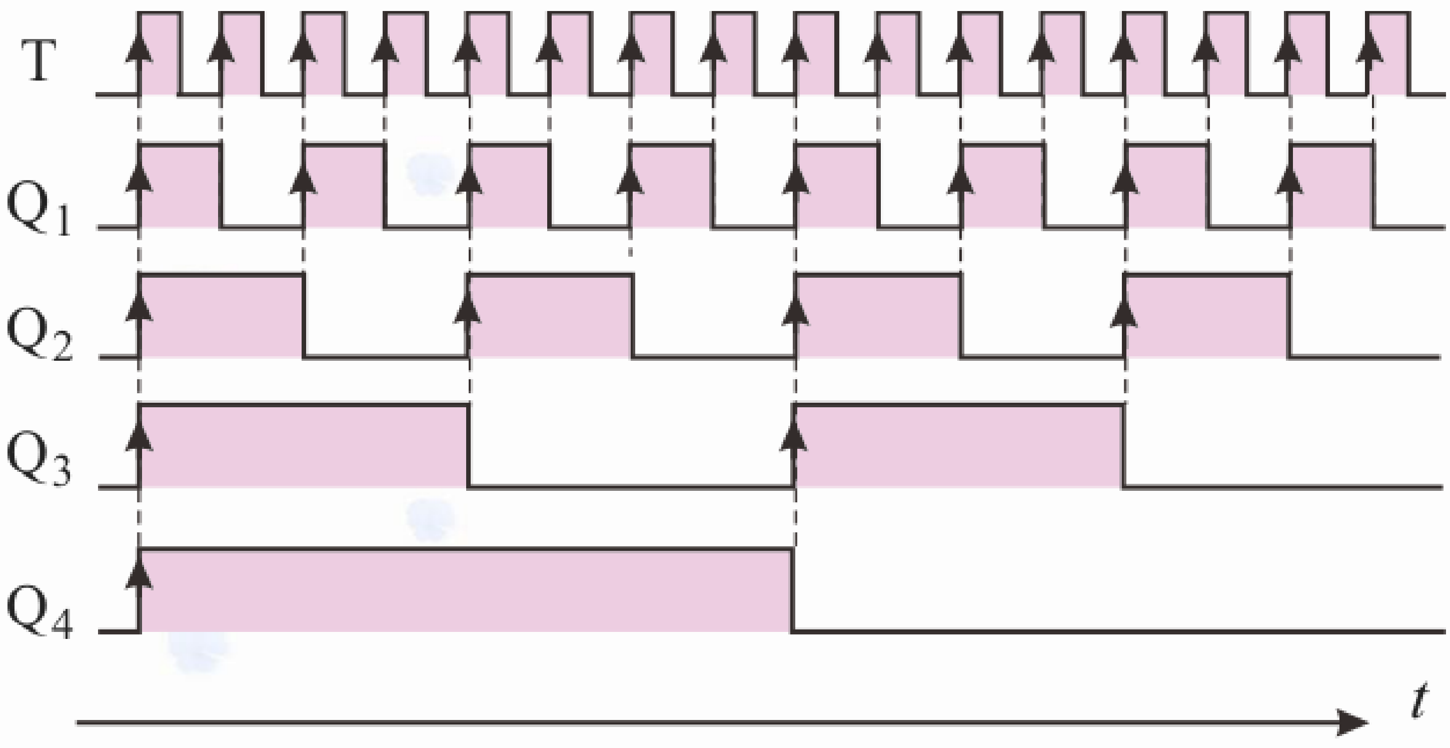
T	Q	Q
↓	0	0
↓	1	1
↑	0	1
↑	1	0

7.2.4 Sequential logic: Counters

Unsynchronized counter (ripple counter)



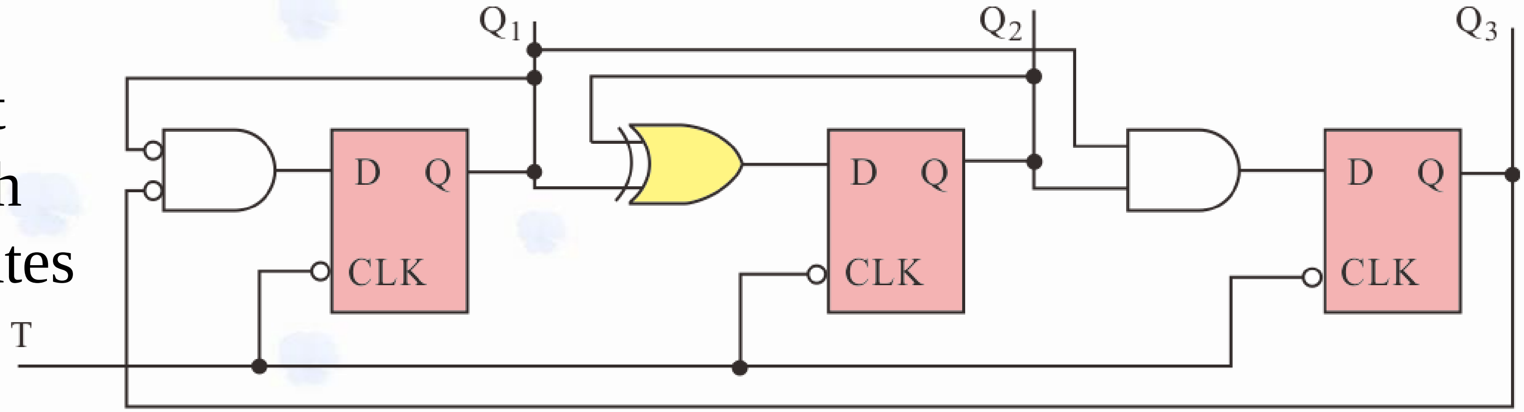
Timing chart



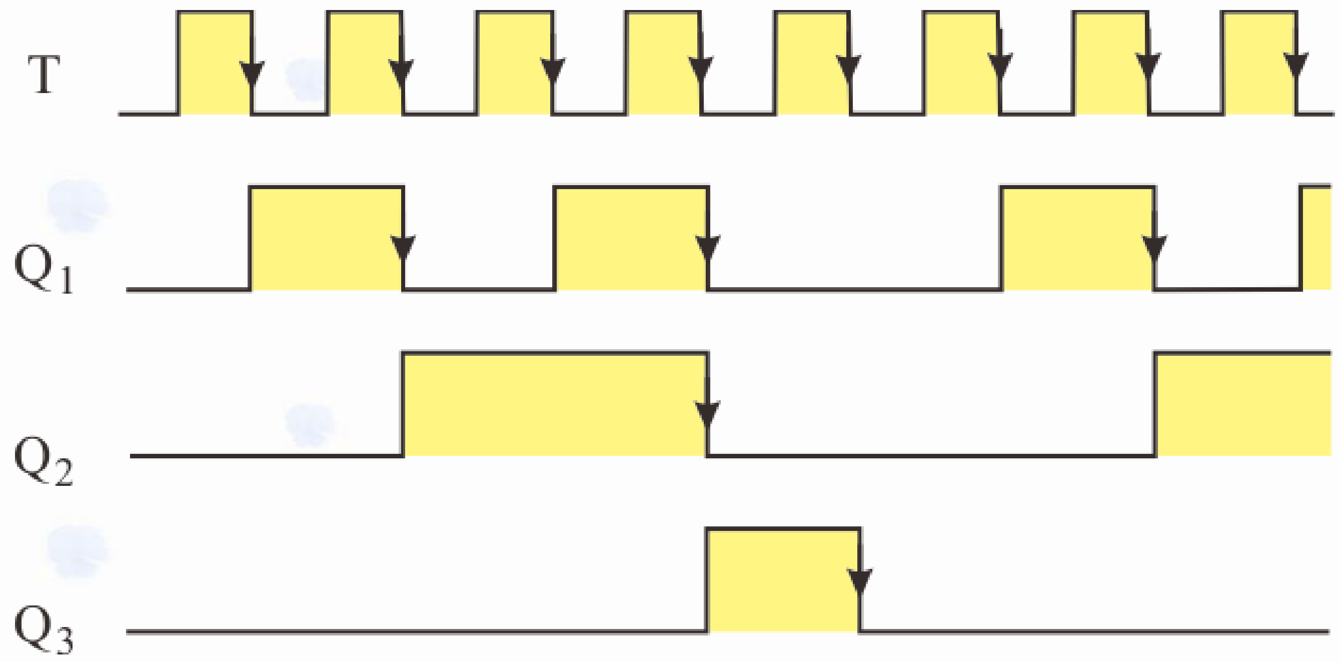
7.2.4 Sequential logic: Counters

Synchronized counter

Equivalent circuit with discrete gates

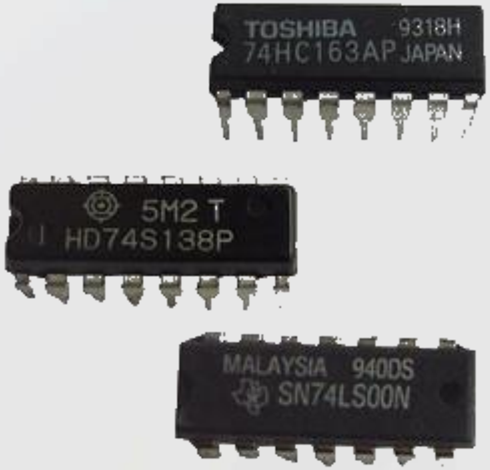


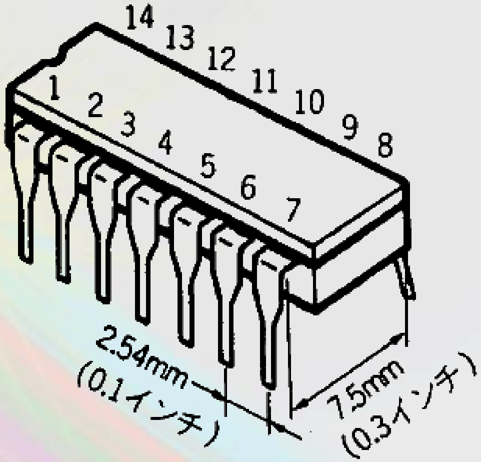
Timing chart



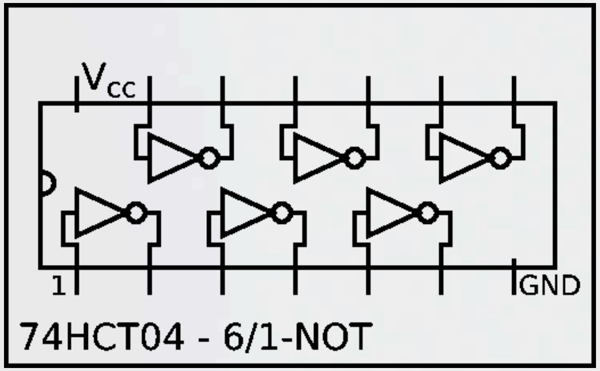
Standard gate logic IC packaging

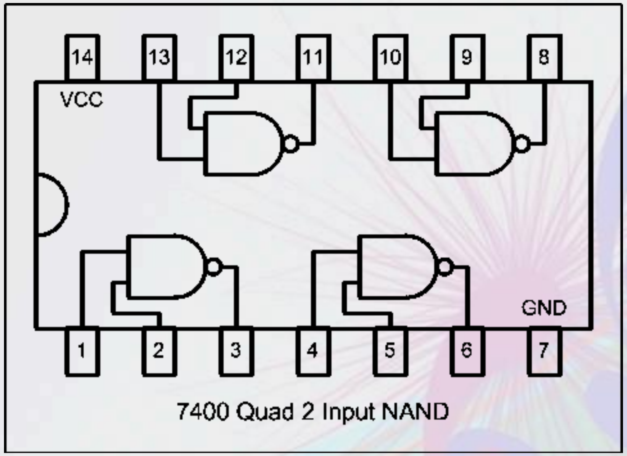
Full pitch



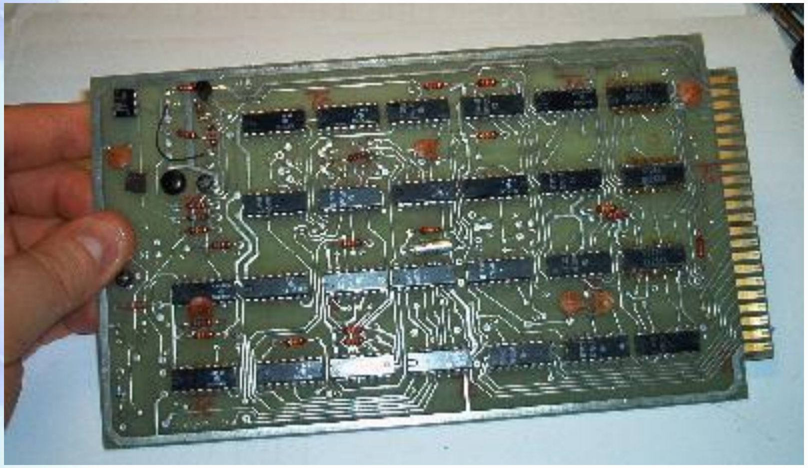


Half pitch surface mount



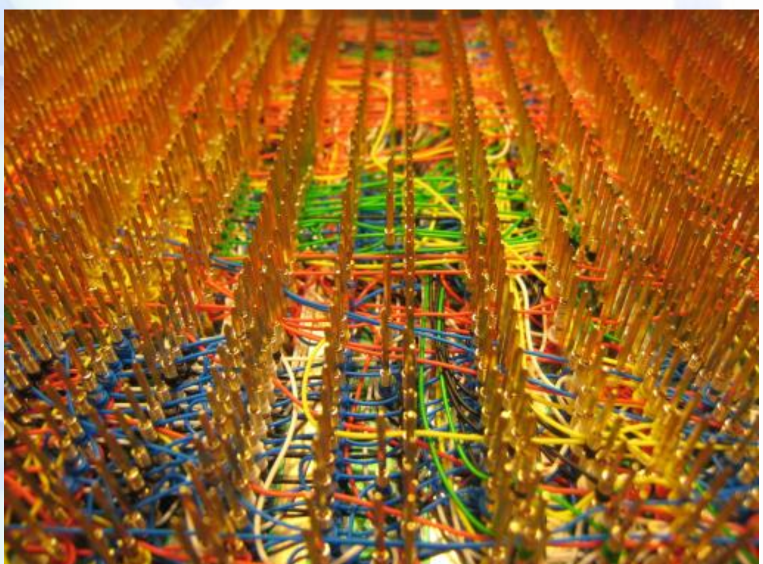


Mounting of logic ICs

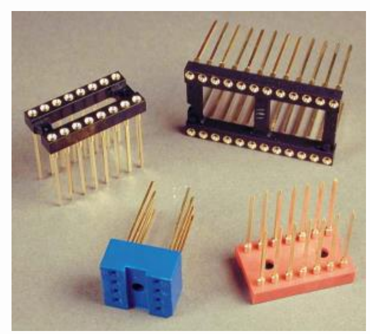


Printed board with soldering

Surface mounting

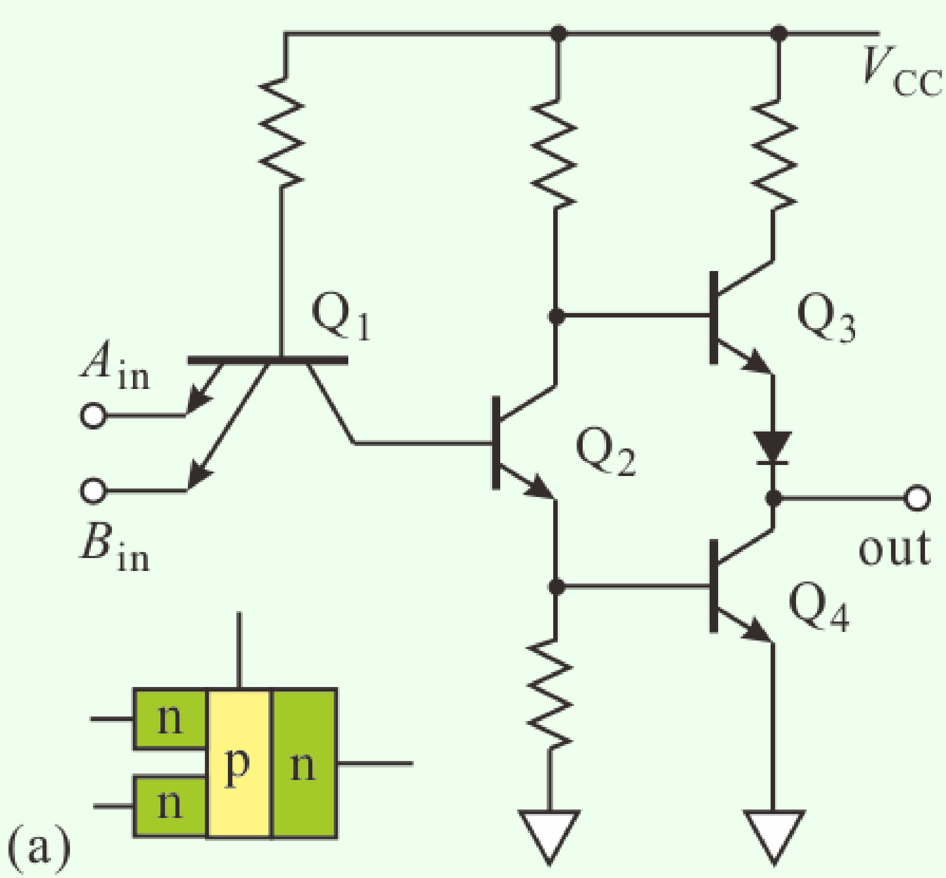


Wire wrapping

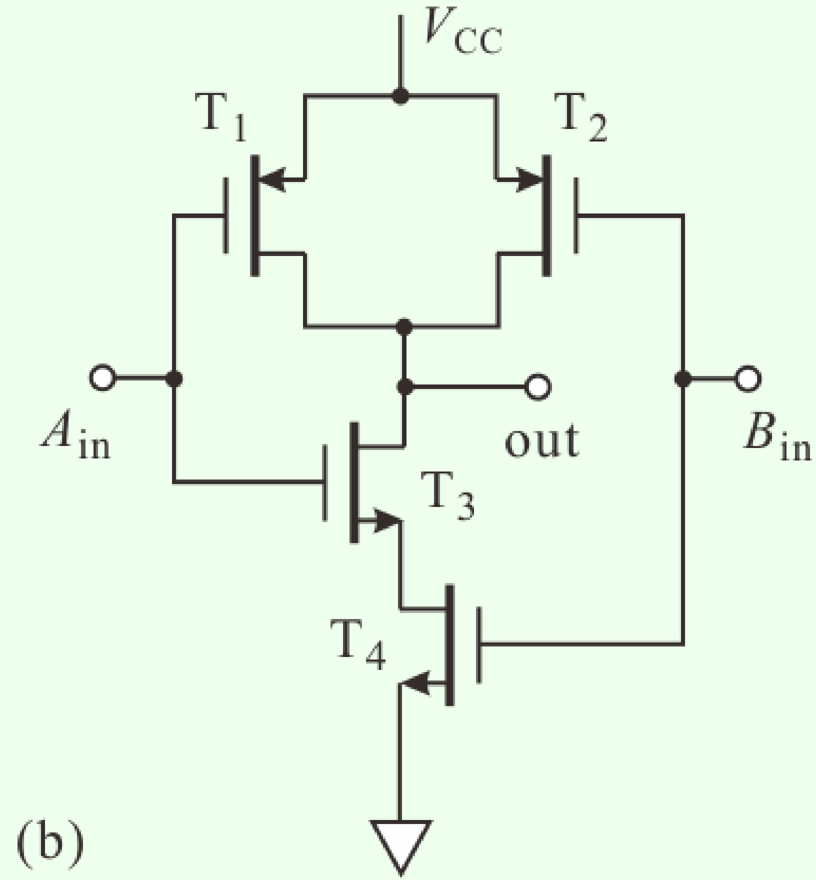


7.3 Implementation of logic gates

NAND gates



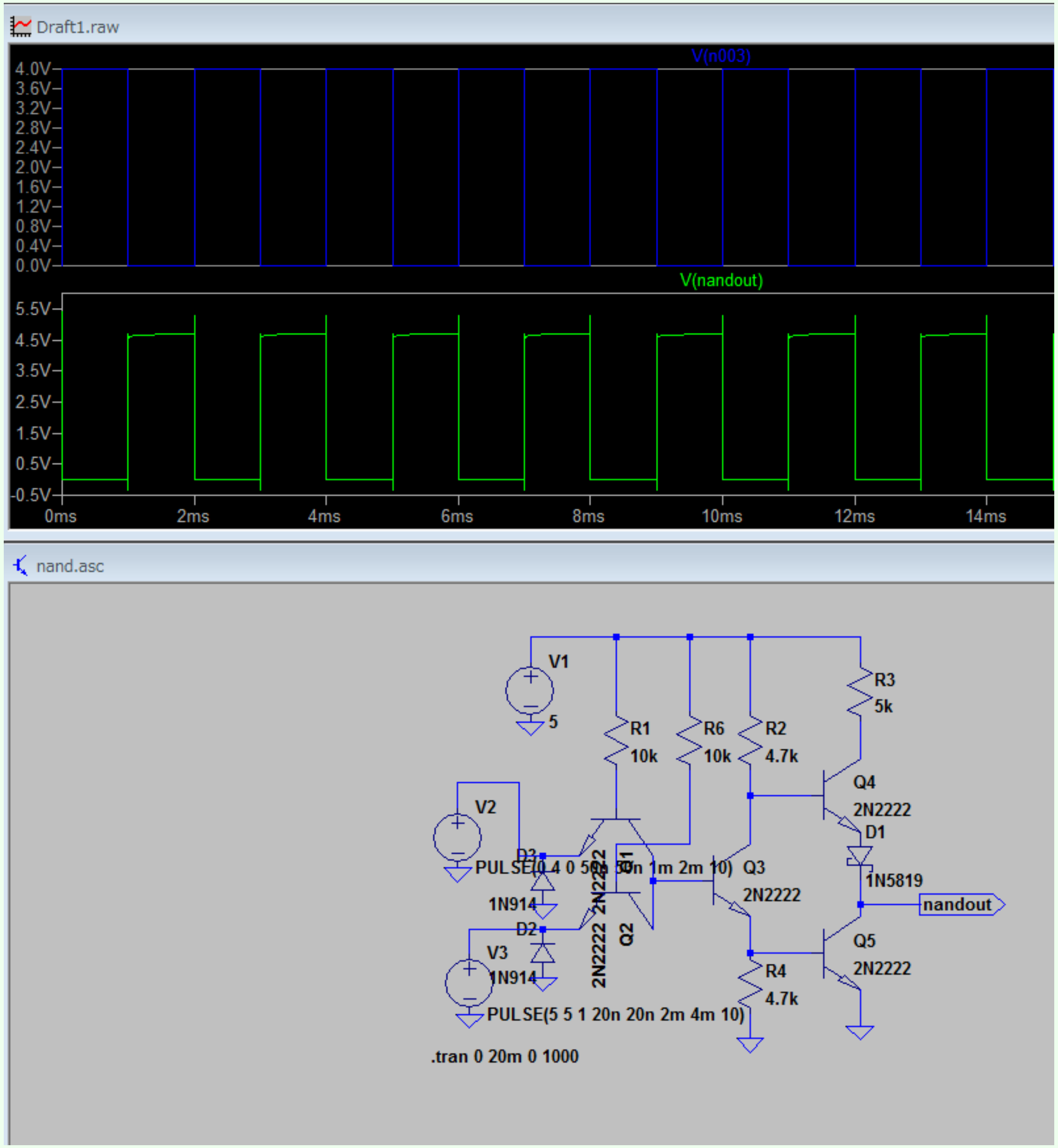
TTL (transistor-transistor logic)



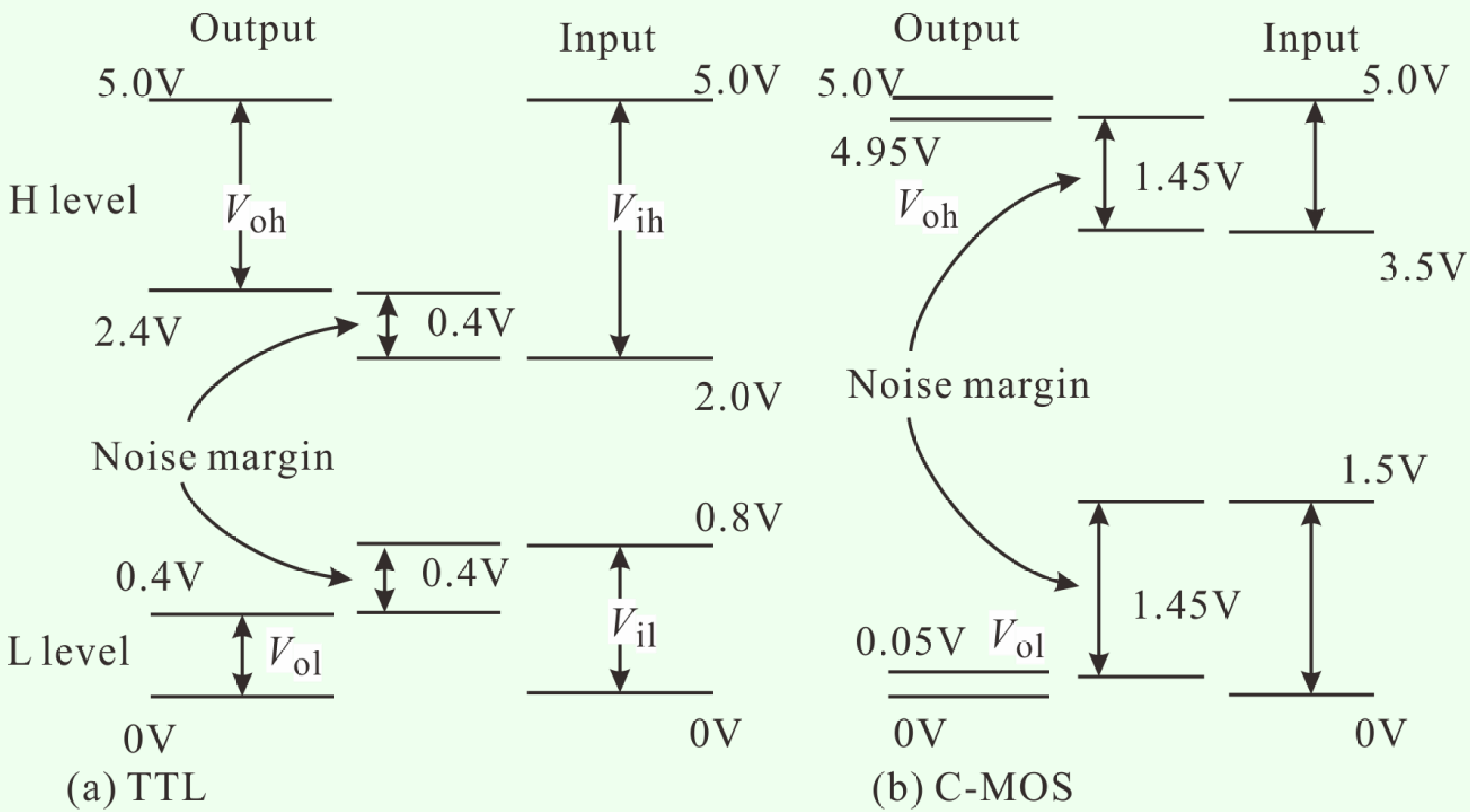
CMOS (complimentary MOS)

7.3 Implementation of logic gates

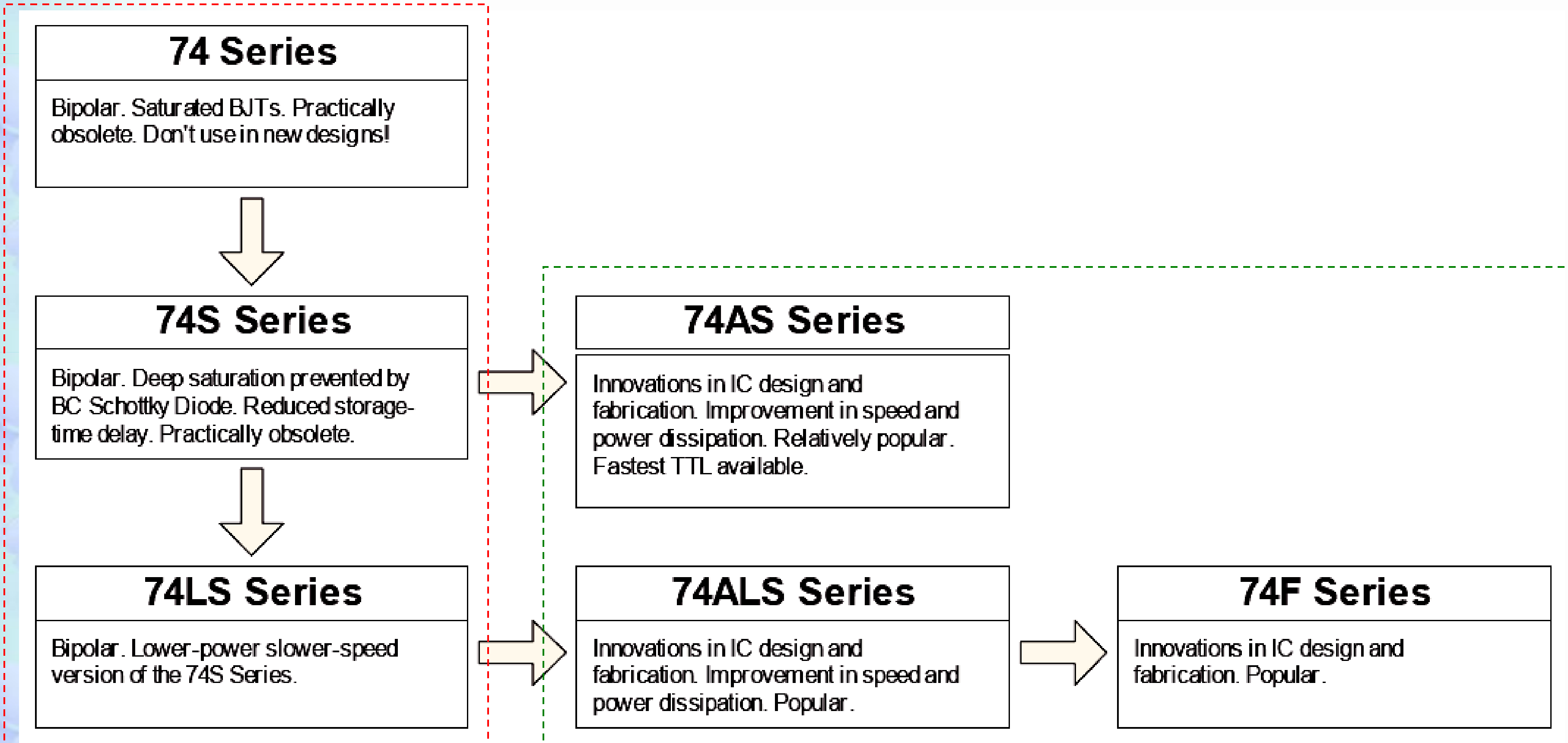
LT Spice simulation



Voltage levels diagram



TTL logic family evolution



Legacy: don't use in new designs

Widely used today

CMOS logic family evolution

obsolete

4000 Series

CMOS. Wide supply voltage range. High noise margin. Low speed. Weak output drive. Practically obsolete.

↓

74C Series

CMOS. Pin-compatible with TTL devices. Low speed. Obsolete. Replaced by HC/HCT family.

↓

74HC/HCT Series

CMOS. Drastic increase in speed. Higher output drive capability. HCT input voltage levels compatible with TTL.

↓

74AC/ACT Series

CMOS. Functionally compatible, but not pin-compatible to TTL. Improved noise immunity and speed. ACT inputs are TTL compatible.

General trend:

- Reduction of dynamic losses through successively decreasing supply voltages:

12V → 5V → 3.3V → 2.5V → 1.8V

CD4000 LVC/ALVC/AVC

- Power reduction is one of the keys to progressive growth of integration

74AHC/AHCT Series

CMOS. Improved speed, lower power, lower drive capability.

BiCMOS Logic

CMOS/Bipolar. Combine the best features of CMOS and bipolar. Low power high speed. Bus interfacing applications (74BCT, 74ABT)

74LVC/ALVC/LV/AVC

CMOS. Reduced supply voltage.
LVC: 5V/3.3V translation
ALVC: Fast 3.3V only
AVC: Optimised for 2.5V, down to 1.2V

Summary

TTL

Logic Family	T_{PD}	$T_{rise/fall}$	$V_{IH,min}$	$V_{IL,max}$	$V_{OH,min}$	$V_{OL,max}$	Noise Margin
74	22ns		2.0V	0.8V	2.4V	0.4V	0.4V
74LS	15ns		2.0V	0.8V	2.7V	0.5V	0.3V
74F	5ns	2.3ns	2.0V	0.8V	2.7V	0.5V	0.3V
74AS	4.5ns	1.5ns	2.0V	0.8V	2.7V	0.5V	0.3V
74ALS	11ns	2.3ns	2.0V	0.8V	2.5V	0.5V	0.3V
ECL	1.45ns	0.35ns	-1.165V	-1.475V	-1.025V	-1.610V	0.135V
4000	250ns	90ns	3.5V	1.5V	4.95V	0.05V	1.45V
74C	90ns		3.5V	1.5V	4.5V	0.5V	1V
74HC	18ns	3.6ns	3.5V	1.0V	4.9V	0.1V	0.9V
74HCT	23ns	3.9ns	2.0V	0.8V	4.9V	0.1V	0.7V
74AC	9ns	1.5ns	3.5V	1.5V	4.9V	0.1V	1.4V
74ACT	9ns	1.5ns	2.0V	0.8V	4.9V	0.1V	0.7V
74AHC	3.7ns		3.85V	1.65V	4.4V	0.44V	0.55V

CMOS

7.4 Circuit implementation and simplification of logic operation

Truth table → Simplification → Circuit diagram

Simplification
- Visual method: Karnaugh mapping
- Quine-McClusky algorithm

Product of all the logic variables: canonical expansion

principal disjunctive canonical expansion (主加法標準展開)

$$Y = \sum_j \prod_{i=1}^{n} g_i(a_{ij})$$

Ex) $Y = \bar{A}\cdot\bar{B}\cdot C\cdot D + B\cdot C\cdot D + A\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot C\cdot D$

$$\begin{aligned} Y &= \bar{A}\cdot\bar{B}\cdot C\cdot D + (A+\bar{A})\cdot B\cdot C\cdot D + A\cdot B\cdot\bar{C}\cdot(D+\bar{D}) + A\cdot\bar{B}\cdot C\cdot D \\ &= \bar{A}\cdot\bar{B}\cdot C\cdot D + A\cdot B\cdot C\cdot D + \bar{A}\cdot B\cdot C\cdot D + A\cdot B\cdot\bar{C}\cdot D \\ &+ A\cdot B\cdot\bar{C}\cdot\bar{D} + A\cdot\bar{B}\cdot C\cdot D \end{aligned}$$

Or in binary: Y = 0011+1111+0111+1101+1100+1011

Quein-McClusky algorithm

Classification with the number of 1

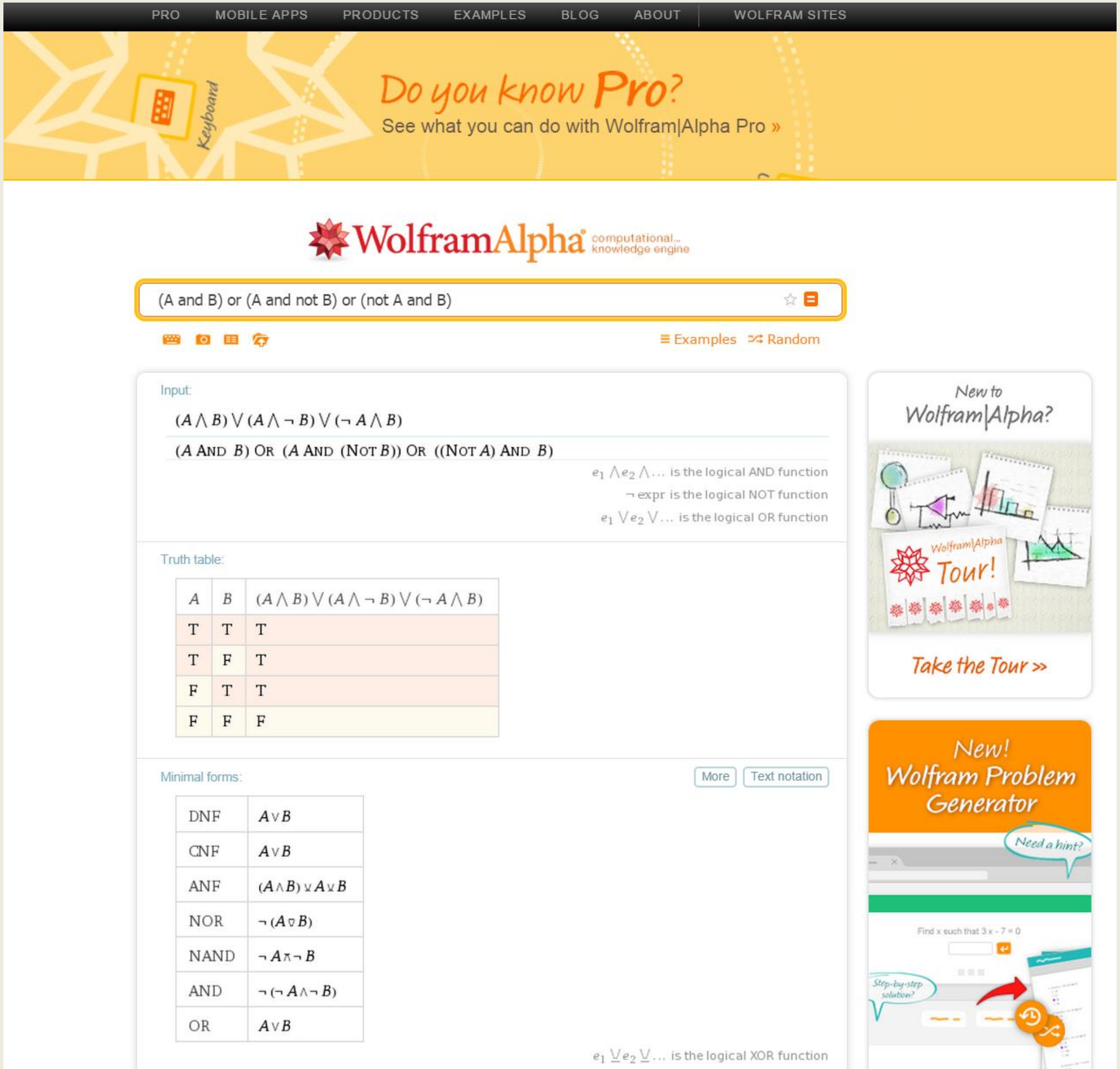
Num.of 1	smallest	compress1	compress2
2	0011	0_11	__11
	1100	_011	__11
3	0111	110_	
	1011	_111	
	1101	1_11	
4	1111	11_1	

Y = __11+110_+11_1 First simplification

	smallest					
	0011	1100	0111	1011	1101	1111
__11	◎		◎	◎		◎
110_		◎			◎	
11_1					○	○

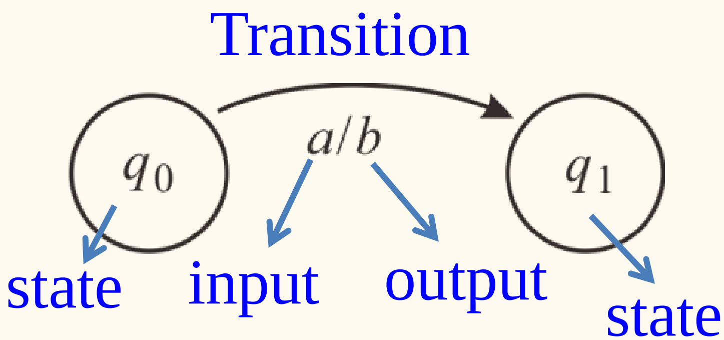
Y = __11+110_ Final form

Wolfram Alpha

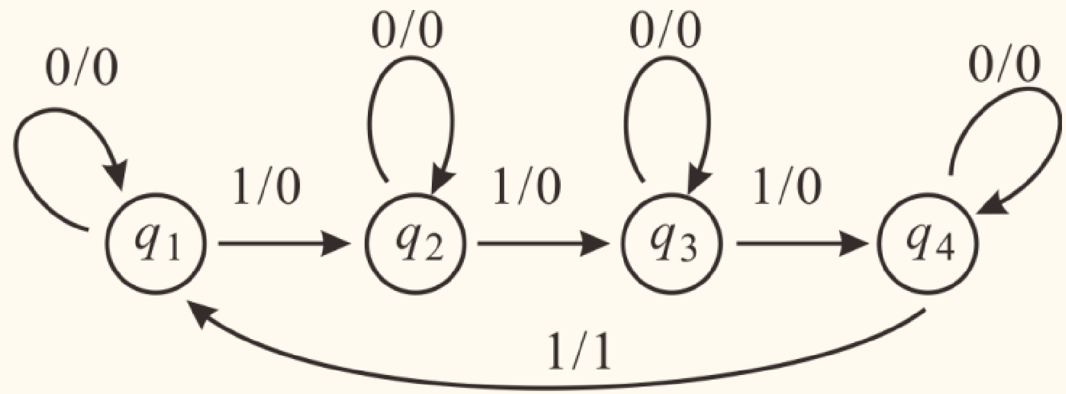


Design of sequential logic circuit: State diagram

State (transition) diagram:



Ex) 2-bit counter with two T-FF



FF output:

$Q_n^{(1)}, Q_n^{(2)}$

Karnaugh map simplification

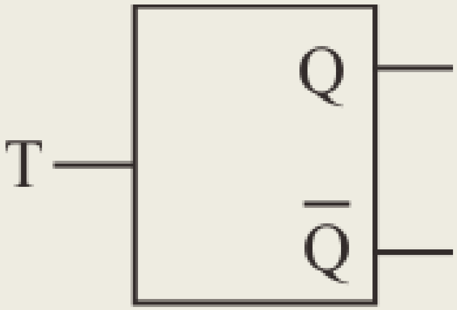
		input x					
		0			1		
		next		out	next		out
$Q_n^{(1)}$	$Q_n^{(2)}$	$Q_{n+1}^{(1)}$	$Q_{n+1}^{(2)}$	y	$Q_{n+1}^{(1)}$	$Q_{n+1}^{(2)}$	y
0	0	0	0	0	1	0	0
0	1	0	1	0	1	1	0
1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	1

Recursion equation:

$$Q_{n+1}^{(1)} = \overline{x} \cdot Q_n^{(1)} + x \cdot \overline{Q}_n^{(1)},$$

$$Q_{n+1}^{(2)} = \overline{x} \cdot Q_n^{(2)} + Q_n^{(2)} \cdot \overline{Q}_n^{(1)} + x \cdot \overline{Q}_n^{(2)} \cdot Q_n^{(1)}.$$

Design of sequential logic circuit: State diagram



T	Q	Q
↓	0	0
↓	1	1
↑	0	1
↑	1	0

Characteristic equation (recursion equation)

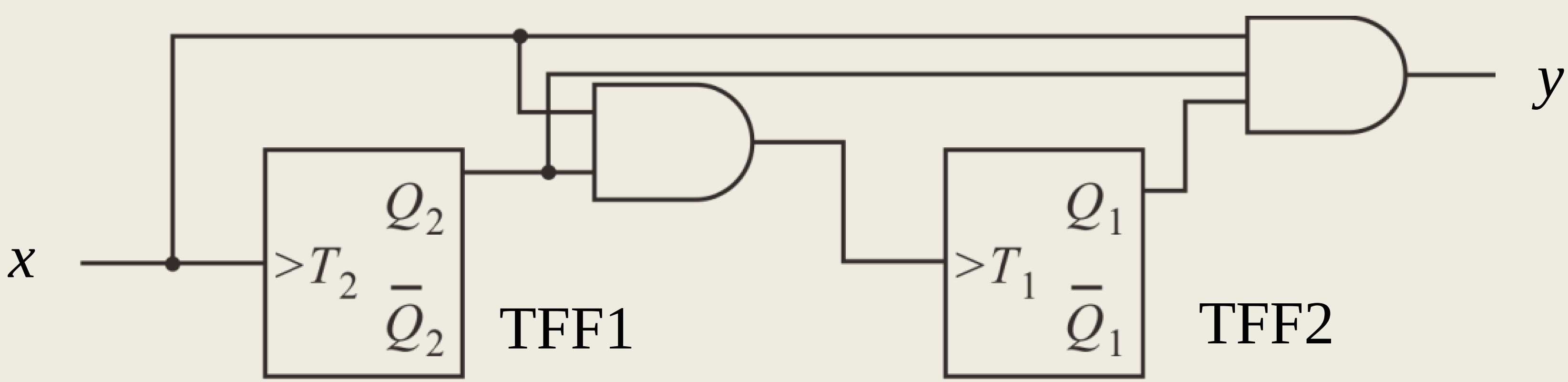
$$Q_{n+1} = \alpha Q_n + \beta \overline{Q}_n$$

T-FF : $Q_{n+1} = \overline{T} Q_n + T \overline{Q}_n$

$$Q_{n+1}^{(1)} = \overline{x} \cdot Q_n^{(1)} + x \cdot \overline{Q}_n^{(1)},$$

$$y = \overline{x Q_n^{(1)} Q_n^{(2)}}$$

$$\begin{aligned} Q_{n+1}^{(2)} &= (\overline{x} + \overline{Q}_n^{(1)}) \cdot Q_n^{(2)} + (x \cdot Q_n^{(1)}) \cdot \overline{Q}_n^{(2)} \\ &= \overline{(x \cdot Q_n^{(1)})} \cdot Q_n^{(2)} + (x \cdot Q_n^{(1)}) \cdot \overline{Q}_n^{(2)} \end{aligned}$$



7.5 AD/DA converter circuit

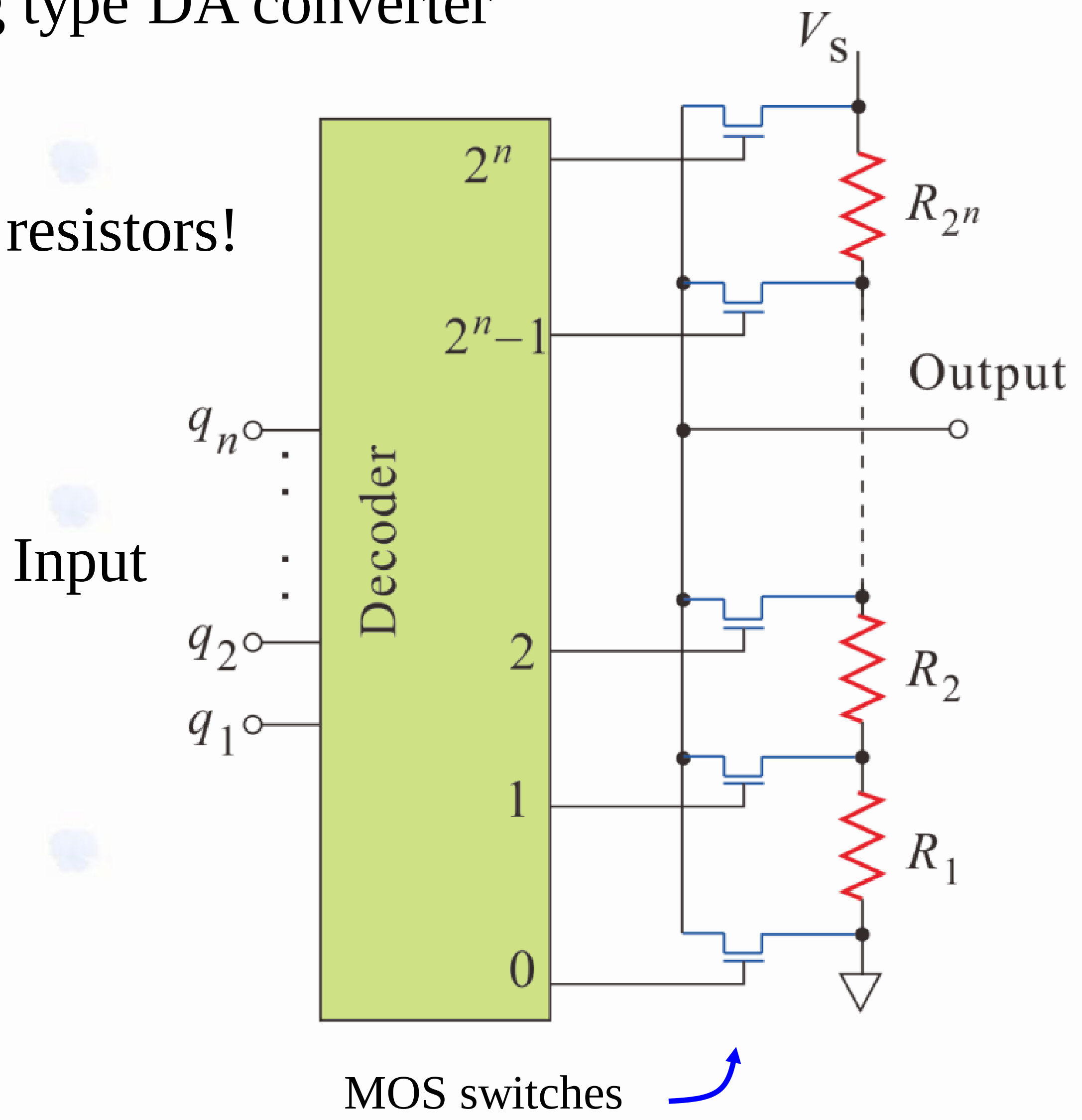
7.5.1 Digital to Analog conversion

Resistor string type DA converter

n bits converter

→ 2^n outputs!, 2^n resistors!

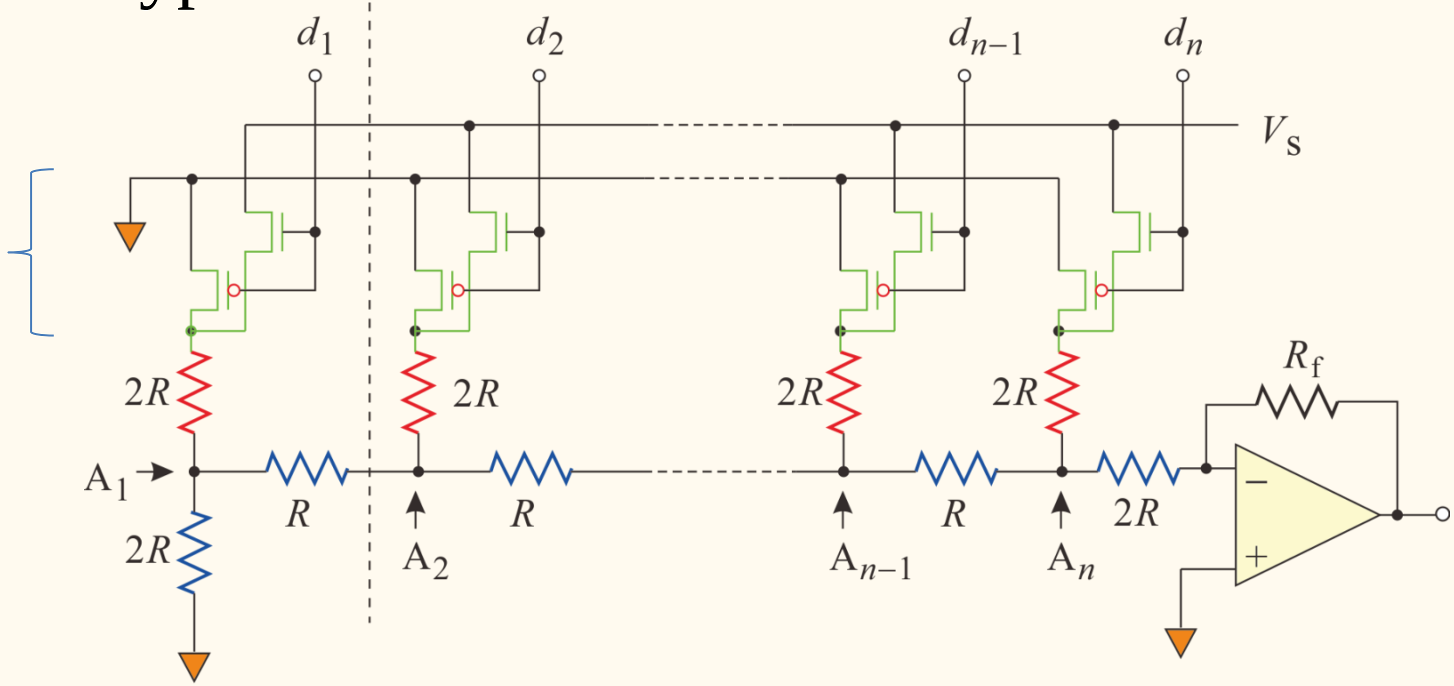
$$V_{\text{out}} = \frac{p_{\text{input}}}{2^n} V_S$$



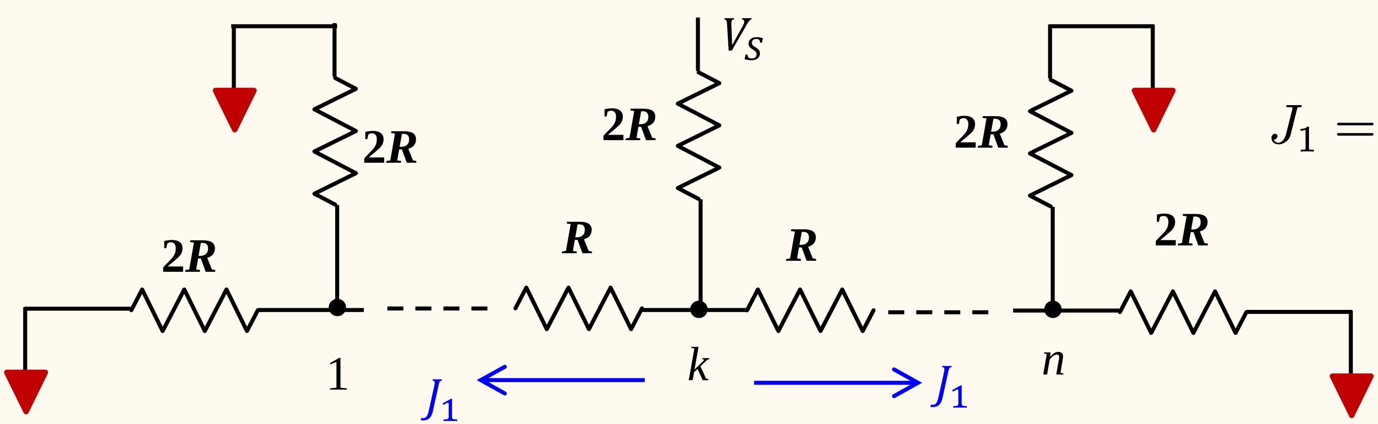
Resistor ladder type DA converter

MOS switch array

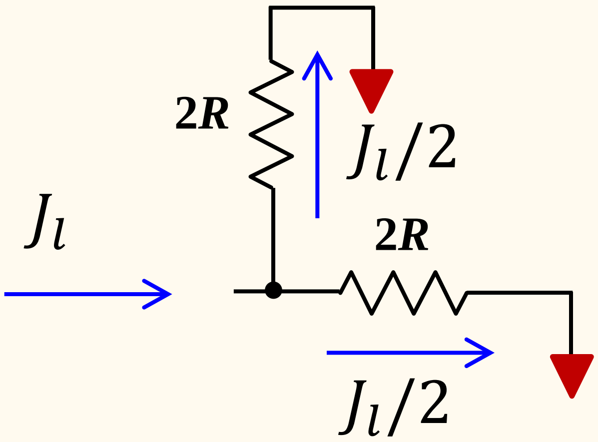
ON: V_S
OFF: ground



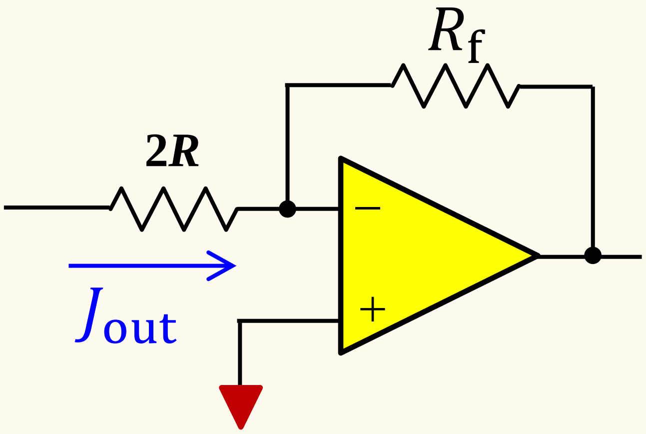
Input $(0,0,\cdots,0,1,0,\cdots,0)$ $\quad d_k = 1$, others = 0



$$J_1 = \frac{V_S}{2 \cdot (2R + R)} = \frac{V_S}{6R}$$



$$J_{l+1} = \frac{J_l}{2}$$



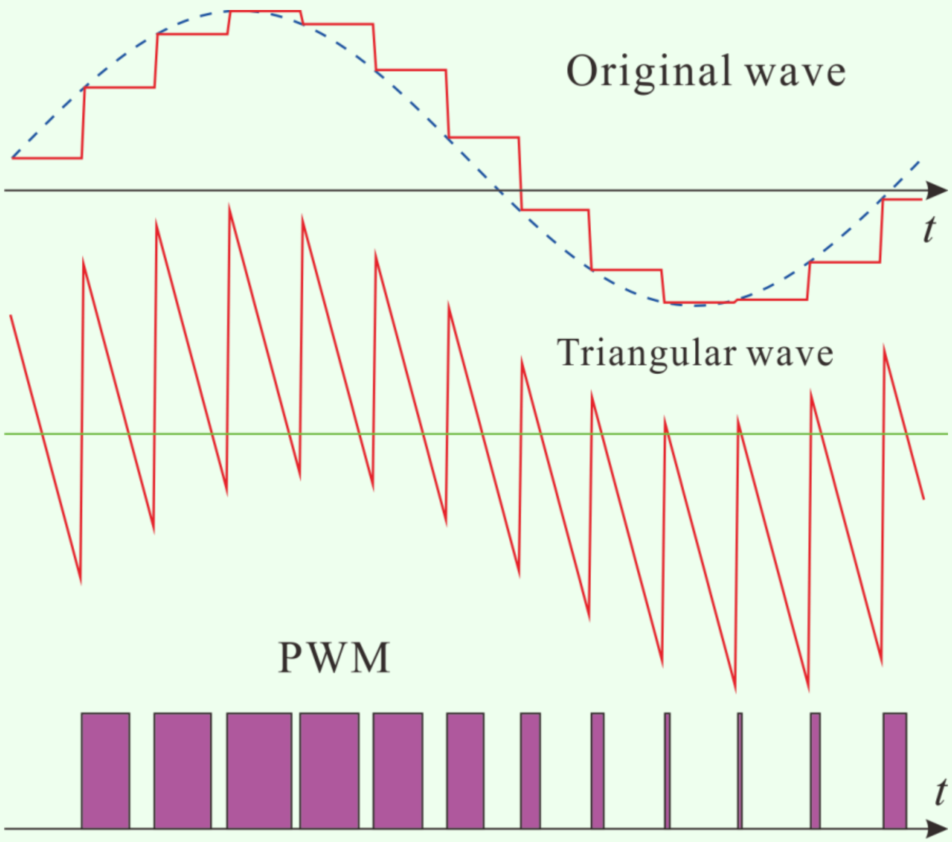
$$J_{out}\begin{pmatrix} 0\cdots 0 & 1 & 0\cdots 0 \\ n & k & 1 \end{pmatrix} = \frac{V_S}{3R}\left(\frac{1}{2}\right)^{n-k+1} = \frac{V_S}{6\cdot 2^n R}2^k$$

From the superposition theorem:

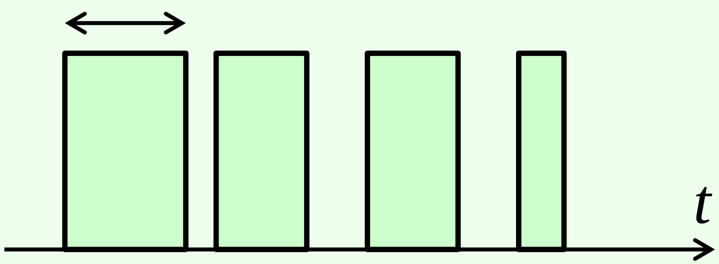
$$V_{out}(\{d_i\}) = -\frac{1}{3\cdot 2^n}\frac{R_f}{2R}V_S\sum_{k=1}^{n}2^k d_k$$

7.5.1 Digital to Analog converter

Pulse width modulation (PWM)



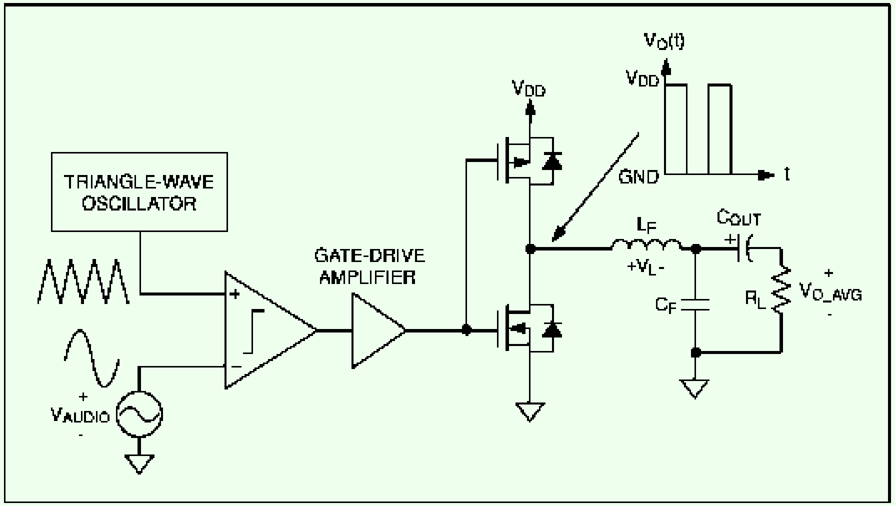
Digital signal →
PWM signal with a counter



Low pass

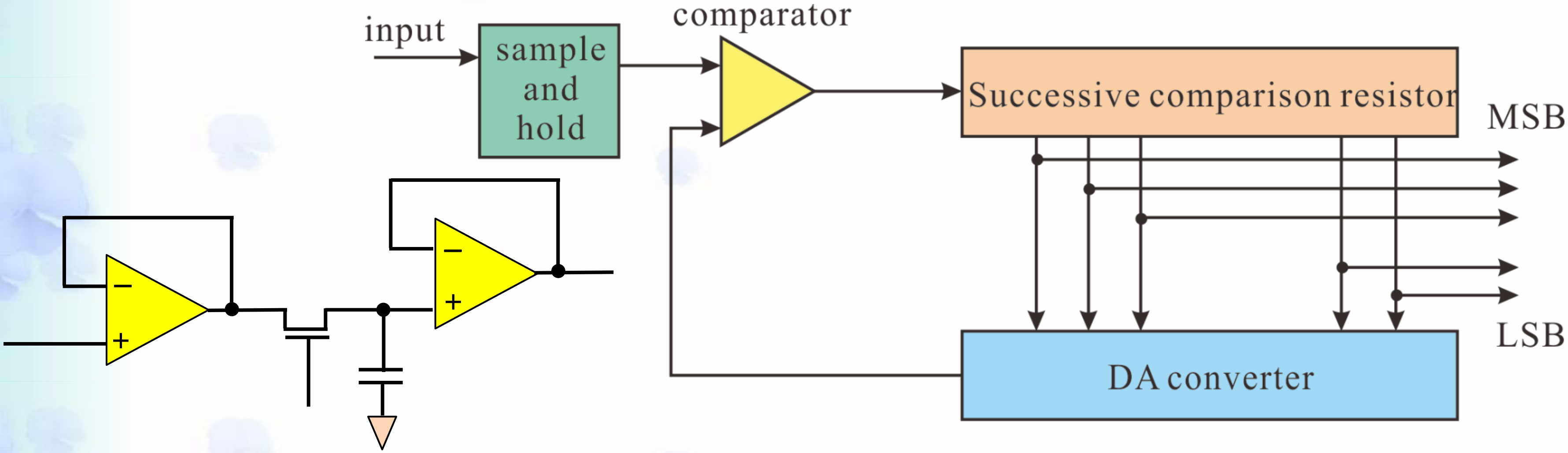
Analog signal

D-class amplifier

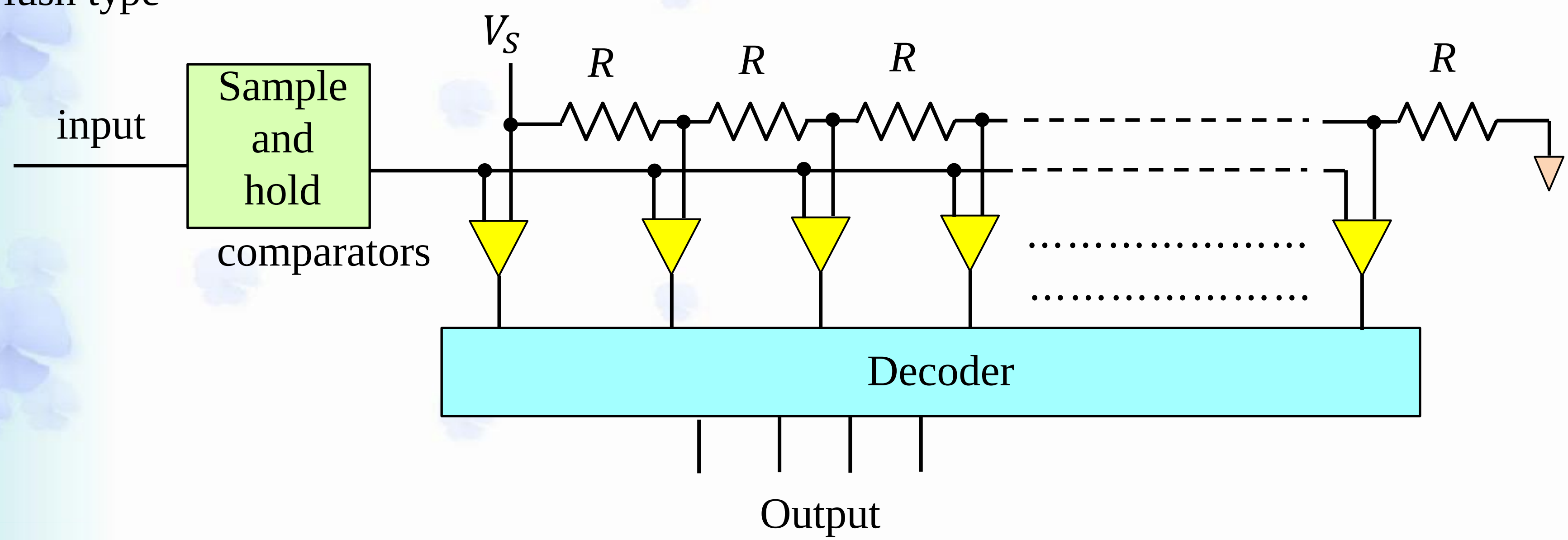


7.5.2 Analog-Digital converter

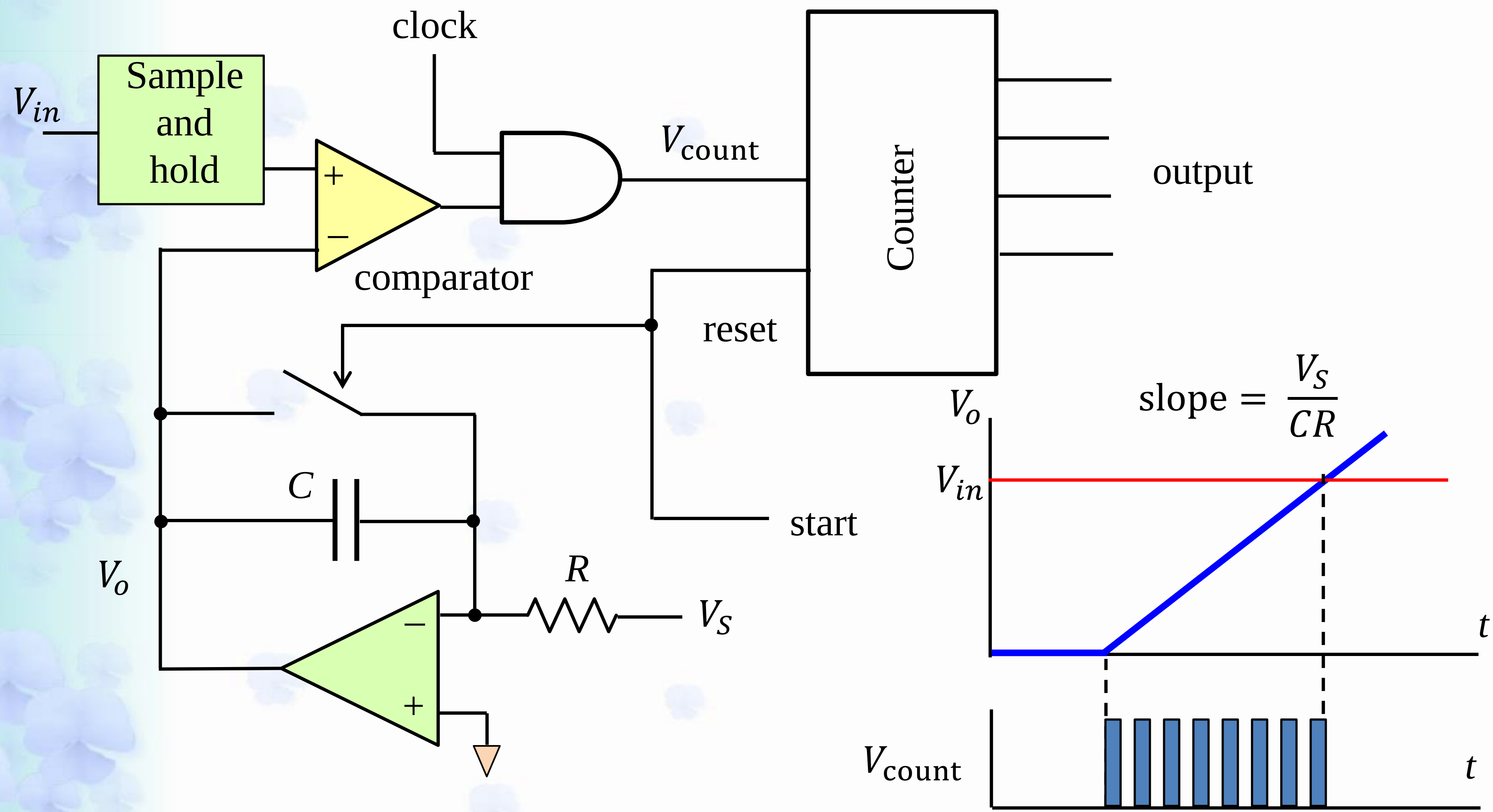
Successive comparison type AD converter



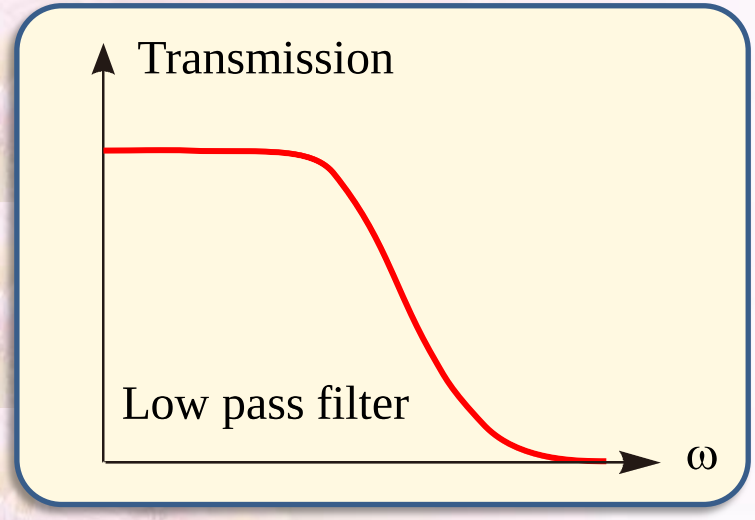
Flush type

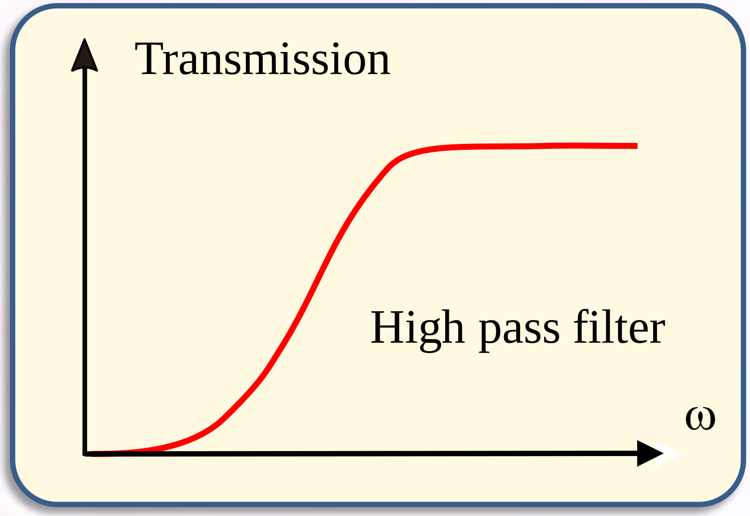


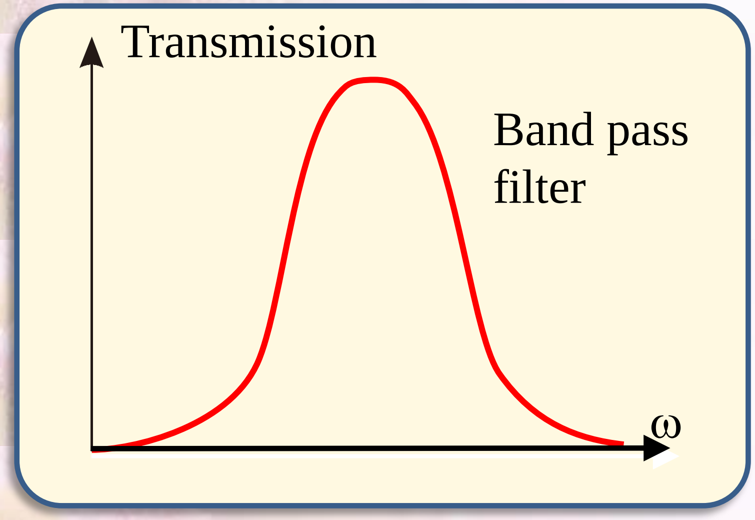
Integrating Analog-Digital Converter

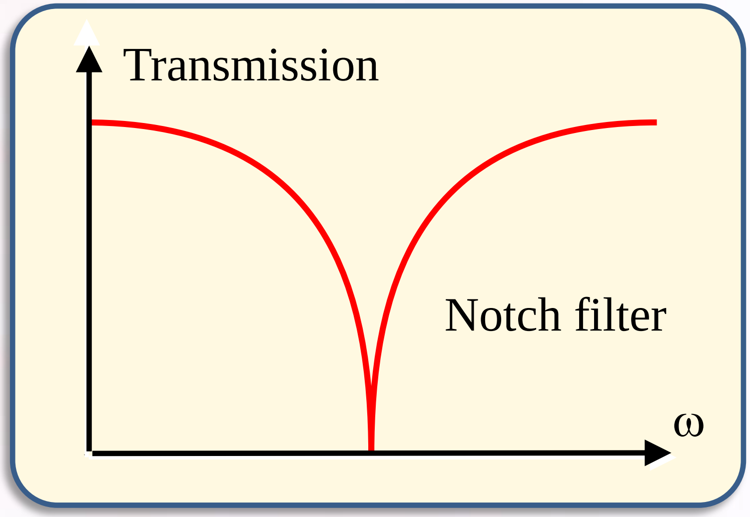


Filter Circuit



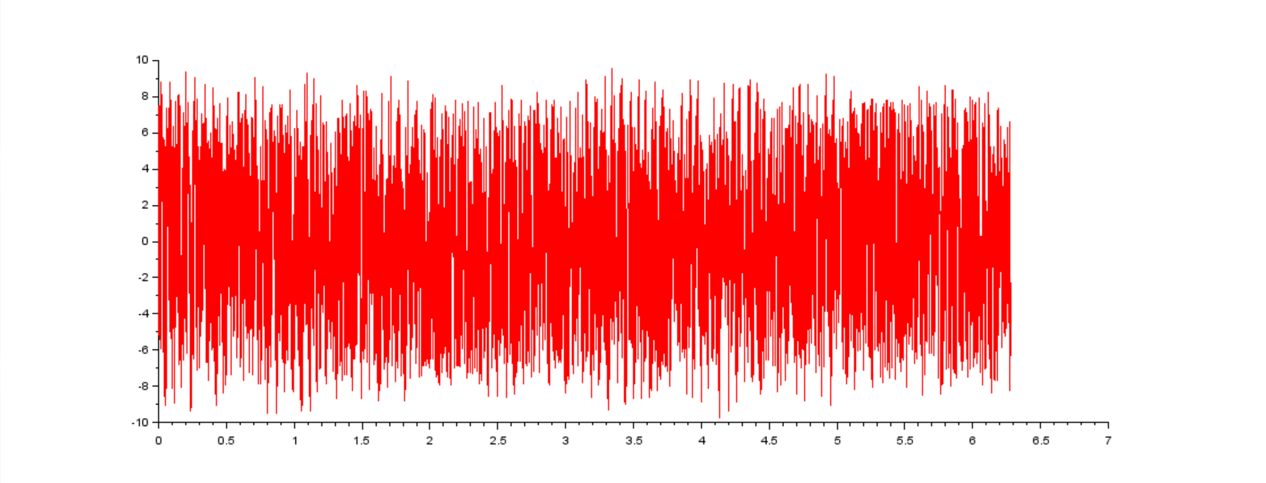




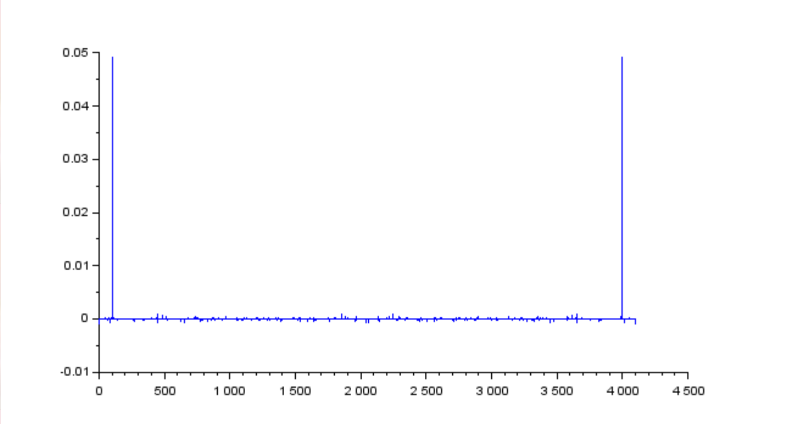


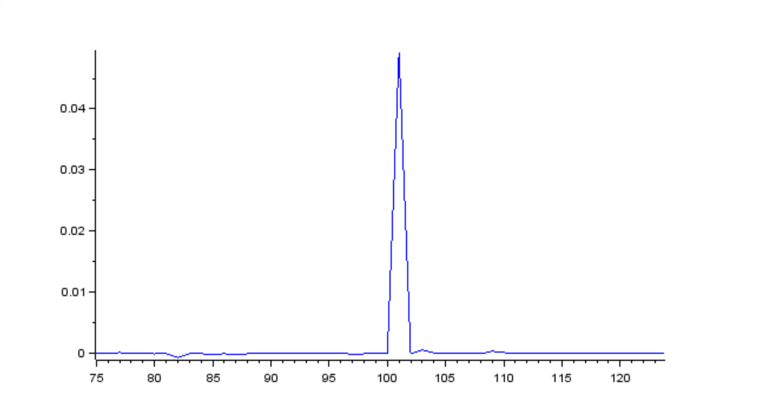
An example for retrieving data from noise

Signal with noise



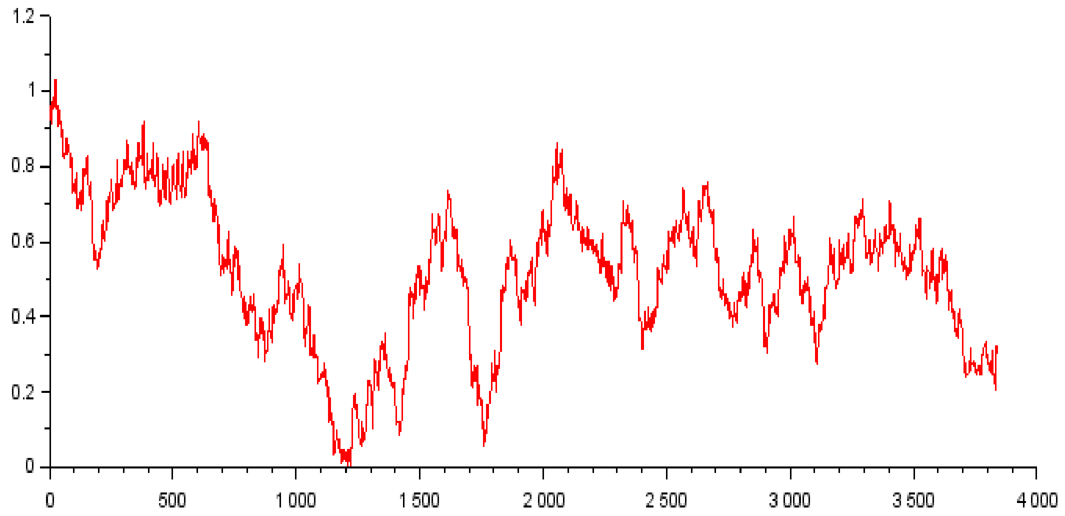
Detection of carrier



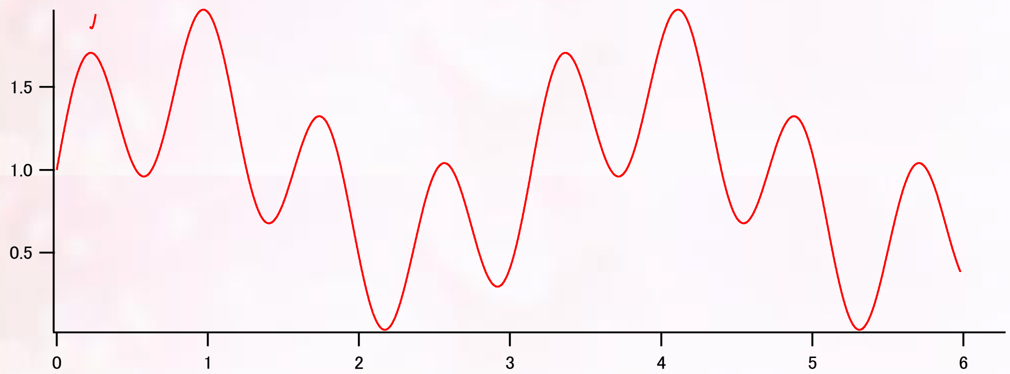


Results

Frequency filter



Original signal



7.6 Digital filter (as a digital signal processing)

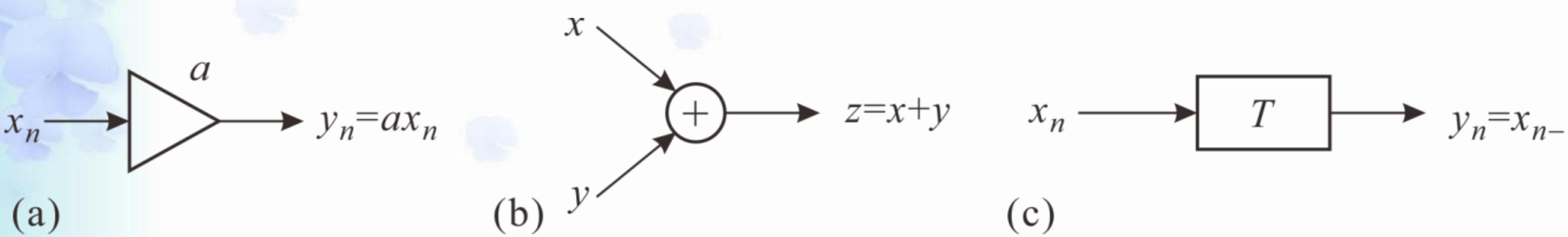
Digital filtering:

$$\{x_i\} = (x_0, x_1, \cdots)$$

$$\{y_i\} = (y_0, y_1, \cdots)$$

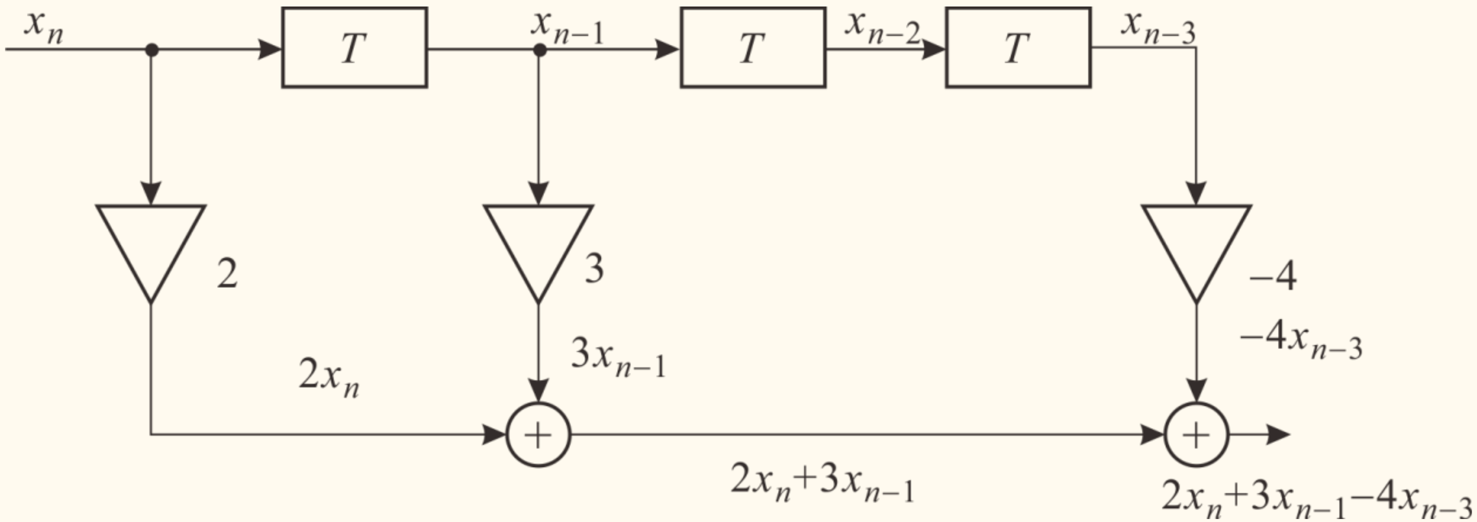
$$y_n = F(x_{n-k}, x_{n-k+1}, \cdots, x_n)$$

Block diagram representation of operations



constant multiplier   adder   delay (shift resistor)

Block diagram example



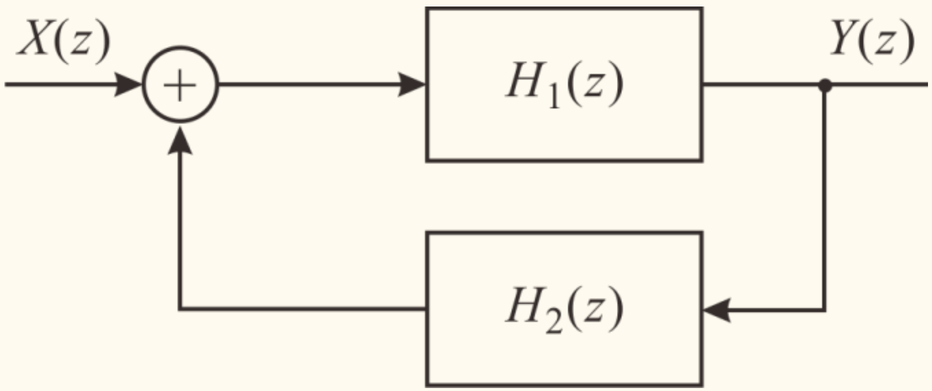
$$y_n = 2x_n + 3x_{n-1} - 4x_{n-3}$$

$$X(z) = \sum_{n=0}^{\infty} x_n z^{-n}, \quad Y(z) = \sum_{n=0}^{\infty} y_n z^{-n}$$

$$\begin{aligned} Y(z) &= 2X(z) + 3z^{-1}X(z) - 4z^{-3}X(z) \\ &= (2 + 3z^{-1} - 4z^{-3})X(z) \end{aligned}$$

$$\therefore H(z) \text{ (transfer function) } = 2 + 3z^{-1} - 4z^{-3}$$

Feedback and transfer function



$$Y(z) = H_1(z)W(z) = H_1(z)(X(z) + H_2(z)Y(z)),$$

$$\therefore Y(z) = \frac{H_1(z)}{1 - H_1(z)H_2(z)} X(z)$$

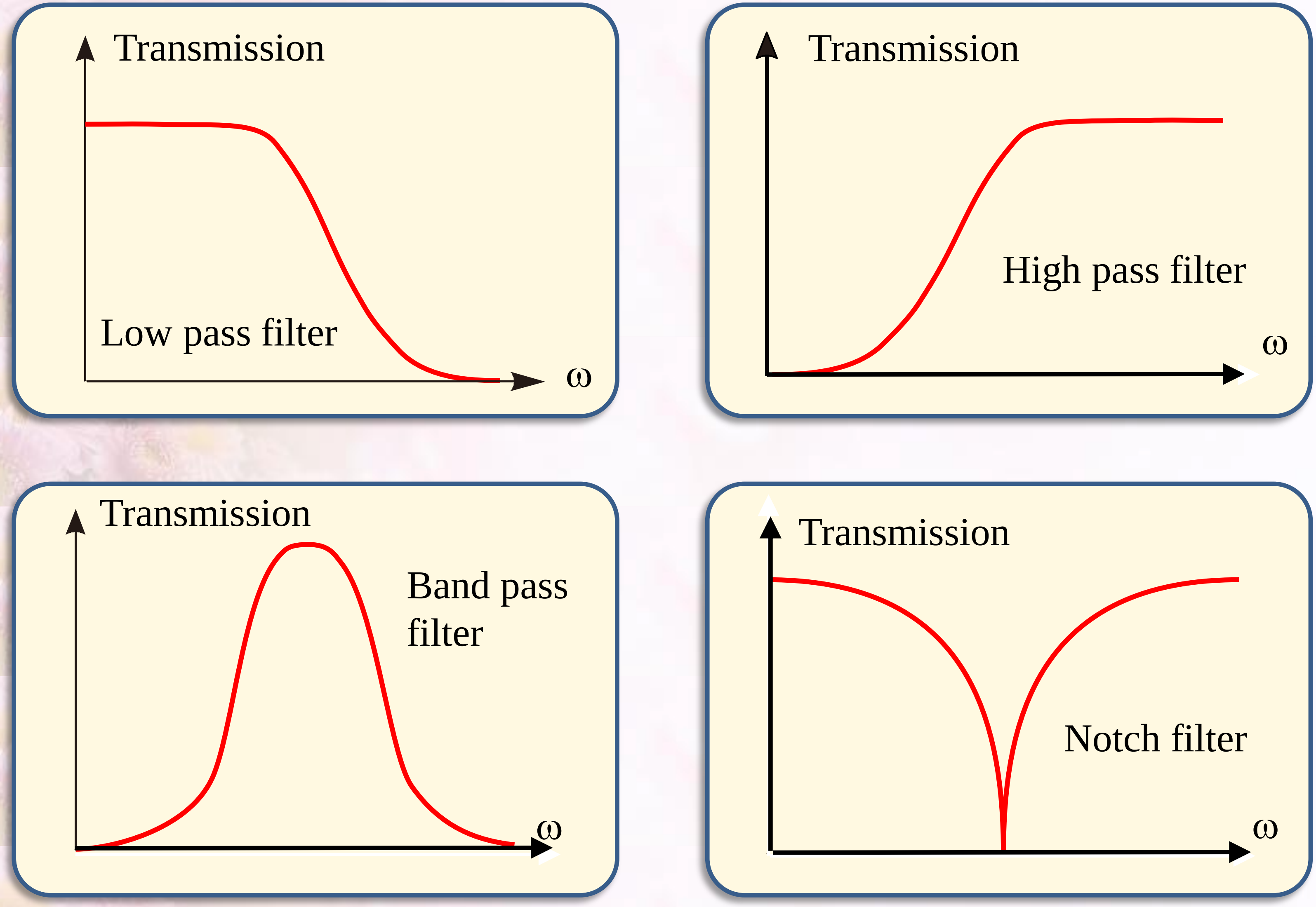
$$H(z) = \frac{H_1(z)}{1 - H_1(z)H_2(z)}$$

$$(\text{transfer function}) = \frac{(\text{direct gain})}{1 - (\text{feedback transfer gain})}$$

電子回路論第14回 (最終回)
Electric Circuits for Physicists

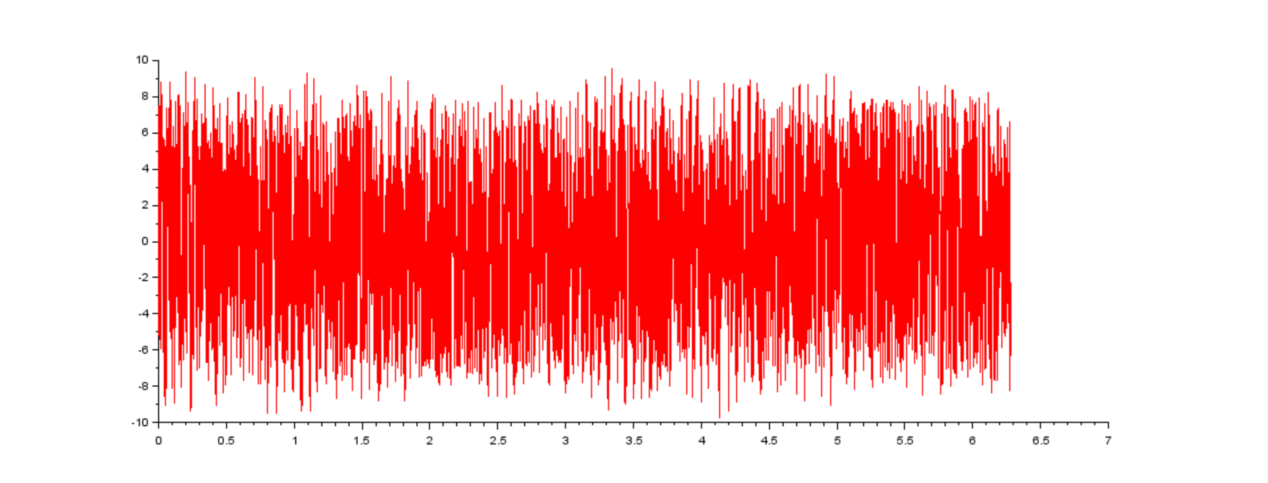
東京大学理学部・理学系研究科
物性研究所
勝本信吾
Shingo Katsumoto

Filter Circuit

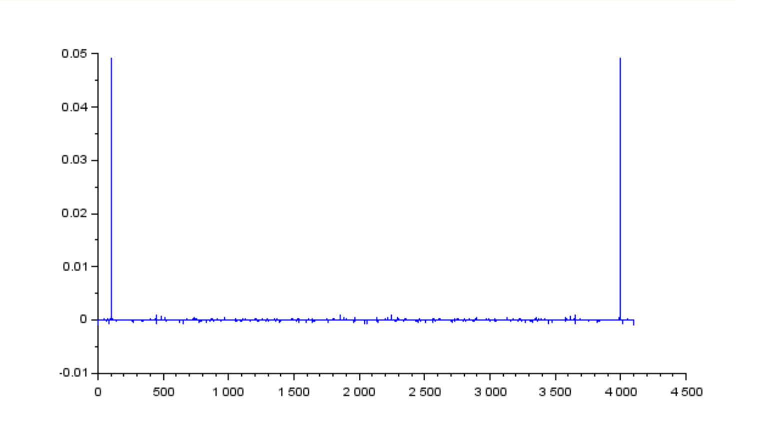


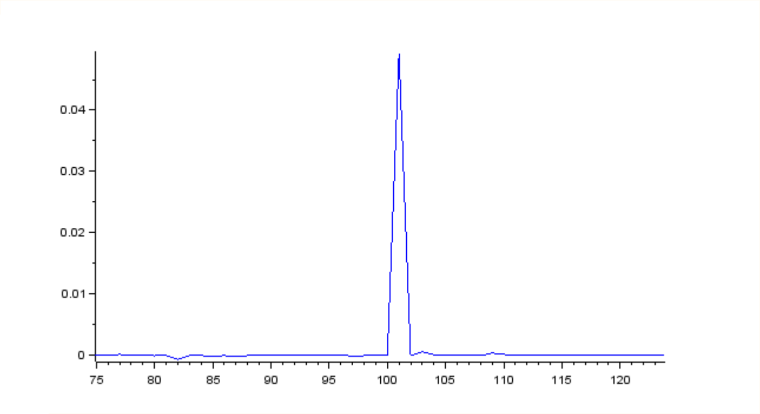
An example for retrieving data from noise

Signal with noise



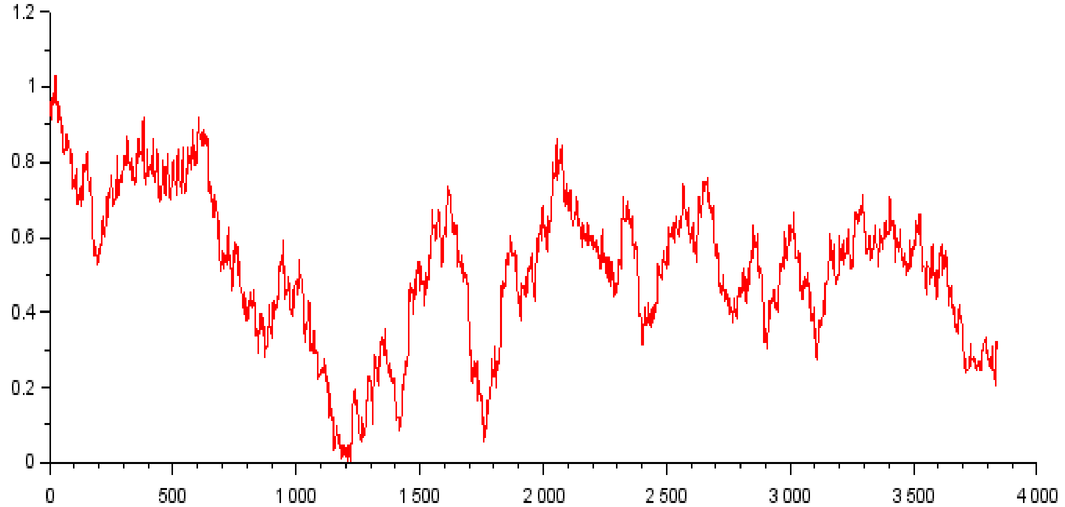
Detection of carrier



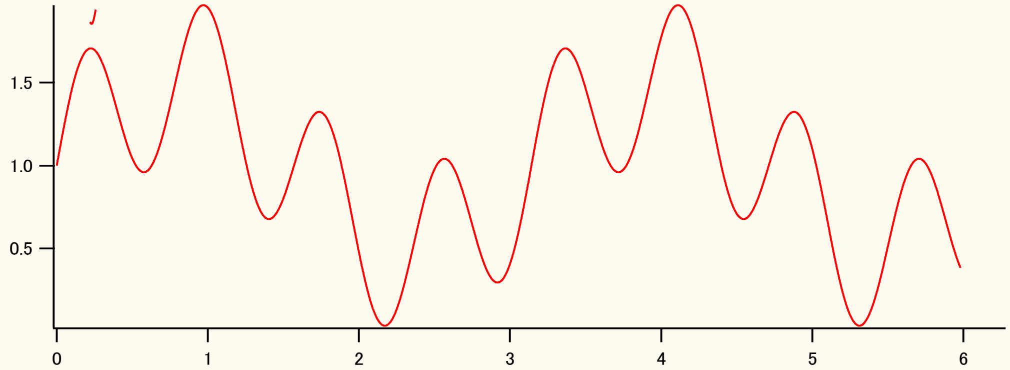


Results

Frequency filter



Original signal



7.6 Digital filter (as a digital signal processing)

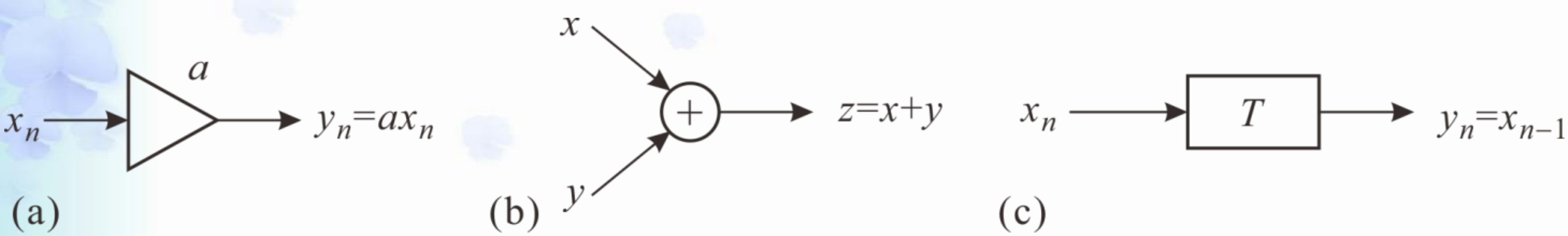
Digital filtering:

$$\{x_i\} = (x_0, x_1, \cdots)$$

$$\{y_i\} = (y_0, y_1, \cdots)$$

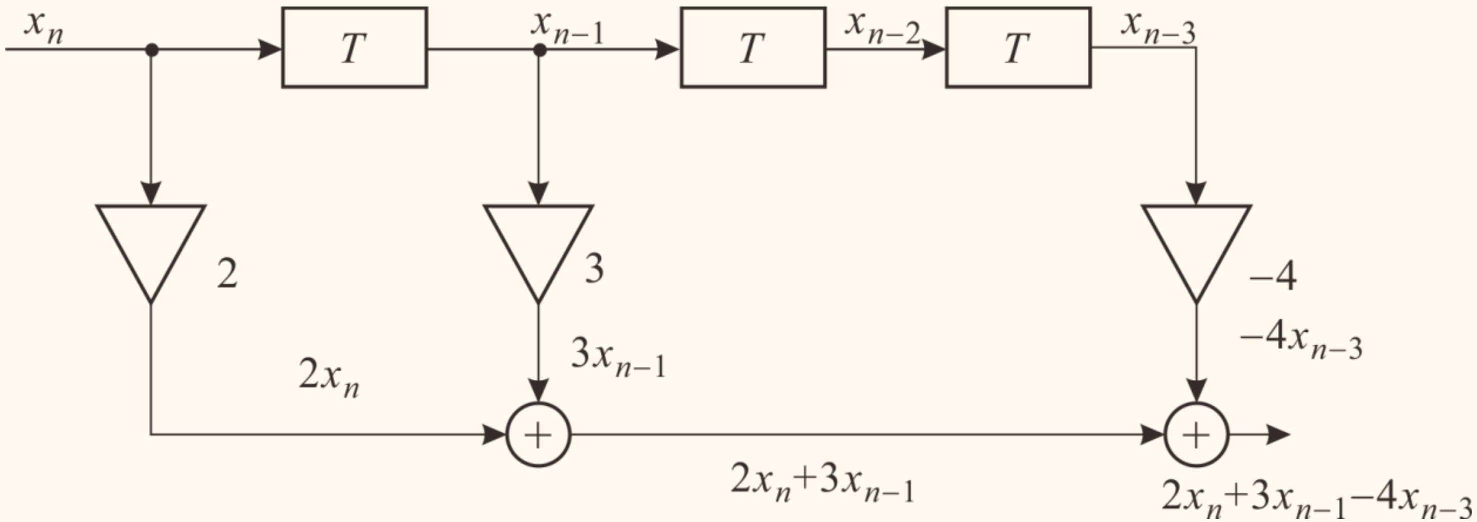
$$y_n = F(x_{n-k}, x_{n-k+1}, \cdots, x_n)$$

Block diagram representation of operations



constant multiplier adder delay (shift resistor)

Block diagram example



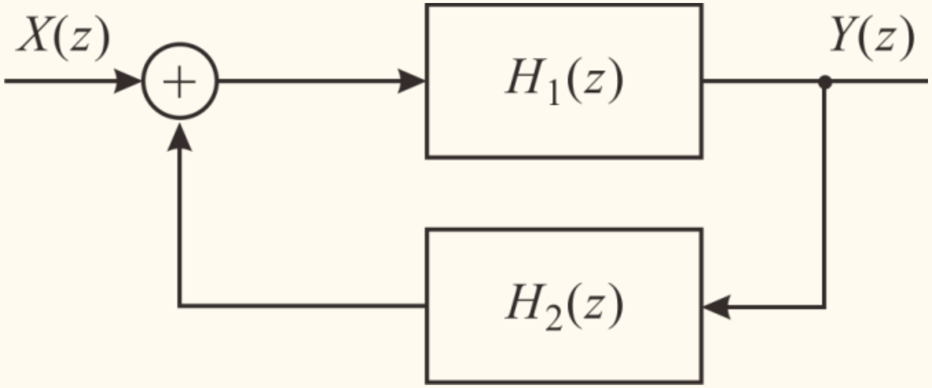
$$y_n = 2x_n + 3x_{n-1} - 4x_{n-3}$$

$$X(z) = \sum_{n=0}^{\infty} x_n z^{-n}, \quad Y(z) = \sum_{n=0}^{\infty} y_n z^{-n}$$

$$\begin{aligned} Y(z) &= 2X(z) + 3z^{-1}X(z) - 4z^{-3}X(z) \\ &= (2 + 3z^{-1} - 4z^{-3})X(z) \end{aligned}$$

$$\therefore H(z) \text{ (transfer function) } = 2 + 3z^{-1} - 4z^{-3}$$

Feedback and transfer function



$$Y(z) = H_1(z)W(z) = H_1(z)(X(z) + H_2(z)Y(z)),$$

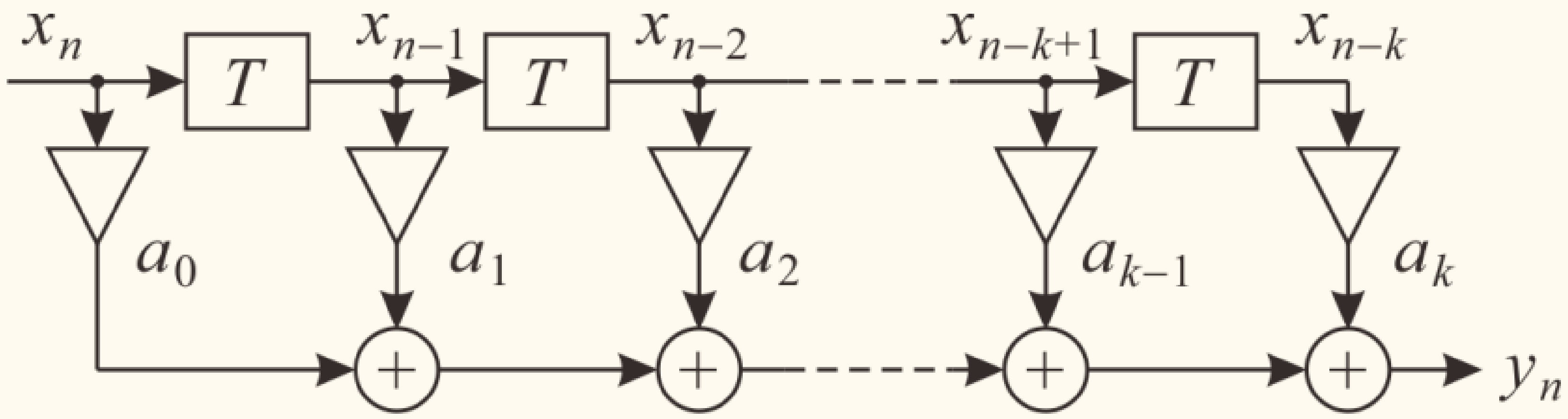
$$\therefore Y(z) = \frac{H_1(z)}{1 - H_1(z)H_2(z)} X(z)$$

$$H(z) = \frac{H_1(z)}{1 - H_1(z)H_2(z)}$$

$$(\text{transfer function}) = \frac{(\text{direct gain})}{1 - (\text{feedback transfer gain})}$$

FIR filter

Finite impulse response (FIR) filter

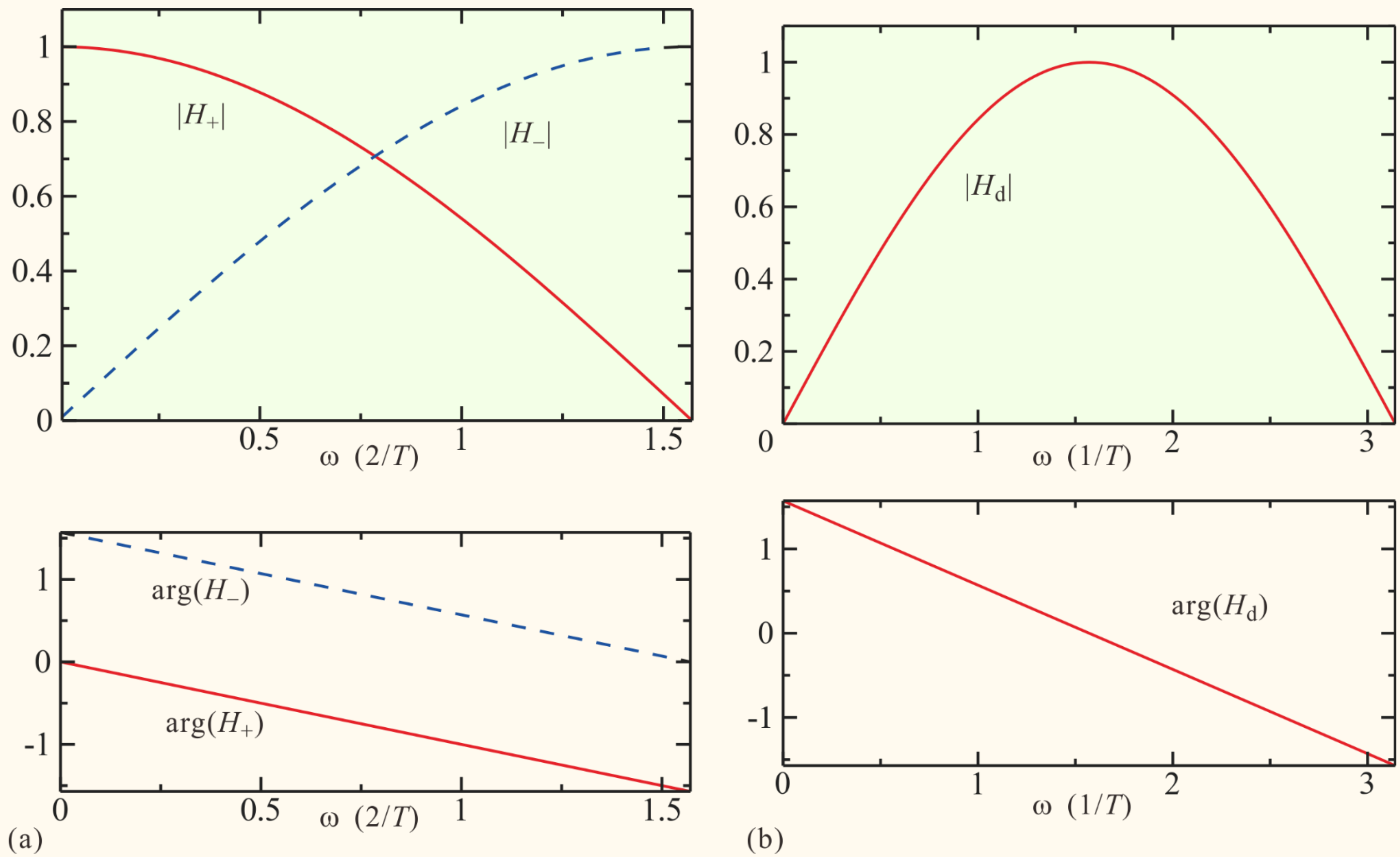


$$H(e^{i\omega\tau}) = \sum_{j=0}^{k} a_j e^{-ij\omega\tau}$$

A Simple example of FIR filter

Moving average, differentiation: $F_{\pm}(x_n, x_{n-1}) = (x_n \pm x_{n-1})/2$

$$H_{\pm}(e^{i\omega\tau}) = e^{-i\omega\tau/2}\begin{pmatrix}\cos(\omega\tau/2)\\ i\sin(\omega\tau/2)\end{pmatrix}$$

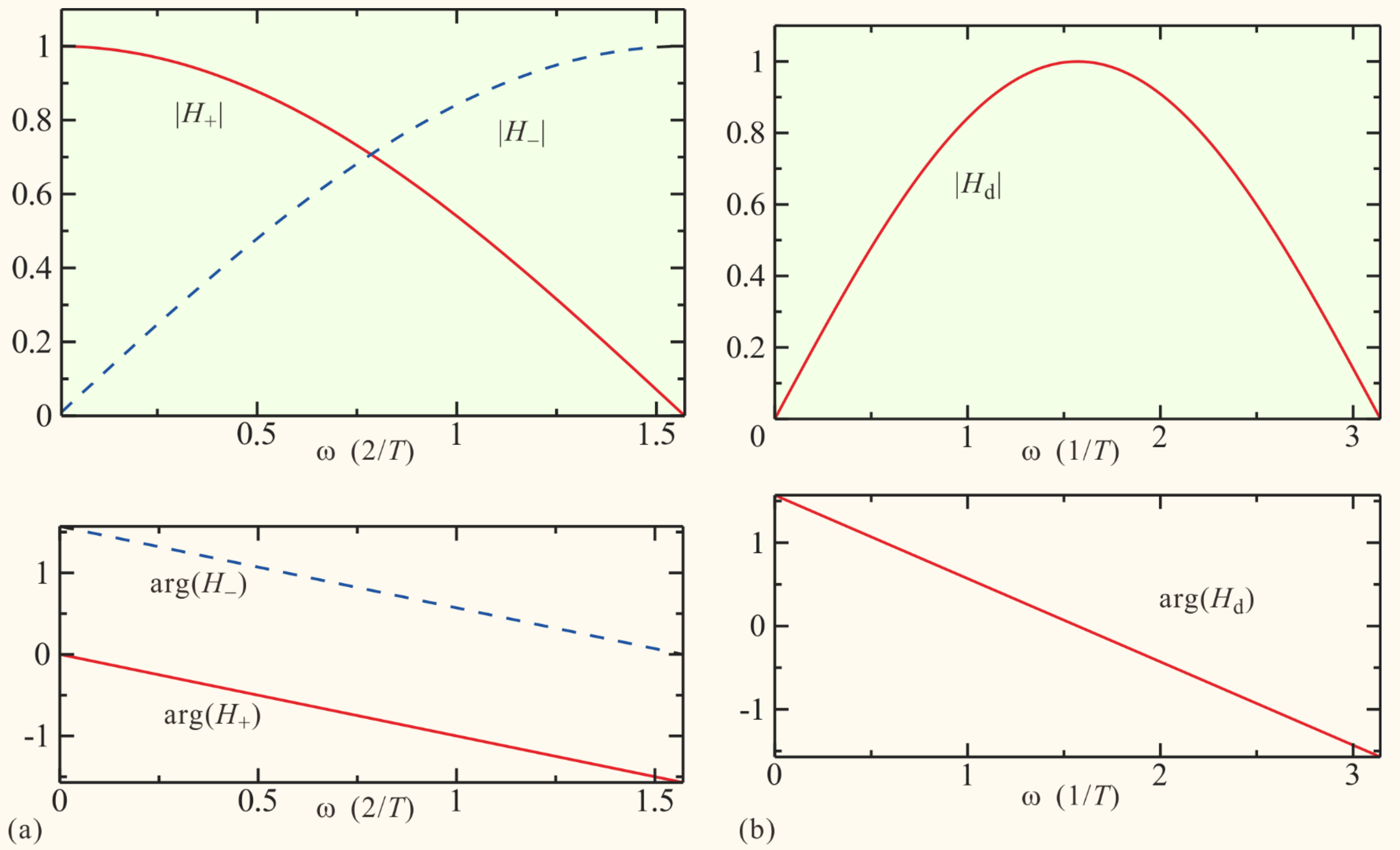


A Simple example of FIR filter

Differentiation of moving average:

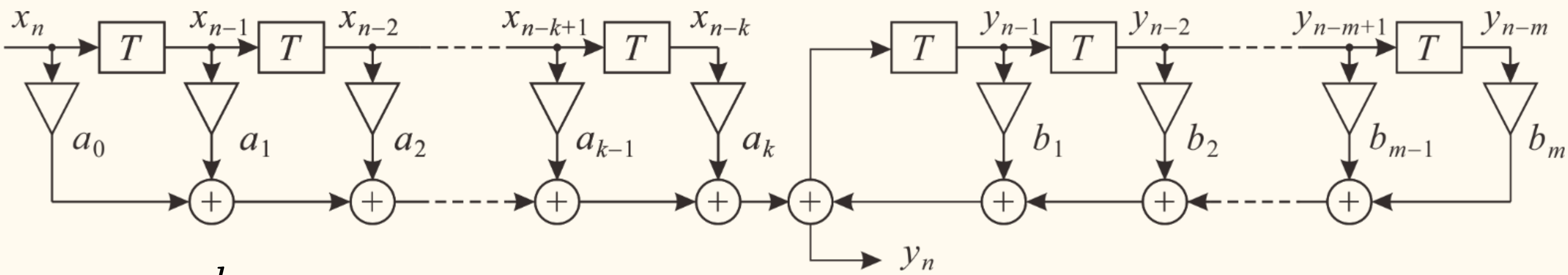
$$F_{\mathrm{d}} = [(x_n + x_{n-1}) - (x_{n-1} + x_{n-2})]/2 = [x_n - x_{n-2}]/2$$

$$H_{\mathrm{d}} = (1 - e^{-2i\omega\tau})/2 = ie^{-i\omega\tau}\sin\omega\tau$$



IIR Filter

Infinite impulse response (IIR) filter:



$$y_n = \sum_{l=0}^{k} a_l x_{n-l} + \sum_{j=1}^{m} b_j y_{n-j}$$ Stability condition: $\lim_{n\to\infty} y_n = 0$

$$Y(z) = X(z) \sum_{l=0}^{k} a_l z^{-l} + Y(z) \sum_{j=1}^{m} b_j z^{-j}$$

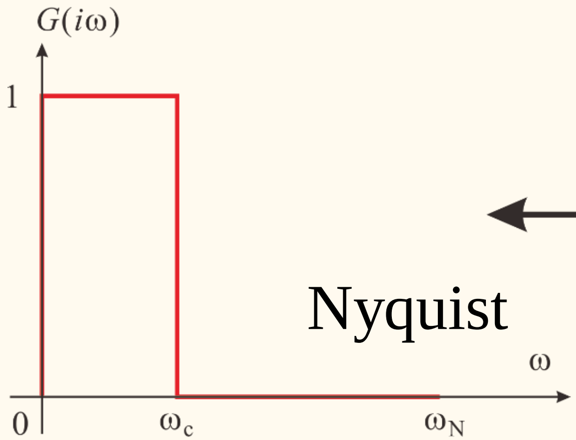
$$H(z) = \frac{Y(z)}{X(z)} = \sum_{l=0}^{k} a_l z^{-l} \Bigg/ \left(1 - \sum_{j=1}^{m} b_j z^{-j}\right)$$

Conversion of z-transform: $|z| > 1$ the poles should be in $|z| < 1$

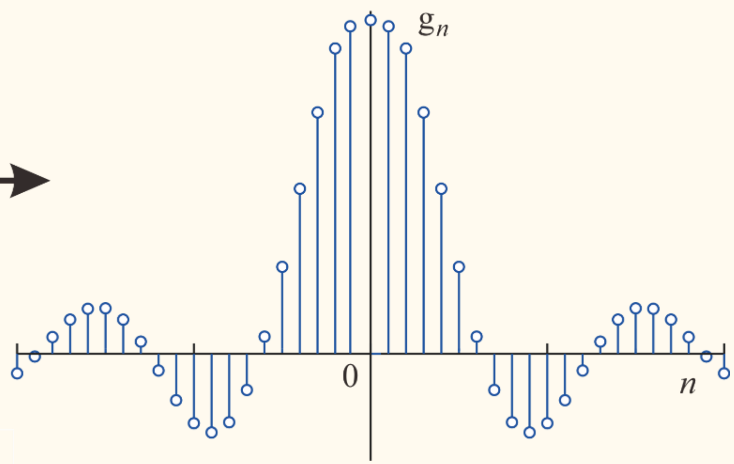
Design of FIR filter: Window function

Ideal low pass filter $G(e^{i\omega\tau}) = \begin{cases} 1, & |\omega| \le \omega_c, \\ 0, & \omega_c < |\omega| \le \omega_N \text{ Nyquist frequency} \end{cases}$

$$G(e^{i\omega\tau}) = \frac{\omega_c}{\omega_N} \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \mathrm{sinc}\left(n\frac{\omega_c}{\omega_N}\right) e^{-ni\omega\tau} = \gamma_c \sum_{n=-\infty}^{\infty} \frac{1}{n\pi} \mathrm{sinc}(n\gamma_c) z^{-n}$$

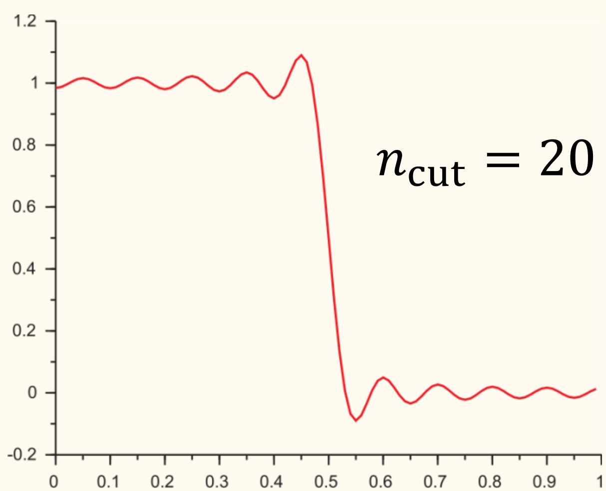


Nyquist



Cut the series at a finite number

$|G(e^{i\omega\tau})|$



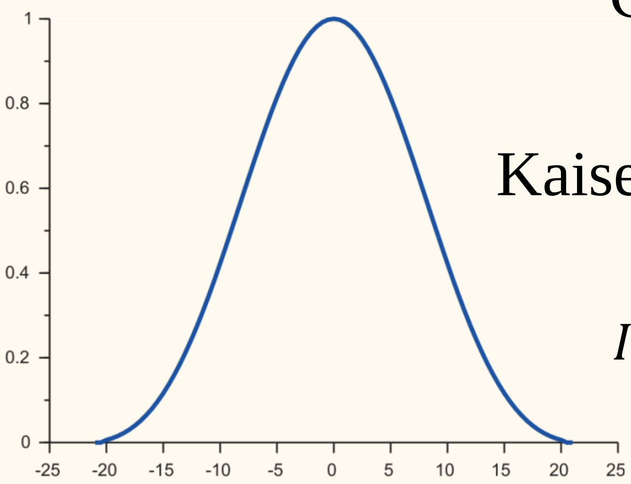
$n_{\mathrm{cut}} = 20$

Ripples in frequency characteristics

ω/ω_N

Design of FIR filter: Window function

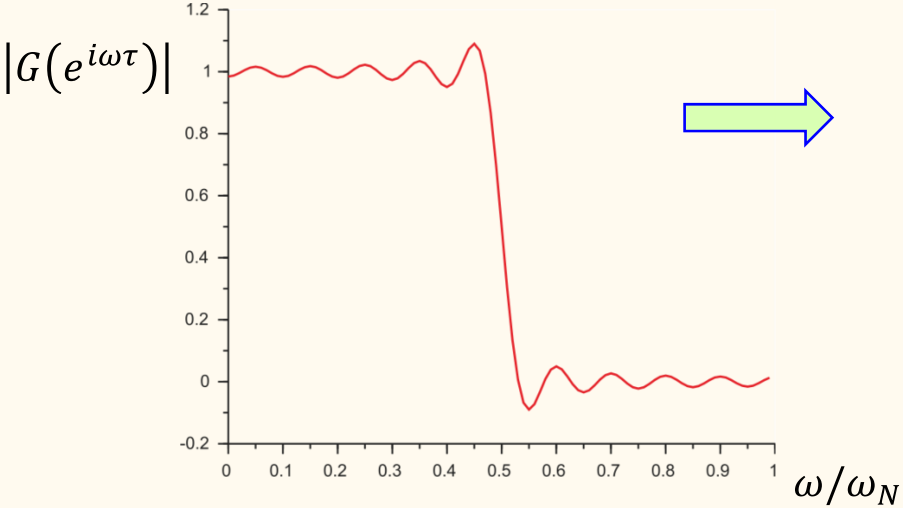
Sudden cutting of z-transform series → Ripples

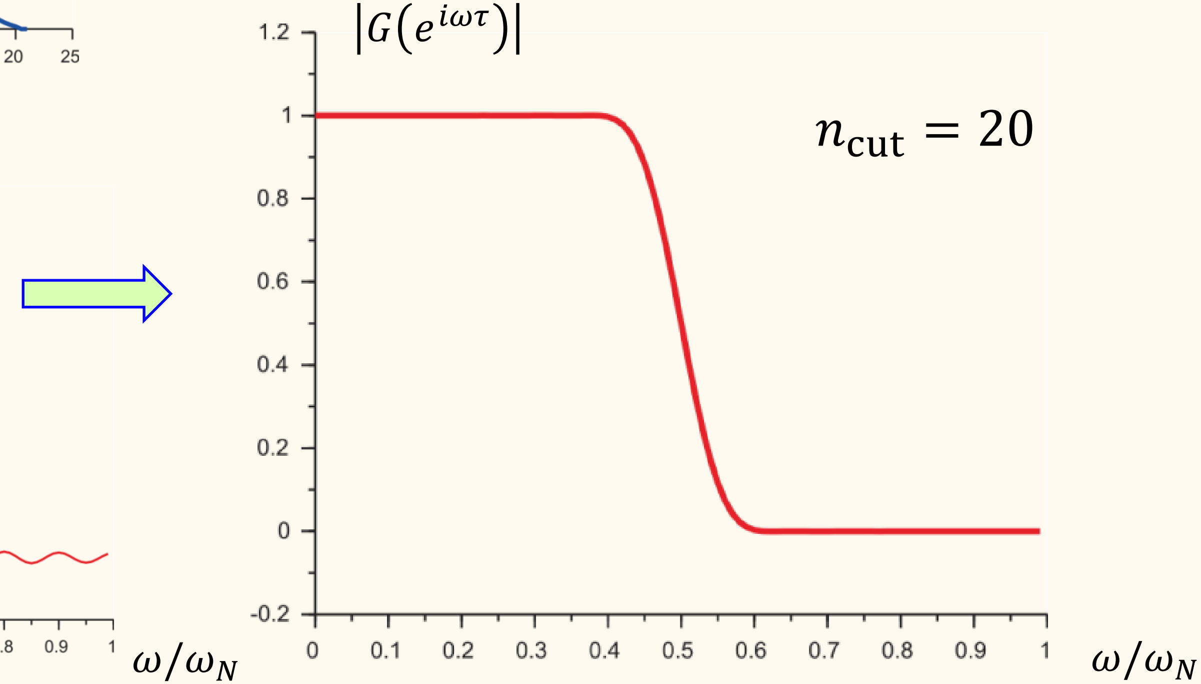


Cut with a smooth function

Kaiser window $w_n = \begin{cases} \dfrac{I_0\left(\alpha\sqrt{1-(n/L)^2}\right)}{I_0(\alpha)} & |n| \le L, \\ 0 & |n| > L \end{cases}$

I_0 : 0th order 1st type modified Bessel function





Design of IIR filter

Transfer function: a rational function (有理式)

A way to design IIR filter: modification of analog filter transfer function

Remember: Butterworth filter

$$\Xi(s) = \sum_{k=0}^{N-1} \frac{\omega_k}{s - s_k}, \quad s_k = r_\mathrm{c} \exp\left[i \left\{ \frac{\pi}{2} + \frac{(2k+1)\pi}{2n} \right\} \right]$$

$$\xi(t) = u_\mathrm{H}(t) \sum_{k=0}^{n-1} w_k \exp(s_k t)$$

Heaviside function

$$h_n = h_{\mathrm{H}n} \sum_{k=0}^{n-1} w_k e^{n s_k},$$

Time discretization with $\tau = 1$:

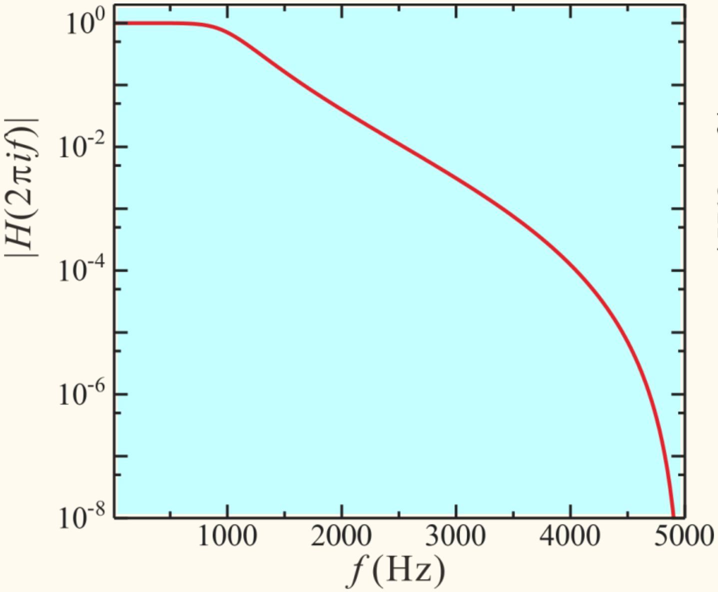
$$\therefore H(z) = \sum_{k=0}^{n-1} \frac{w_k}{1 - \exp(s_k) z^{-1}}$$

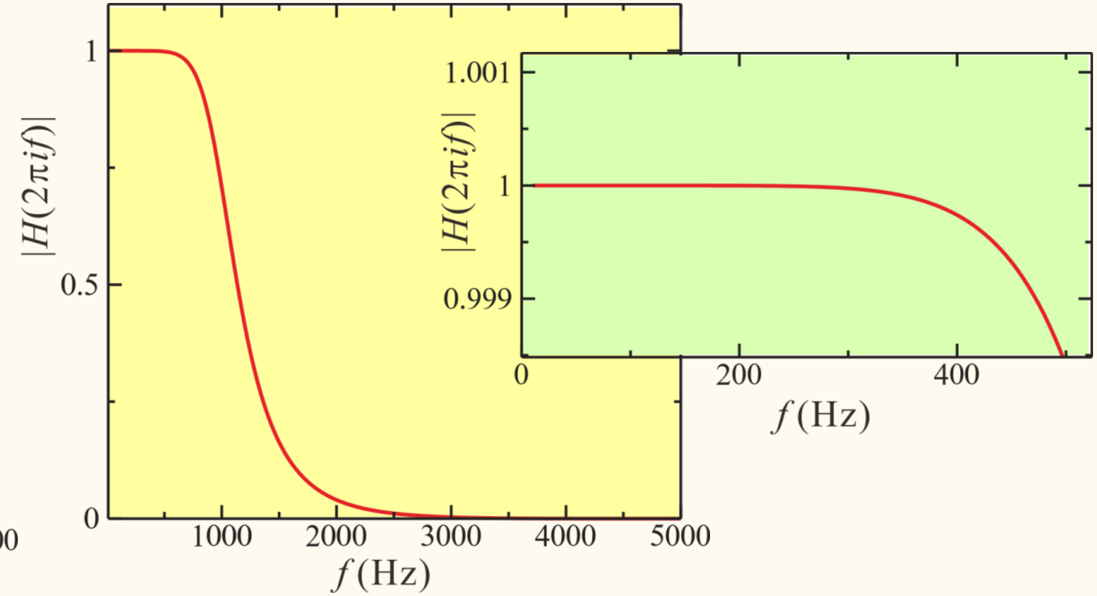
Design of IIR filter

Impulse invariant method: $$\frac{1}{s - s_k} \to \frac{1}{1 - \exp(s_k) z^{-1}}$$

Bilinear z-transform (双一次z変換法) : $$s \to \frac{1 - z^{-1}}{1 + z^{-1}}$$

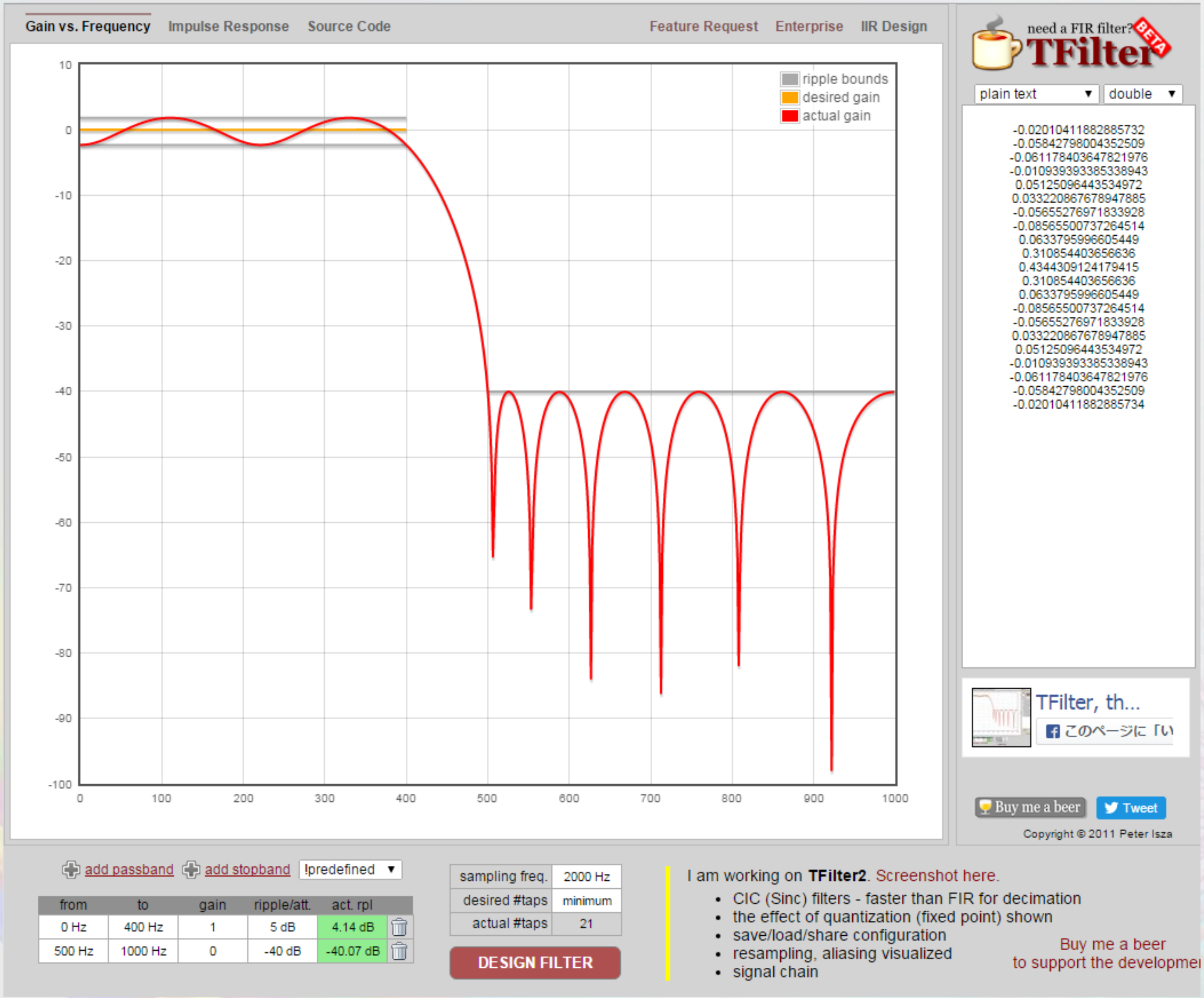
4th Butterworth: $$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 - a_1 z^{-1} - a_2 z^{-2} - a_3 z^{-3} - a_4 z^{-4}}$$



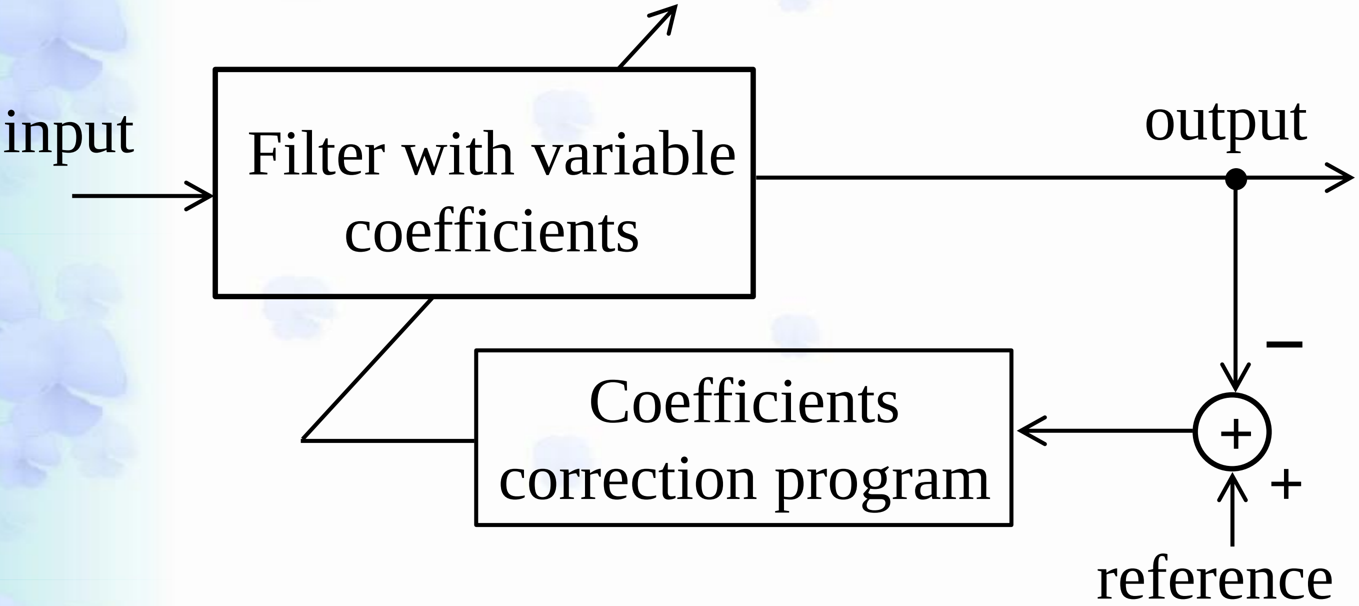


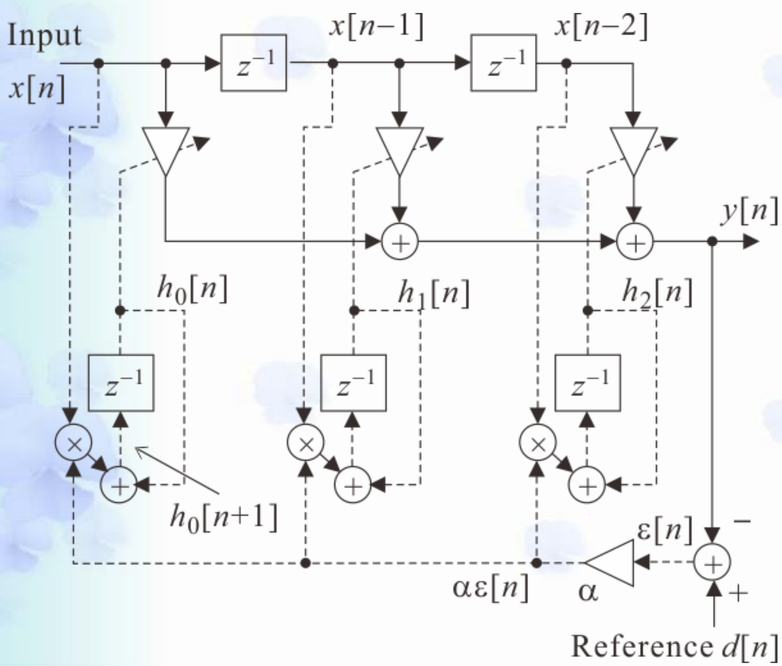
Digital filter design web application

http://t-filter.engineerjs.com/



Adaptive filter

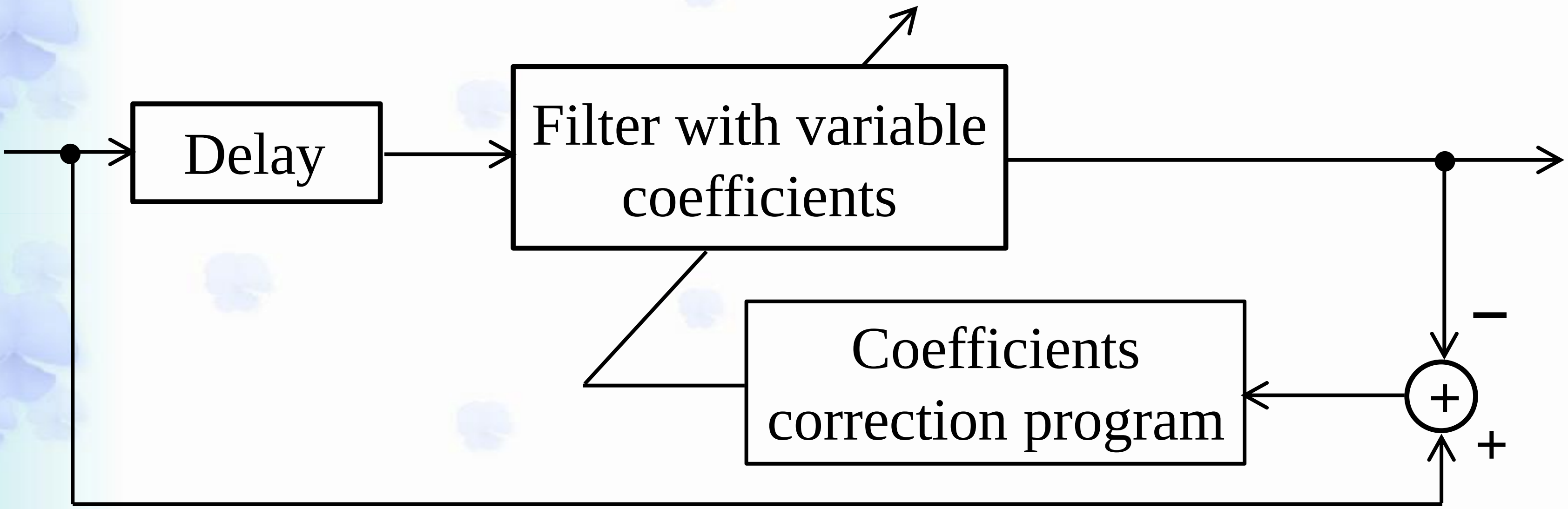


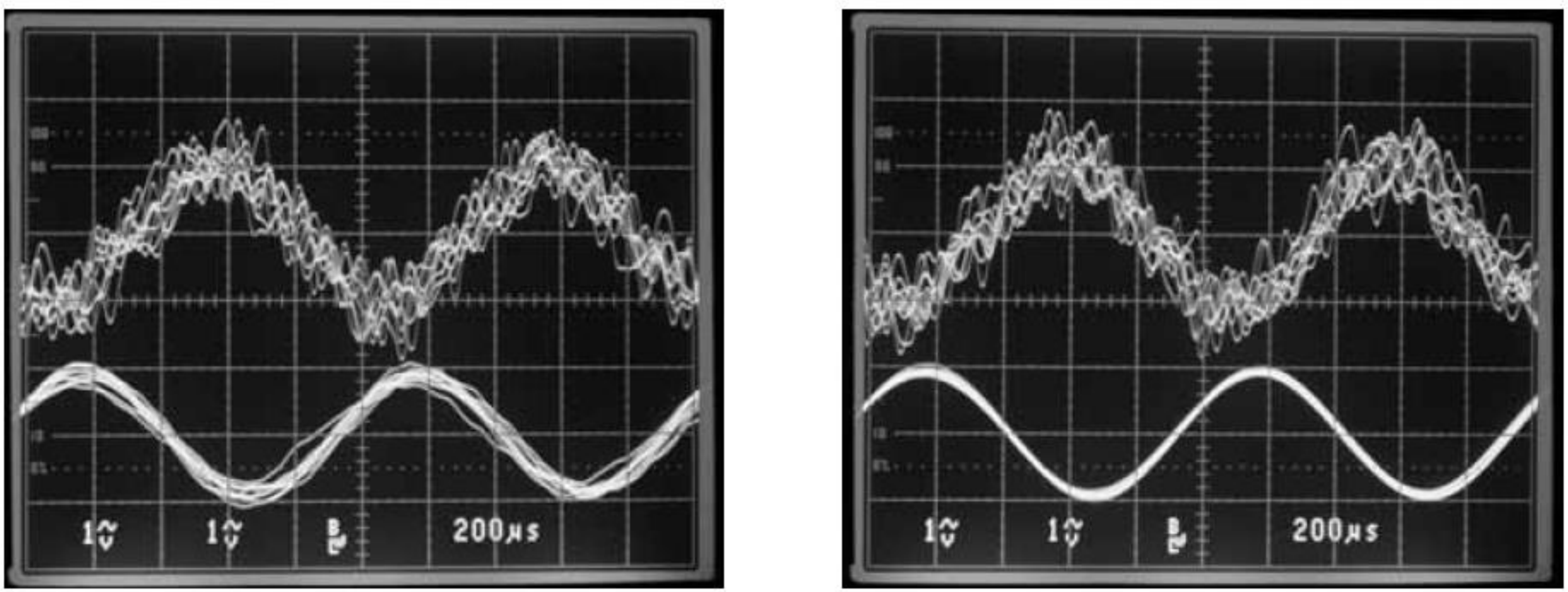


Least mean square method:

$$h_k[l+1] = h_k[l] + 2\alpha\epsilon[l]x[l-k]$$

Adaptive filter (adaptive line enhancer)

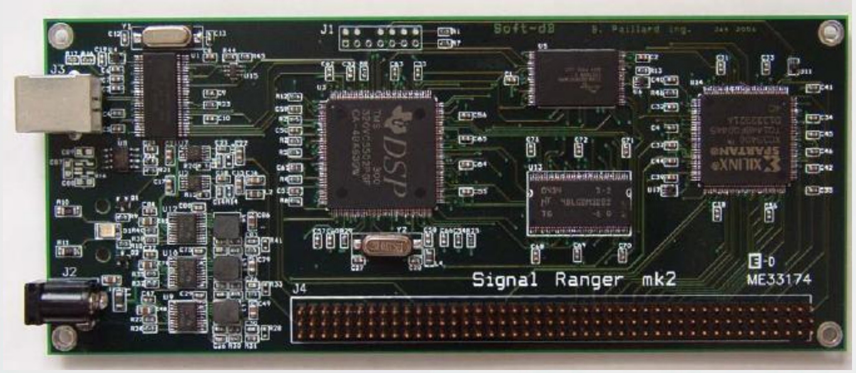


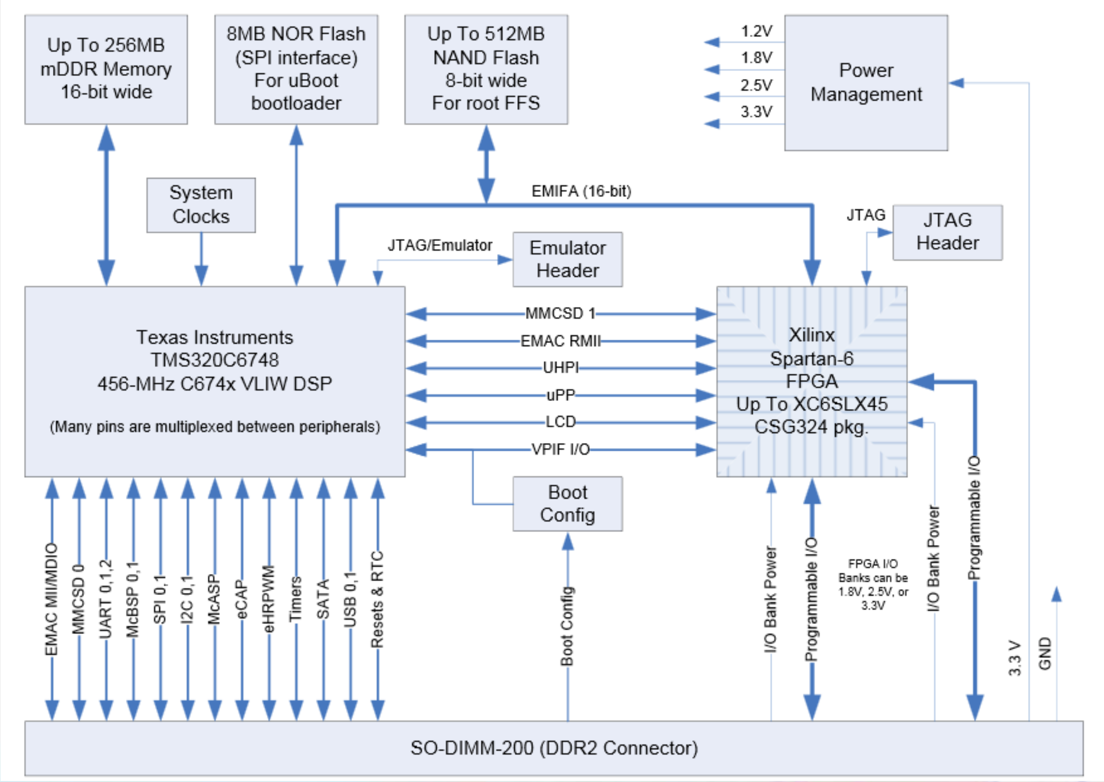


(a) $\mu = 1 \times 10^{-3}$の場合

(b) $\mu = 1 \times 10^{-5}$の場合

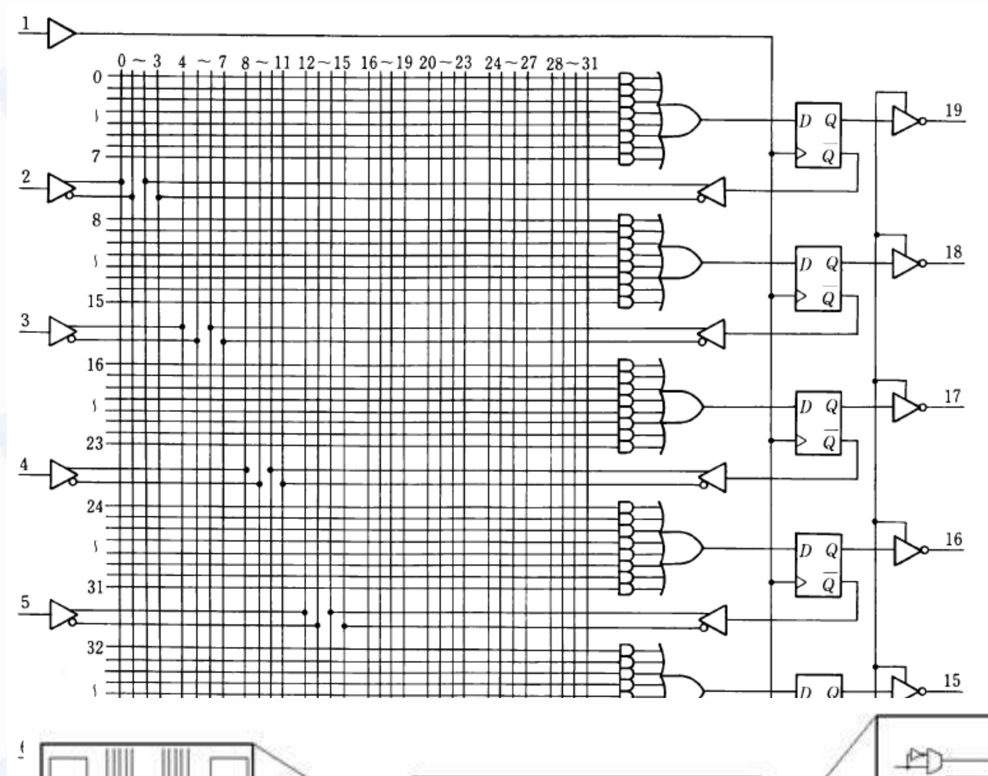
Digital signal processing (DSP) board



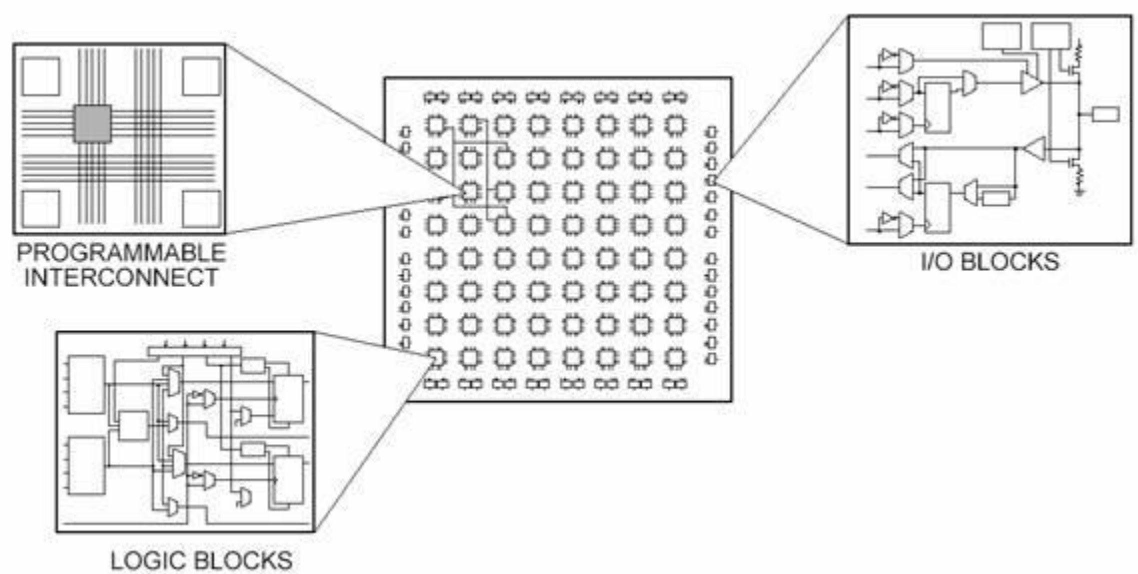


PLD/FPGA with HDL

Example of programmable logic device (PLD) circuit



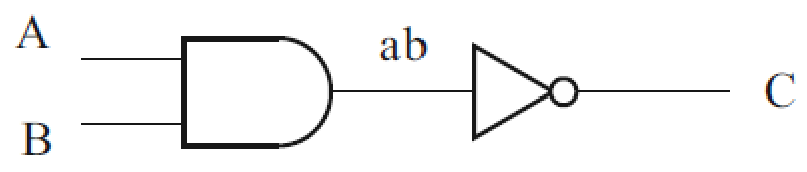
Example of field-programmable gate array (FPGA) circuit



FPGA ∈ PLD

Hardware description language, HDL

```
-- Library declaration --------------
library IEEE;
use IEEE, STD_LOGIC_1164.ALL;
-- Entity declaration ---------------
entity NAND_CIRCUIT is
port(
A : in std_logic;
B : in std_logic;
C : out std_logc
);
end NAND_CIRCUIT;
-- Architecture declaration ----------
architecture RTL of NAND_CIRCUIT is
signal ab : std_logic;
begin
ab <= A and B;
C  <= not ab;
end RTL;
```



RTL: register transfer level

Cf. SPICE

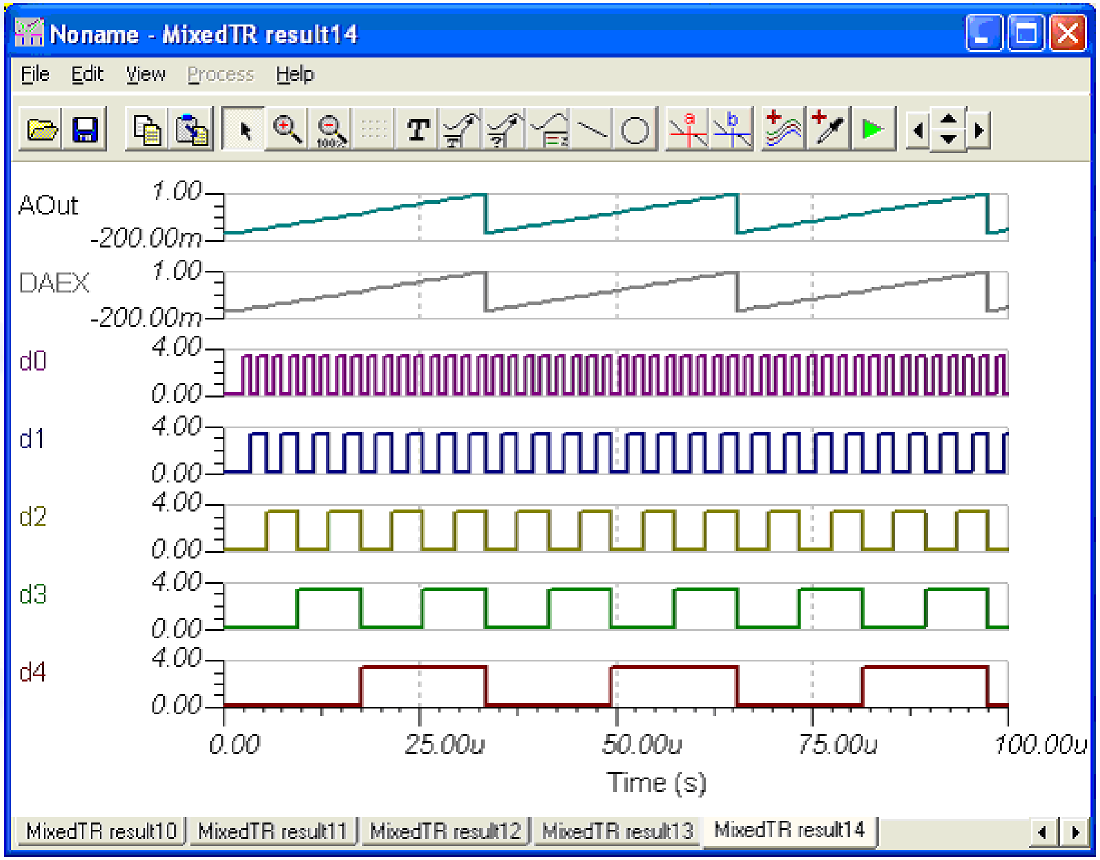
Spice+HDL mixed circuit simulation

Easier logic circuit design

Polyphony: Python ⟶ Verilog HDL http://www.sinby.com/PolyPhony/index.html

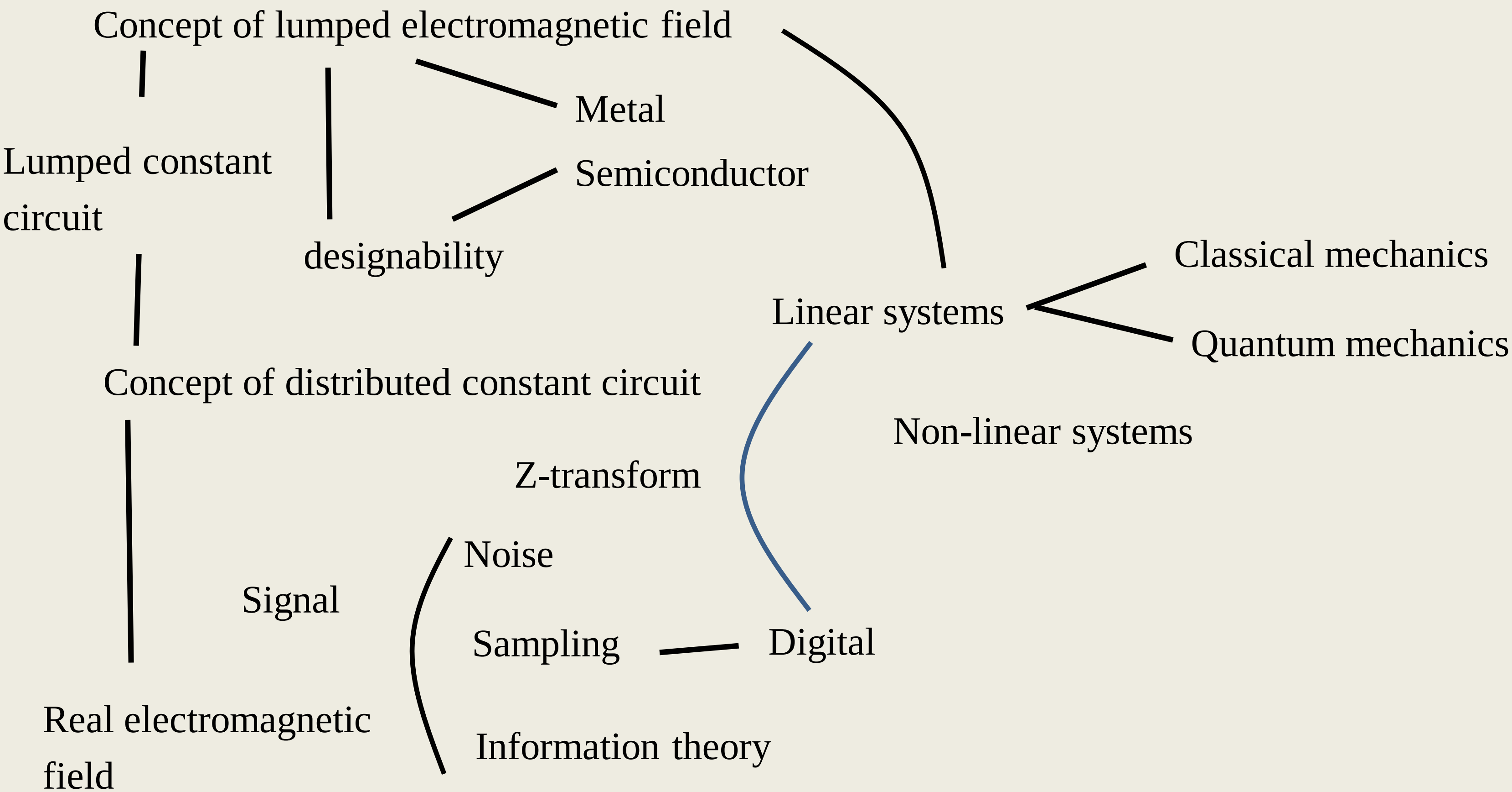
C-to-Hardware compiler (CHC) http://anvil.co.jp/?page_id=296

Mixed circuit simulation



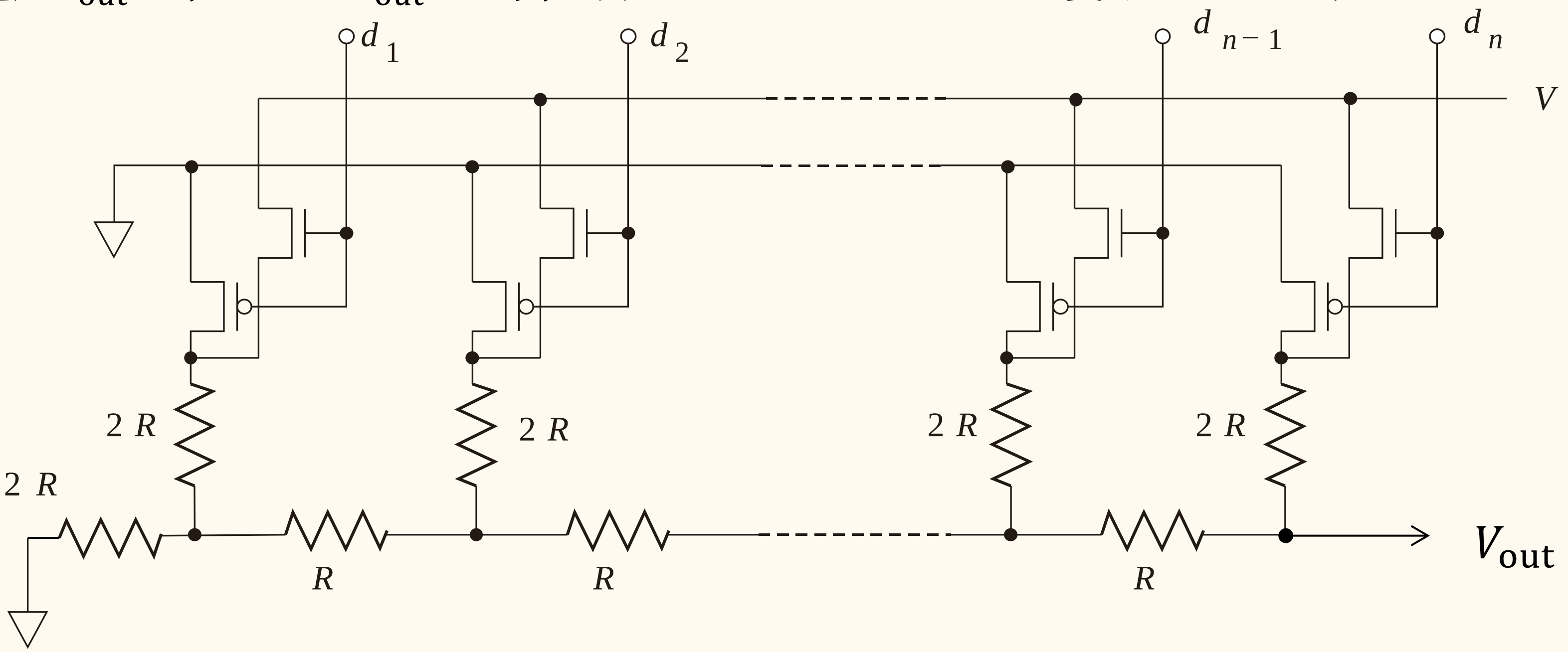
Overview

Electric Circuits: Treasury of Languages and Concepts



電子回路論レポート問題: 1. DA変換回路

(1) 次の図のような抵抗ラダー型DA変換回路を考える．講義で扱ったものと端の処理だけが違っている．この回路に2進数列$\{d_k\}(k = 1, .., n)$が入力されたとき，出力電圧V_{out}を求めよ． V_{out}は，高入力インピダンスアンプで受けるものとする．



(2) 手元に，$\{R_0/2^k\}$ $(k = 0, \ldots, n)$の抵抗値列を持つ抵抗，抵抗値R_f の抵抗，OPアンプ，電圧V_Sの標準電源，$n+1$個のnチャネルMOSスイッチ，同じく$n+1$個のpチャネルMOSスイッチがある．これらを使って，2進数列$\{d_k\}$が入力された時に

[illegible]

を出力するDA変換器回路を考えなさい．

電子回路レポート問題 ： 1. DA変換回路

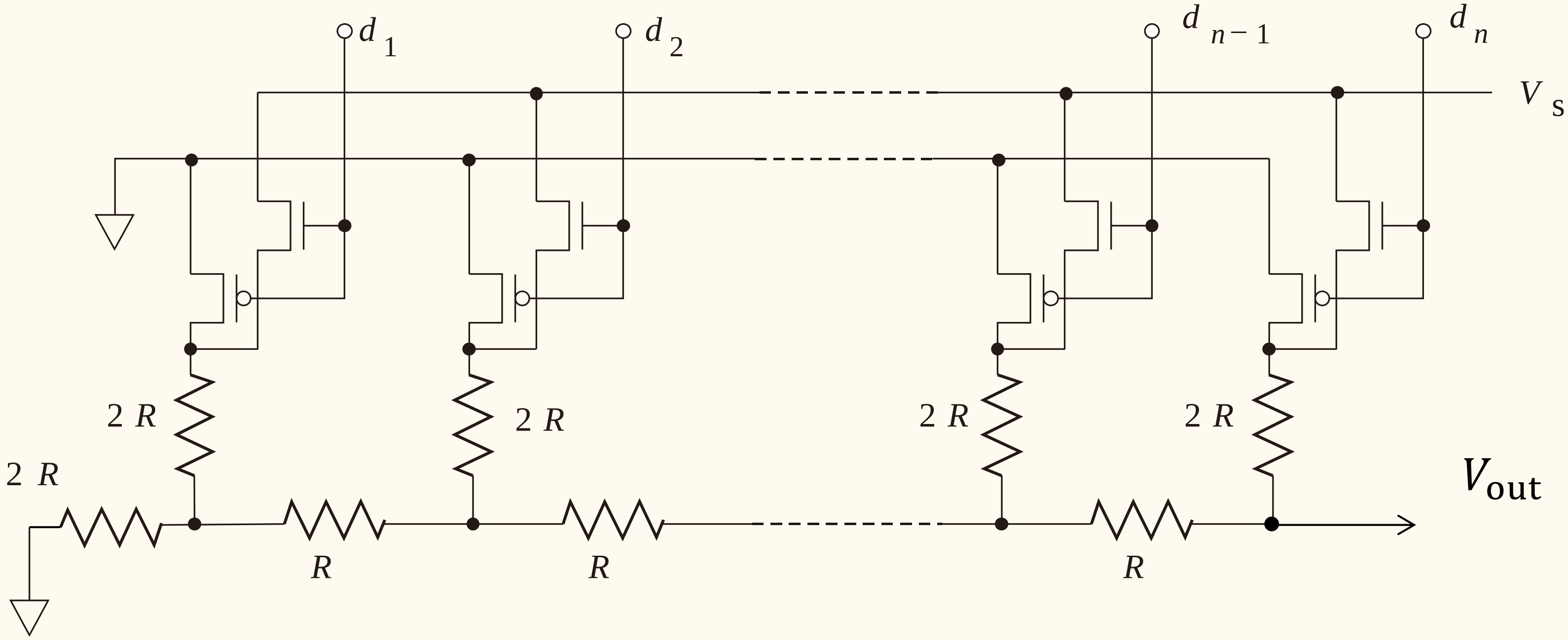
(3) (2) と同様，に手元に，$\{2^k C_0\}$ $(k = 0, \ldots, n)$の抵抗値列を持つキャパシタ，電圧V_Sの標準電源，$n+1$個のnチャネルMOSスイッチ，同じく$n+1$個のpチャネルMOSスイッチがある．これらを使って，2進数列$\{d_k\}$が入力された時に

$$V_{\mathrm{out}} = \frac{V_{\mathrm{S}}}{2^{n+1}-1}\sum_{k=0}^{n} d_k 2^k$$

という出力が得られるようなDA変換回路を考えなさい．ただし，出力は入力インピダンスが高くバイアス電流が無視できるような増幅器で受けることとする．

Problems for the final report: 1. DA conversion circuits

(1) Let us consider the following resistance ladder DA conversion circuit. The right end is a bit different from the one we treated in the lecture. Calculate the output voltage V_{out} for the input $\{d_k\}$ $(k = 1, \cdots, n)$.



(2) We have resistors with values $\{R_0/2^k\}$ $(k = 0, \cdots, n)$, and R_{f}, an OP amp., a standard voltage source of the voltage V_{S}, $n + 1$ n-channel MOS switches, $n + 1$ p-channel MOS switches. With these components, design a DA conversion circuit which has the output

$$V_{\mathrm{out}} = -V_{\mathrm{S}} \frac{R_{\mathrm{f}}}{R_0} \sum_{k=0}^{n} d_k 2^k$$

for the binary input $\{d_k\}$.

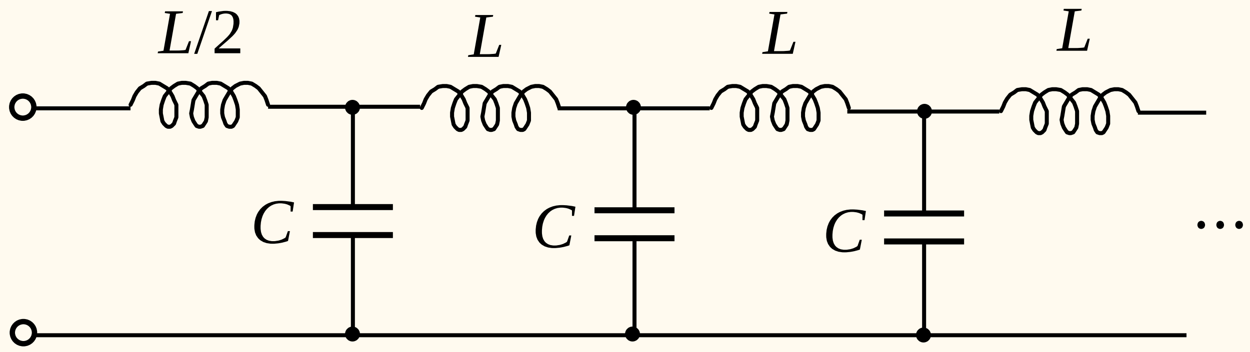
Problems for the final report: 1. DA conversion circuits

(3) We have capacitors with values $\{2^k C_0\}$ $(k = 0, \cdots, n)$, a standard voltage source of the voltage V_S, $n + 1$ n-channel MOS switches, $n + 1$ p-channel MOS switches. With these components, design a DA convertor circuit which has the output

$$V_{\text{out}} = \frac{V_{\text{S}}}{2^{n+1} - 1} \sum_{k=0}^{n} d_k 2^k$$

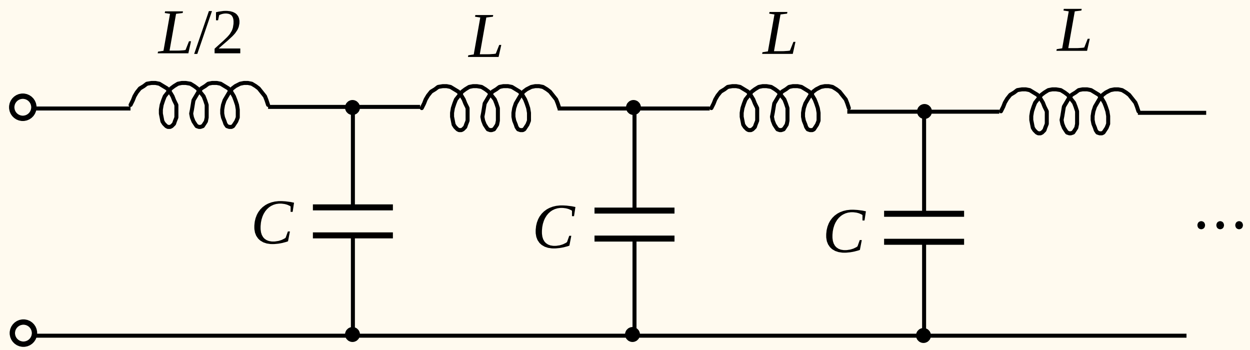
for the binary input $\{d_k\}$.

電子回路論レポート問題：2. 分布定数回路



上図のように，端のインダクタンス$L/2$を除いてLとCが無限に繰り返す回路がある．この回路の，周波数軸上での透過域と減衰域を求めよ．また，左の端子から見たインピーダンスの周波数特性(周波数ωに対するインピーダンス)を求めよ．

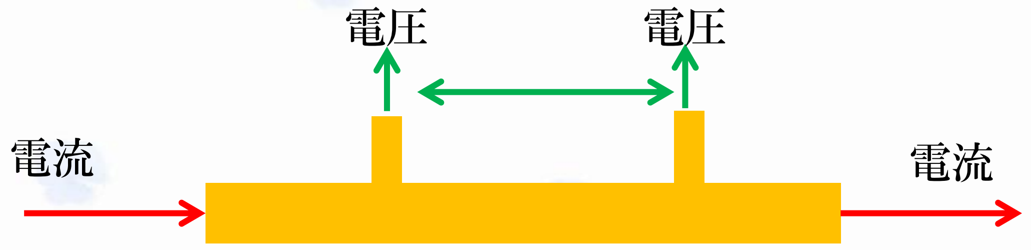
Problems for the final report: 2. Distributed constant circuit



Consider the above circuit with *L*, *C* infinite repetition to the right and the inductor *L*/2 at the left end. Obtain the transmission range and the attenuation range in the frequency domain. And what is the total impedance from the left end for the frequency ω.

電子回路論レポート問題：3. OPアンプ回路

次のような回路部品がある．
高精度OPアンプ ・・・　4個
定電圧ダイオード (逆方向に電圧を印加するとツェナートンネルにより一定電圧を発生する)　2.5V ・・・　1個
これらを使って，低温で試料の電気抵抗を測定するための回路を構成しなさい．
試料は下の図のように，電流端子，電圧端子が別に出た構造をしている．

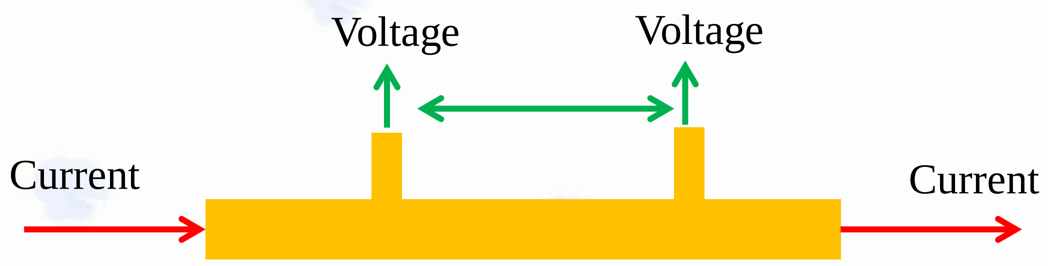


ただし，

(a) 抵抗，キャパシタ，インダクタの類の受動素子は適当に追加してよい．
(b) OPアンプの電源を供給するためのトラッキング電源は準備されているものとする．
(c) 測定抵抗の範囲は100Ω～10kΩで，接触抵抗を含めて50kΩ以内である．
(d) OPアンプのオフセット電圧，バイアス電流は無視できるとする．従ってオフセット調整回路を入れる必要はない．

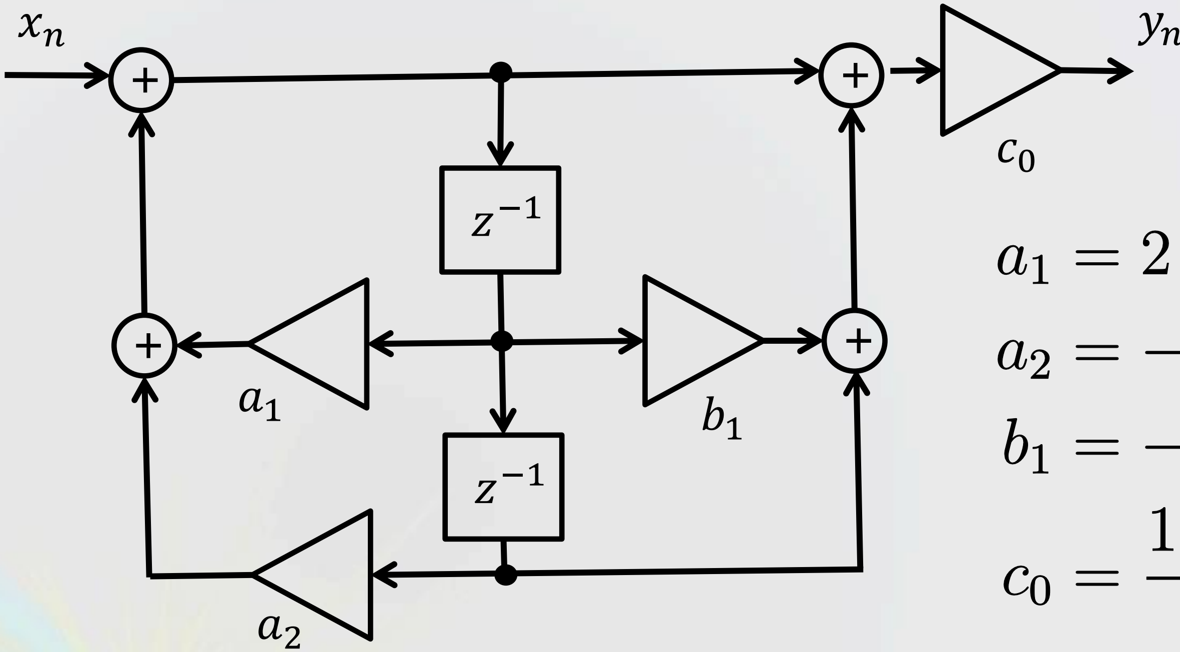
Problems for the final report: 3. OP amp. circuit

We have the following components:
4 high precision operational amplifiers,
1 high precision Zener tunnel diode with the constant voltage 2.5V (this diode provide precise 2.5V for the reverse bias).
With these components, design a circuit to measure the electric resistance of a sample at low temperatures. The shape of the sample is shown below:



(a) You can add any passive elements (resistors, capacitors, inductors).
(b) The power supply for the OP amps. is ready.
(c) The sample resistance range is from 100Ω to 10kΩ, lower than 50kΩ including the contact resistance.
(d) The offset voltages, the bias currents of the OP amps. can be ignored. No need for the offset cancellation circuit.

電子回路論レポート問題：4. ディジタル・フィルター



$$a_1 = 2\exp(-\pi g_0 \tau)\cos(2\pi f_0 \tau)$$

$$a_2 = -\exp(-2\pi g_0 \tau)$$

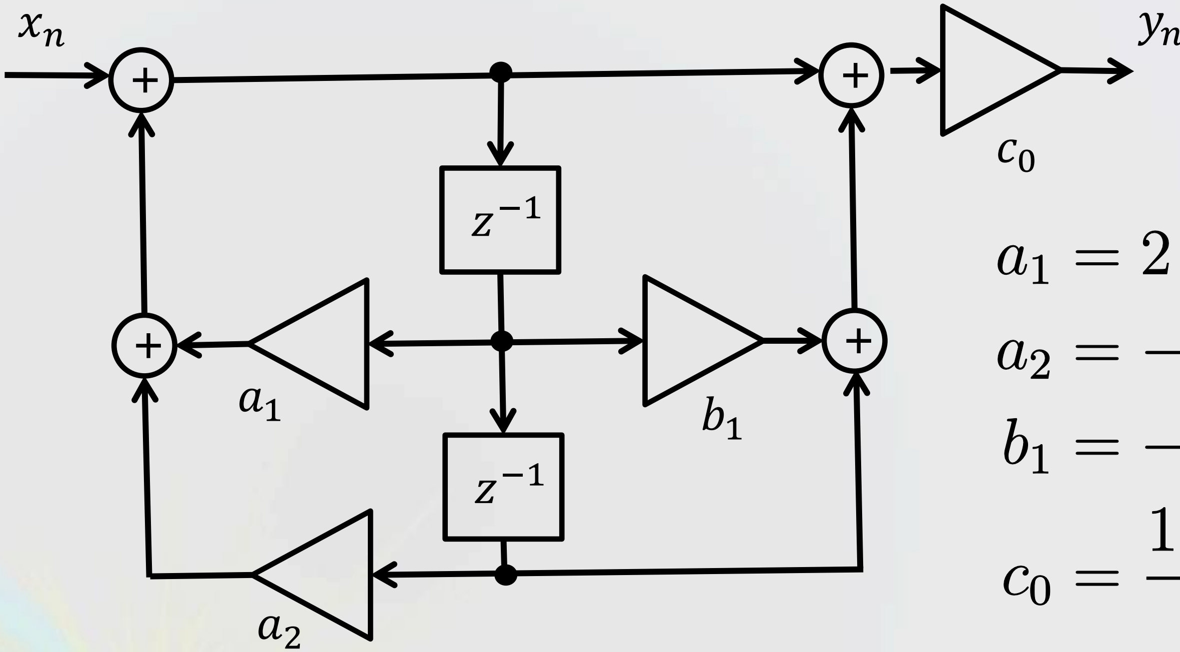
$$b_1 = -2\cos(2\pi f_0 \tau)$$

$$c_0 = \frac{1 - a_1 - a_2}{2 + b_1}$$

(1) 上のブロックダイアグラムから，出力y_n, y_{n-1}, y_{n-2}と入力x_n, x_{n-1}, x_{n-2}との関係式を示せ．

(2) 係数a_1， a_2， b_1， c_0 が右上のような関係を満たすとき，このフィルターはどのような周波数特性を示すか．ただし，$g_0 < f_0 < f_s/2$ (サンプリング周波数)を満たすとする．($20g_0 = 10f_0 = f_s$ としてグラフを描いてみよ．)

Problems for the final report: 4. Digital filter



$$a_1 = 2\exp(-\pi g_0 \tau)\cos(2\pi f_0 \tau)$$
$$a_2 = -\exp(-2\pi g_0 \tau)$$
$$b_1 = -2\cos(2\pi f_0 \tau)$$
$$c_0 = \frac{1 - a_1 - a_2}{2 + b_1}$$

(1) From the above diagram, write down the relation between the output y_n, y_{n-1}, y_{n-2} and the input x_n, x_{n-1}, x_{n-2}.

(2) When the coefficients a_1, a_2, b_1, c_0 satisfy the above relations, obtain the frequency characteristics of this filter. Draw a rough sketch of the graph for $20g_0 = 10f_0 = f_s$.

電子回路論レポート問題: 5 離散フーリエ変換

http://kats.issp.u-tokyo.ac.jp/kats/electroniccircuit/report/reportdata.txt

に，4096点のデータが入っている．各行に x $f(x)$ という形で納められており，セパレーターはTAB記号 (ASCII番号9番)である．

このデータは，ある特定の周波数で励起した出力を取ったもので，信号は特定の周波数成分で応答する．ただし，その振幅は完全に一定とは限らない．

(1) 離散フーリエ変換 (現実的には高速フーリエ変換，FFTをかけることになると思われる)を施し，パワースペクトルの主ピークから信号の周波数を特定せよ．

(2) 特定の窓 (幅 256点程度が適当)を使って部分的にフーリエ変換を施し，上で求めた周波数に最も近い周波数成分振幅を
求める．窓の位置を x が小さい位置から100点刻み程度に動かし，各点での振幅を窓位置に対してプロットせよ．

解答のみ記せばよい．プログラム等を示す必要はない．

Problems for the final report: 5 Discrete Fourier Transform

http://kats.issp.u-tokyo.ac.jp/kats/electroniccircuit/report/reportdata.txt

contains data of 4096 points. Each line has a point in the form $x \;\; f(x)$.
The separator between x and $f(x)$ is TAB (ASCII No.9).

The data are signal responding to an excitation with a particular frequency. Hence the signal has the same central frequency but the amplitude is not necessarily constant.

(1) Apply discrete Fourier transformation (DFT, practically fast Fourier transformation) to the signal and extract the central frequency from the main peak.

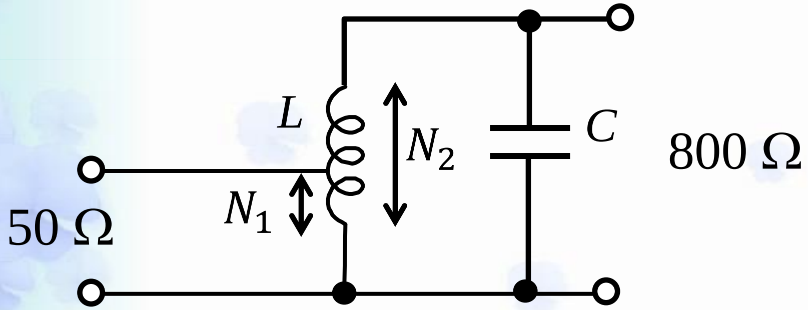
(2) Carry out DFT to a window with a shorter period (256 points is appropriate) and obtain the amplitude of the central frequency component. Shift the position of the window with a step size about 100 points. Plot the amplitude as a function of the window position.

You do not need to show your program codes for the analysis.

電子回路レポート問題：6 インピーダンス整合

FETを用いたアンプで信号ラインの特性インピーダンスとFETの入力インピーダンスとの間で整合を取りゲインを最大化すると，雑音も増幅され，雑音指数 (Noise figure, NF) が悪くなる．

そこで，50 Ω ラインの信号をFETから見たときのインピーダンスが800 Ω程度になるように変換する．入力は80 MHz～90 MHzの共鳴フィルターを構成する．



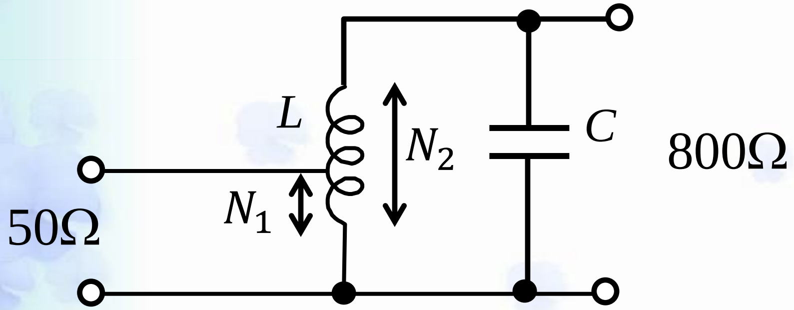
これを左図のようにコイルに中間タップを出すことで行う．全体のコイル巻き数N_2に対してタップの巻き数位置はN_1とする．入力中心周波数は85 MHz，共鳴幅(半値幅)は10 MHzとする．

CとL及びN_1のN_2に対する比を求めよ．(有効数字3桁)

ここでは，コイルのインダクタンスは巻き数の2乗に比例するとする．

Problems for the final report: 6 Impedance matching

In amplifiers with FETs, the noise matching condition, that optimizes the noise figure, usually deviates from the power matching one. The impedance conversion circuit shown below thus converts the transmission line characteristic impedance 50 Ω into 800 Ω. The input to FET also constitutes a resonance filter for 80 MHz to 90 MHz.



The coil in the left has a tap at the winding number N_1 in the total winding number N_2. The central frequency of input is 85 MHz, the resonance width (peak width at half height) is 10 MHz. Calculate C, L, and the ration of N_1 to N_2. (significant digits =3) Assume the inductances are proportional to squares of the winding numbers.